CHAPTER IV
PRICING SCHEMES FOR MULTISERVICE NETWORKS

4.1 Overview

The contributions in this chapter will address solutions to internet-charging schemes of multiservice in single and multiple links. Sain & Herpers (2003) attempted to formulate the network charging schemes into optimization models and solve it to obtain maximum profits by considering the price, total network capacity of services offered and QoS levels for each service offered. The attempt is to apply optimization techniques to solve these problems. These problems are considered as MINLP, which can be solved by using optimization tools. Transformation of the internet pricing problem in multiple services into optimization models is created and the solution is obtained. This solution will help us interpret the current issues involving pricing, network sharing and QoS levels.

Meanwhile, Byun & Chatterjee (2004) attempted to formulate internet pricing into optimization models and solve the problem not as an optimization problem but by using the simulation approach. The formulation is useful in helping in our next proposed model although their proposed model lacks information on how much QoS level can be adopted in each service, total network shared and number of users allowed for using the services.

4.2 A New Approach to Pricing Schemes in Single Link Multiservice Networks

The first contribution is the attempt to solve pricing schemes in multiservice networks to seek better solutions compared to Sain & Herpers (2003) that has been processed and analyzed by LINGO 11.0. The second contribution is to derive a new formulation involving the base price, quality premium and QoS level that does not exist in the formulations by Sain & Herpers (2003) and our third contribution in multiservice networks will be a newly improved formulation that will work on multiple bottleneck link networks.

4.2.1 Model Formulation of a New Approach to Pricing Schemes in Single Link Multiservice Networks

The idea was derived from Sain & Herpers (2003) and were analyzed by comparing it with the results from the new models. Same models, parameters and decision variables were adopted but with small differences on the integer constraints and on the approach used. LINGO 11.0 and the Branch and Bound solver was used to process our computations. Branch and Bound solver is chosen since it is a systematic method for implicitly enumerating all possible combinations of integer variables (LINGO, 2008).

Parameters and decision variables used in Sain and Herpers (2004) are as follows and the parameter values are presented in Table 3.

Parameters:

C : Total capacity

d_s : Capacity needed to provide full QoS unit of service s

m_s : minimum level of QoS needed for service s

n_s : maximum number of users for service s

p_s : user price sensitivity of service s

S : number of service

TABLE 3: Parameter Values Using the New Approach of Sain & Herper's (2003) Model

Service	1	2	3
d_s	60	750	330
m_s	3	45	15
n_s	0.8	0.8	0.5
p_s	10	10	10
S	3		
C	5000		

Decision variables:

a_s : Reserved share of the total capacity for service s

q_s : QoS level for service s

x_s : number of users of service s

The optimization model proposed by Sain & Herpers (2003) is to maximize the objective function (1) subject to Eq(2) to Eq(7).

4.2.2 Optimal Solution of a New Approach to Pricing Schemes in Single Link Multiservice Networks

Table 4 shows the results obtained from the model. The problem is considered as MINLP since at least one of the expressions (objective function or the constraints) is nonlinear and the subset of the variables should be integers whereas the rest is permitted to have continuous values. The solver of LINGO 11.0 attempts to find local optimum solutions. It means that for the case of MINLP, several solutions are available and are locally optimal, thus making the MINLP solution locally optimal. When finding this local optimum, the problems that will have better objective values or there may be no existing solutions sometimes arise. Several solutions obtained were locally optimal, whereby some solutions were better in one circumstance and the same exact solution was given for another circumstance.

From Table 4, solution 1 and 2 showed differences in some aspects such as in the QoS level, number of concurrent users, and profit per service. Three (3) services are offered both in solution 1 and 2. In solution 1, for profit maximization reasons, the service provider did not offer services 1 and 3. For service 2, a QoS level of 95% is optimal by allowing 7 concurrent users to apply the service with a profit of 299.25. For this service, 100% of the network was reserved and used. When adding up the profits it was found that only service 2 yielded a profit of 299.25 while using 4987.5 of the total capacity of 5000, which is 99.75%.

In solution 2, also for profit maximization reasons, the service provider did not offer service 1 and 3. In service 2, the QoS level of 83% is optimal by allowing 8 concurrent users to use the service with a profit of 298.8. This service also reserves and uses 100% of the network and yields 298.8 profit and the usage of 4980 is of the total capacity of 5000, which has a utilization degree of 99.6%.

According to Sain & Herpers (2003), the ratio for service 2 is 0.06, which is the highest price per capacity ration, so this service is offered first until the total network capacity is filled up. This implies that the QoS level is 100% and there are 7 users applying the service at a capacity of 4987.5. After offering this service, there exists an idle capacity of 12.5, which cannot utilize for other services. The complete comparison between Sain & Herpers (2003) results and our solution is presented in Table 4.

Observing the total capacity used, it is obtained that solution 1 is a better solution than solution 2 because it equals a utilization degree of 99.75% while in solution 2, the utilization degree is only 99.6%, which is the same result obtained by Sain & Herpers (2003). Thus, solution 1 gives a better result compared to Sain & Herper's (2003). GMU shows the amount of LINGO' generated from LINGO's available memory. Solution 1 is obtained by gaining GMU = 25K and ER = 1 sec with ESS= 4 and TSI=115. Meanwhile, for solution 2, the GMU was 24K, ER=0 sec, ESS=1 and TSI= 28.

TABLE 4: Comparison of Results between Sain and Herpers (2003) and This Study

	Sain and Herpers (2003)			**Solution 1**			**Solution 2**		
Service	1	2	3	1	2	3	1	2	3
Share of total network capacity (a_s)	0.1	0.9	0	0	1	0	0	1	0
QoS level (q_s)	0.8	1	0.5	0.9	0.95	0.75	0.8	0.83	0.5
No. of concurrent users(X_s)	10	6	0	0	7	0	0	8	0
Used capacity per service ($q_s*d_s*X_s$)	480	4500	0	0	4987.5	0	0	4980	0
Total capacity used $\sum q_s*d_s*X_s$	4980			4987.5			4980		
Profit per service $q_s*p_s*X_s$	24	270	0	0	299.25	0	0	298.8	0
Total profit $\sum q_s*p_s*X_s$	294			299.25			298.8		

By comparing the computational time taken to achieve the solution, a better time taken to finish the iterations were reached by solution 2 and since it only took 28 iterations, but solution 1 took 115 iterations as shown in Table 5.

TABLE 5: Solver Status of Sain and Herpers' Model by Using LINGO 11.0

Solver Status	Solution 1	Solution 2
Model Class	MINLP	MINLP
State	Local Optimal	Local Optimal
Objective	300	300
Infeasibility	0	0
Iterations	115	28
Extended Solver Status		
Solver Type	Branch and Bound	Branch and Bound
Best Objective	300	300
Objective Bound	300	300
Steps	4	1
Update Interval	2	2
Generated Memory Used(K)	25	24
Elapsed Time(S)	1	0

4.2.3 Analysis of a New Approach on Pricing Schemes in Single Link Multiservice Networks

Finally, what the ISPs desire is better profits with lesser number of iterations and only slightly different values of objective function (equals to 0.45).

However, since the provider's profit are considered, the new result is better than that in Sain & Herpers (2003) model. The new results showed better profit gain but had differences in the number of services offered. They offered two types of services with the total number of users, while the new approach only offered one kind of service.

4.3 A New Improved Model on Pricing Schemes in Single Link Multiservice Networks

Yang (2004), Byun & Chatterjee (2004) and Yang et al. (2004; 2003) formulated a pricing strategy for differentiated service networks. Sain and Herpers (2003) also tried to formulate a network charging scheme using the optimization model and solving it to obtain maximum profit by considering the price, total network capacity of services offered and QoS levels for each service offered. It is attempted to compare Sain & Herpers (2003) results with the proposed results. The new improved internet pricing models are adopted and modified from Yang (2004), Sain & Herpers (2003) and Byun & Chatterjee (2004). The models are created to show that better results were obtained with additional parameters, decision variables and constraints. Modification of their model were performed by also considering the base price and quality premium of service.

Optimization techniques were applied to solve the problem in this study. Furthermore, optimization problem as MINLP was considered to be solved by using optimization tools. The internet pricing problem will be transformed in multiservice networks into optimization models and attempt to solve it to get an optimal solution. This solution will help interpreting the current issues involving pricing, network sharing, base price, quality premium and also QoS levels. ISP's point of view will be the main concern to obtain maximum revenue by gaining prices for services available, capacity allocation for each service, determination of base price and quality premium for each service and QoS level for services offered.

Let α_i as the base price and β_i as the quality premium to be fixed or vary depending on what target the ISP would achieve. Additional parameter values are set, in addition to parameters set by Sain & Herpers (2003), which are presented in Table 6. The solution for proposed improved model of MINLP were tested. By applying each case, it is clear how profit attained. The analysis previously described by Sain & Herpers (2003) can also be adopted in the proposed model. The ISPs can also possibly achieve their target by considering each case.

4.3.1 Model Formulation of a Newly Improved Model on Pricing Schemes in Single Link Multiservice Networks

The model formulations of the new improved model for internet pricing schemes for multiservice networks in Single Link QoS networks were presented. The proposed model formulations are considering for four possible cases, set up with different sets of base price and quality premium.

TABLE 6: Additional Parameters for the New Modified Model of Internet Pricing Scheme in Single Link Multiservice Network

Service	i=1	i=2	i=3
α fixed	0.5	0.5	0.5
β fixed	0.4	0.4	0.4
l_i	0.05	0.02	0.01
b_i	0.8	0.5	0.3
c_i	0	0	0
g_i	0.5	0.7	0.6

New Improved Model of Internet Pricing in Single Link Multi Service Network When Setting up α and β to be fixed

The new improved model formulation proposed will be

$$\text{Max} \sum_{i=1}^{S} (\alpha + \beta I_i) p_i x_i \tag{45}$$

subject to

$$I_i d_i x_i \le a_i C, i = 1, 2, \ldots, S \tag{46}$$

$$\sum_{i=1}^{S} I_i d_i x_i \le C \tag{47}$$

$$\sum_{i=1}^{S} a_i = 1 \tag{48}$$

$$0 \le a_i \le 1, i = 1, 2, \ldots, S \tag{49}$$

$$m_i \le I_i \le 1, i = 1, 2, \ldots, S \tag{50}$$

$$0 \le x_i \le n_i, i = 1, 2, \ldots, S \tag{51}$$

$$\{x_i\} \text{ integer}, i = 1, 2, \ldots, S \tag{52}$$

Where Eq(46) to Eq(52) are adopted from Sain & Herpers (2003) and

Parameters when α and β are fixed

C	:	Total capacity
m_i	:	Minimum QoS level of service i
n_i	:	Number of users applying service i
p_i	:	User's price sensitivity
α	:	Base price for each service
β	:	Quality premium for each service
S	:	Service number
d_i	:	Capacity needed to provide full QoS unit of service s

Decision variables when α and β are fixed

a_i	:	Reserved total capacity share for service i
I_i	:	Quality index of the ith service class
x_i	:	Number of users applying service i

The following are descriptions of the model formulations. ISPs desire to obtain maximum revenue by setting up prices chargeable for a base price and quality premium and QoS levels to recover cost and to enable the users to choose services based on their preferences are stated in the objective function (45). Constraint (46) describes that the required capacity for service does not exceed the network capacity reserved. Constraint (47) explains that required capacity is not greater than total network capacity C. Constraint (48) guarantees that network capacity has different allocation for each service that lies between 0 and 1 (Constraint (49). Constraint (50) states that the QoS level should lie between the determined QoS level for each service. Constraint (51) tells us that users apply the service is nonnegative and is not more than the largest possible number of users determined by the service provider. Constraint (52) tells us that there are limitations to the number of users and it should be positive integers.

The Solution result is presented in Table 7, Service 1 has shown optimal QoS level of 0.8. One user allows applying the service and only 0.9% of the network is reserved for 48. Service 2 obtained optimal QoS level of 0.807, for six users using the service and 72.64% of network capacity is reserved in which of 3631.5 is used. For Service 3, a QoS level of 50% is obtained, giving the chance for 8 users apply the service and 26.4% network is reserved and it use 1320. ISP targets to be able to recover their cost of service and also can enable the users to select the services based on their budget and preferences. If summing up all services then a total revenue of 308.616 was obtained and usage capacity of 4999.5 or degree of utilization of 99.99 %. So there is only 0.5 idle capacity that cannot be utilized for other services.

TABLE 7: Solution of a New Improved Model on Pricing Scheme of Single Link Multiservice Network when α and β are Fixed

Service	i=1	i=2	i=3
Share of total network capacity(a_i)	0.0096	0.7264	0.264
QoS level(I_i)	0.8	0.807	0.5
No. of concurrent users(x_i)	1	6	8
Used capacity per service ($I_i*d_i*x_i$)	48	3631.5	1320
Total capacity used($\sum I_i*d_i*x_i$)	4999.5		
Profit per service(($\alpha+\beta*I_i$)$*p_i*x_i$)	2.46	222.156	84
Total profit ($\sum (\alpha+\beta*I_i)*p_i*x_i$)	308.616		

New Improved Model of Internet Pricing in Single Link Multi Service Network When Setting up α to be fixed and β varies

The new proposed model formulation will be

$$\text{Max} \sum_{i=1}^{S} (\alpha + \beta_i I_i) p_i x_i \tag{53}$$

subjected to constraints (46)-(52). Adding the new proposed constraints as follows.

$$\beta_i I_i \geq \beta_{i-1} I_{i-1}, i > 1 \tag{54}$$

$$l_i \leq \beta_i \leq b_i, i = 1, 2, \ldots S \tag{55}$$

where

Parameters when α fixed and β varies

- C : Total capacity
- m_i : Minimum QoS level of service i
- n_i : Number of users applying service i
- p_i : User's price sensitivity
- α : Base price for each service

S : Service number

l_i : Minimum quality premium for service i

b_i : Maximum quality premium for service i

Decision variables when α is fixed and β varies

a_i : Reserved total capacity share for service i

I_i : Quality index of the ith service class

x_i : Number of users applying service i

β_i : Quality premium for service i

The descriptions of the constraints are as follows. Objective function (53) shows that ISPs obtain maximum revenue by setting the price chargeable for a base price, quality premium and QoS level to recover cost, promote certain services (Sain and Herpers, 2003) and increase the number of users over all other services. Constraint (54) explains that quality premium has different levels for each service which is at least the same level or a lower level. Constraint (55) shows that the quality premium does not fall out of determined quality premium set up by ISPs.

For cases where α=0.5 (fixed) and β varies as shown in Table 8, the different solution was obtained. For the reason of revenue maximization, ISP does not offer service 3. Service 1, with QoS level of 80% is obtained with four active users applying the service and 3.84% network is reserved of where 192 is used. For service 2, a QoS level of 80% is obtained with eight active users using the service and 96.16% network is reserved with 8 customers apply the service. By setting up α to be fixed and β varies, ISP can target to recover cost of service and promote certain services which are service 1 and service 2 by no promoting for service 3. By summing up all revenues, the revenue of 297.6 and utilization degree of 99.84% were obtained.

TABLE 8: Solution of a New Improved Model on Pricing Scheme of Single Link Multiservice Network when α is Fixed and β Varies

Service	i=1	i=2	i=3
Premium quality (β_i)	0.375	0.375	0.3
Share of total network capacity(a_i)	0.0384	0.9616	0
QoS level(I_i)	0.8	0.8	1
No. of concurrent users(x_i)	4	8	0
Used capacity per service ($I_i*d_i*x_i$)	192	4800	0
Total capacity used($\sum I_i*d_i*x_i$)		4992	
Profit per service ($(\alpha+\beta_i*I_i)*p_i*x_i$)	9.6	288	0
Total profit ($\sum (\alpha+\beta_i*I_i)*p_i*x_i$)		297.6	

New Improved Model of Internet Pricing in Single Link Multi Service Network When Setting up α and β vary

The new proposed model formulation when α and β vary will be

$$\text{Max} \sum_{i=1}^{S} (\alpha_i + \beta_i I_i) p_i x_i \tag{56}$$

subject to constraints (46)-(52) and (54)-(55). By adding the new proposed constraints then we have as follows.

$$\alpha_i + \beta_i I_i \geq \alpha_{i-1} + \beta_{i-1} I_{i-1}, i > 1 \tag{57}$$

$$c_i \leq \alpha_i \leq g_i, i = 1, 2, \ldots, S \tag{58}$$

where

Parameters when α and β vary

- C : Total capacity
- m_i : Minimum QoS level of service i
- n_i : Number of users applying service i
- p_i : User's price sensitivity
- S : Service number
- l_i : Minimum quality premium value for service i
- b_i : Maximum quality premium value for service i
- c_i : Minimum base price value for service i
- g_i : Maximum base price value for service i

Decision variables when α and β vary

- a_i : Reserved total capacity share for service i
- I_i : Quality index of the ith service class
- x_i : Number of users applying service i
- α_i : Base price for service i
- β_i : Quality premium for service i

The descriptions of the constraints are as follows. Objective function (56) explains that ISPs want to obtain maximum revenue by setting up the price chargeable for a base price and quality premium to compete in the market competition if there is a chance and in the meantime, ISPs can promote certain services for a number of users over all other services. Constraint (57) guarantees that the base price and quality premium has at least the same level or lower level value for each service and that the base price should lie between the prescribed base price set up by ISP (Constraint 58).

In Table 9, The ISP targets on getting the market under a stiff competition and try to promote certain services which are service 1 and 2 to gain revenue maximization. This is the reason why ISP has chosen to differentiate the value of α and β. For revenue reason, ISP does not offer service 3. In service 1, a QoS level of 80% is offered with four active users using the service and 3.84% network is reserved in which 192 is used. Service 2 obtain a QoS level of 80% with eight active users using the service and 96.16 % of network is reserved with 4800 is used. Total of revenue of 334.8 is obtained with utilization degree of 99.84%. So, the idle capacity left enough for 8 only but not to be used by other services.

TABLE 9: Solution of a New Improved Model on Pricing Scheme of Single Link Multiservice Network when α and β Vary

Service	***i*=1**	***i*=2**	***i*=3**
Base price(α_i)	0.26	0.6301951	0.6
Premium quality (β_i)	0.8	0.3372562	0.3
Share of total network capacity(a_i)	0.0384	0.9616	0
QoS level(I_i)	0.8	0.8	1
No. of concurrent users(x_i)	4	8	0
Used capacity per service ($I_i*d_i*x_i$)	192	4800	0
Total capacity used($\sum I_i*d_i*x_i$)	4992		
Profit per service (($\alpha_i+\beta_i*I_i$)$*p_i*x_i$)	10.8	[illegible]	0
Total profit ($\sum$ ($\alpha_i+\beta_i*I_i$)$*p_i*x_i$	334.8		

New Improved Model of Internet Pricing in Single Link Multi Service Network When Setting up α varies and β is fixed

The new proposed model formulation when α varies and β is fixed will be

$$\text{Max} \sum_{i=1}^{S} (\alpha_i + \beta I_i) p_i x_i \tag{59}$$

subject to constraints (46)-(52), (57)-(58).

where

Parameters when α varies and β is fixed

- C : Total capacity
- m_i : Minimum QoS level of service i
- n_i : Number of users applying for service i
- p_i : User's price sensitivity
- S : Service number
- β : Quality premium for each service
- c_i : Minimum base price value for service i
- g_i : Maximum base price value for service i

Decision variables when α vary and β is fixed

- a_i : Reserved total capacity share for service i
- I_i : Quality index of the ith service class

x_i : Number of users applying service i

α_i : Base price for service i

The descriptions of the constraints are as follows. Objective function (59) states that ISPs seek to maximize the price chargeable for a base price and quality premium, QoS level to have market competition if there is a chance and users are able to choose the service according to their budget and preference and for the number of users over all other services.

The last case as shown in Table 10 is when α varies, β is set to 0.4 (fixed) ISP can gain market competition when there is chance meanwhile it can allow users to choose the service that suitable with their needs and budgets. For revenue reason, ISP does not offer service 3. Service 1 obtains a QoS levels of 80%, with 4 users apply the service, 3.84 % networks are reserved and 192 is used. In service 2, QoS level of 80.13% is offered and users of 4 is allowing to use the services, with 8 users use the service and 96.16% networks is reserved and 4808 is used. Total revenue is 77.232 which use of 4999.99 ≈5000 of total capacity. It means that there is almost no idle capacity to waste.

TABLE 10: Solution of a New Improved Model on Pricing Scheme of Single Link Multiservice Network when α Varies and β is Fixed

Service	i=1	i=2	i=3
Base price(α_i)	0.5	0.7	0.6
Share of total network capacity(d_i)	0.0384	0.9616	0
QoS level(I_i)	0.8	0.8013	0.9013
No. of concurrent users(x_i)	4	8	0
Used capacity per service (I_i*d_i*x_i)	192	4807.999	0
Total capacity used($\sum I_i$*d_i*x_i)	4999.999		
Profit per service ((α+β_i*I_i)*p_i*x_i)	7.92	367.391	0
Total profit ($\sum$ (α+β_i*I_i)*p_i*x_i)	377.232		

Table 11 basically summarizes the solver status of four cases using LINGO 11 (2011). Some cases such as the case when α, β are fixed, it took a long time to finish the iterations but it managed to obtain the local optimal for the solutions. Some cases also showed a very small infeasibility, which can be called zero infeasibilities.

TABLE 11: Solver Status of the New Improved Model on Pricing Schemes for Single Link Multiservice Networks

	α, β fixed	**α fixed β vary**	**α, β vary**	**α vary β fixed**
Solver Status				
Model Class	MINLP	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal	Local optimal
Objective	308.628	297.6	334.8	377.23
Infeasibility	7×10^{-15}	2×10^{-14}	0	0
Iterations	528	150	128	147
Extended Solver status				
Solver Type	Branch and Bound	Branch and Bound	Branch and Bound	Branch and Bound
Best Objective	308.628	297.6	334.8	377.23
Objective bound	308.628	297.6	334.8	377.23
Steps	12	5	5	4
Active	0	1	0	3
Update interval	2	2	2	2
GMU(K)	25	27	28	26
ER(sec)	1	0	0	0

4.3.2 Analysis of The New Improved Model on Pricing Schemes of Single Link Multiservice Network

To sum it up, better results were obtained in all cases compared to Sain and Herpers (2003) as shown in Table 12. The advantage of the new model is that an ISP can set its base price and quality premium based on the ISPs preferences. For some cases related to revenue maximization not all the services were offered but only a few. It means that some resources can be saved.

TABLE 12: Comparison between the New Improved Model in Single Link Multiservice Networks and Sain & Herper's (2003) Results

Comparison	Sain & Herpers (2003) Model	Our model			
		α, β fixed	α fixed β vary	α, β vary	α vary β fixed
Total capacity used($\sum I_i{*}d_i{*}x_i$)	4980	4999.5	4992	4992	4992
Total profit	294	308.616	297.6	297.6	334.8

4.4 An Improved Model for Internet Pricing Schemes for Multiple Link Multiservice Networks

The next contribution will be a new modified model for solving internet charging schemes of multiservice networks in multiple bottleneck link networks. Generalization of our model into multiple bottleneck links we performed. Again, the proposed problem was formulated as MINLP, which can be solved by the nonlinear programming method, to obtain exact solutions.

The proposed model is generalized by modifying internet pricing based on Sain & Herpers (2003). By formulating the model into MINLP, the model can be solved using the Branch and Bound Solver by LINGO 11.0. Consider cases of α, the base price or β, the quality premium, to be fixed or vary depending on what targets ISPs would achieve. The objective of the ISPs is to obtain maximum revenue subject to constraints based on system available resources.

4.4.1 Model Formulation of an Improved Model for Internet Pricing Schemes of Multiple Link Multiservice Networks

The model formulations of the new improved model for internet pricing schemes for multiservice networks in multi link networks were presented. The proposed model formulations are considering for four possible cases, set up with different sets of base price and quality premium.

New Improved Model of Internet Pricing in Multi Link Multi Service Network When Setting up α and β to be fixed

The new proposed improved model formulation will be as follows.

$$\text{Max} \sum_{l=1}^{L} \sum_{i=1}^{S} (\alpha + \beta I_i) p_{il} x_{il} \tag{60}$$

such that

$$I_i d_{il} x_{il} \le a_{il} C_l, i = 1, 2, \ldots S, l = 1, \ldots, L \tag{61}$$

$$\sum_{l=1}^{L} \sum_{i=1}^{S} I_i d_{il} x_{il} \le C_l, i = 1, 2, \ldots S, l = 1, \ldots, L \tag{62}$$

$$\sum_{l=1}^{L} \sum_{i=1}^{S} a_{il} = 1 \tag{63}$$

$$0 \le a_{il} \le 1, i = 1, 2, \ldots S, l = 1, \ldots, L \tag{64}$$

$$m_i \le I_i \le 1, i = 1, 2, \ldots S \tag{65}$$

$$0 \le x_{il} \le n_i, i = 1, 2, \ldots S, l = 1, \ldots, L \tag{66}$$

With m_i and n_i are prescribed positive integer numbers.

$$\{x_{ij}\} \text{ integer} \tag{67}$$

where

Parameters when α and β are fixed

C_l : Total capacity in link l

m_i : Minimum QoS level of service i

n_i : Number of users applying service i

p_{il} : User's price sensitivity in link l

α : Base price for each service

β : Quality premium for each service

S : Service number

d_s : Capacity needed to provide full QoS unit of service s

Parameter values for α and β are fixed case are presented in Table 13.

TABLE 13: Parameter Values of Internet Pricing in Multi Link Multi Service Network When Setting up α and β to be Fixed

$l=1$			
Service	**1**	**2**	**3**
C	6000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	3	45	15
α	0.5		
β	0.01		
S	3		
d_s	60	750	330
$l=2$			
Service	**1**	**2**	**3**
C	4000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	6	56	24
α	0.5		
β	0.01		
S	3		
d_s	60	750	330

Decision variables when α and β are fixed

- a_{il} : Reserved total capacity share for service i in link l
- I_i : Quality index of the ith service class
- x_{il} : Number of users applying service i in link l

Following are the explanation of the model formulations. Hence, ISPs want to obtain maximum revenue by setting up prices chargeable for a base price, quality premium and QoS level in order to recover cost and to enable users to choose services based on their preferences as stated in Objective Function (60). Constraint (61) shows that the required capacity of service cannot exceed the network capacity reserved for services with respective to QoS levels since all services cannot exceed the total network capacity C in each link l. Constraint (62) explains that the required capacity cannot be greater than the network capacity C in link l. Constraint (63) guarantees that network capacity has different location for each service that lies between 0 and 1(constraint (64). Constraint (65) shows that users applying the service are non-negative and cannot be greater than the highest possible users determined by service provider (constraint (66)). Constraint (67) states that the number of users should be in positive integers.

New Improved Model of Internet Pricing in Multi Link Multi Service Network When Setting up α to be fixed and β varies

The new improved mathematical formulation of internet pricing scheme in multi link multi service network when α is fixed and β vary is as follows.

$$\text{Max} \sum_{l=1}^{L} \sum_{i=1}^{S} (\alpha + \beta_i I_i) p_{il} x_{il} \tag{68}$$

subject to constraints (61)-(67) and additional constraints as follows.

$$\beta_i I_i \geq \beta_{i-1} I_{i-1}, i > 1 \tag{69}$$

$$k \leq \beta_i \leq q, [k,q] \in [0,1] \tag{70}$$

where

Parameters when α is fixed and β varies

- C_l : Total capacity in link l
- m_i : Minimum QoS level of service i
- n_i : Number of users applying service i
- p_{il} : User's price sensitivity in lik l
- α : Base price for each service
- S : Service number
- k_i : Minimum quality premium for service i
- q_i : Maximum quality premium for service i

Parameter values for α fixed and β vary case are presented in Table 14.

TABLE 14: Parameter Values of Internet Pricing in Multi Link Multi Service Network When Setting up α to be Fixed and β Varies

$l=1$			
Service	**1**	**2**	**3**
C_l	6000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	3	45	15
α	0.5		
S	3		
d_i	60	750	330
k_i	0.05	0.02	0.01
q_i	0.8	0.5	0.3
$l=2$			
Service	**1**	**2**	**3**
C_l	4000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	6	56	34
α	0.5		
S	3		
d_i	60	750	330
k_i	0.05	0.02	0.01
q_i	0.8	0.5	0.3

Decision variables when α is fixed and β vary

a_{il} : Reserved total capacity share for service i in link l

I_i : Quality index of the ith service class

x_{il} : Number of users applying service i in link l

β_i : Quality premium for service i

Following are the descriptions of the constraints. Objective function (68) explains that ISPs want to obtain maximum revenue by setting up the prices chargeable for a base price, quality premium and QoS level to recover cost and to enable users to choose services based on their preferences. Constraint (69) explains that quality premium has different levels for each service, which is at least at the same or lower level. Constraint (70) states that value of quality premium lies between two prescribed values.

New Improved Model of Internet Pricing in Multi Link Multi Service Network When Setting up α and β vary

The proposed mathematical formulation will be as follows.

$$\text{Max} \sum_{l=1}^{L} \sum_{i=1}^{S} (\alpha_i + \beta_i I_i) p_{il} x_{il} \tag{71}$$

subject to constraints (61)-(67), constraints (69)-(70) and additional constraints

$$\alpha_i + \beta_i I_i \geq \alpha_{i-1} \beta_{i-1} I_{i-1}, i > 1 \tag{72}$$

$$y \leq \alpha_i \leq z, [y, z] \in [0,1] \tag{73}$$

where

Parameters when α and β vary

- C_l : Total capacity in link l
- m_i : Minimum QoS level of service i
- n_i : Number of users applying service i
- p_{il} : User's price sensitivity in link l
- S : Service number
- k_i : Minimum quality premium for service i
- q_i : Maximum quality premium for service i
- y_i : Minimum base price value for service i
- z_i : Maximum base price value for service i

Parameter values for α and β vary case are presented in Table 15.

TABLE 15: Parameter Values of Internet Pricing in Multi Link Multi Service Network When Setting up α and β Vary

$l = 1$			
Service	**1**	**2**	**3**
C_l	6000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	3	45	15
S	3		
d_i	60	750	330
k_i	0.05	0.02	0.01
q_i	0.8	0.5	0.3
y_i	0	0	0
z_i	0.5	0.7	0.6
$l = 2$			
Service	**1**	**2**	**3**
C_l	4000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	6	56	24
S	3		
d_i	60	750	330
k_i	0.05	0.02	0.01
q_i	0.8	0.5	0.3
y_i	0	0	0
z_i	0.5	0.7	0.6

Decision variables when α and β vary

a_{il} : Reserved total capacity share for service i in link l

I_i : Quality index of the ith service class

x_{il} : Number of users applying service i in link l

α_i : Base price for service i

β_i : Quality premium for service i

Following are the descriptions of the constraints. ISPs want to obtain maximum revenue by setting up the prices chargeable for a base price, quality premium and QoS level to recover cost and to enable users to choose services based on their preferences like that stated in objective function (71). Constraint (72) explains that the summation of base cost and quality premium has different levels for each service, which is at least at the same or lower level. Constraint (73) shows that the base price should lie between the prescribed base price set up by ISPs.

New Improved Model of Internet Pricing in Multi Link Multi Service Network When Setting up α vary and β is fixed

The improved model formulation when α vary and β is fixed is as follows.

$$\text{Max} \sum_{l=1}^{L} \sum_{i=1}^{S} (\alpha_i + \beta I_i) p_{il} x_{il} \qquad (74)$$

subject to constraints (61)-(67) and constraints (72)-(73).

where

Parameters when α varies and β is fixed

C	:	Total capacity
d_i	:	Capacity needed to provide QoS unit of service i
m_i	:	Minimum QoS level of service i
n_s	:	Number of users applying service i
p_{il}	:	User's price sensitivity
S	:	Service number
β	:	Quality premium for each service
y_i	:	Minimum base price value for service i
z_i	:	Maximum base price value for service i

Parameter values for α vary and β fixed case are presented in Table 16.

TABLE 16: Parameter Values of Internet Pricing in Multi Link Multi Service Network When Setting up α Vary and β is Fixed

$l=1$			
Service	**1**	**2**	**3**
C_l	6000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	3	45	15
S	3		
d_i	60	750	330
β	0.01		
y_i	0	0	0
z_i	0.5	0.7	0.6
$l=2$			
Service	**1**	**2**	**3**
C_l	4000		
m_i	0.8	0.8	0.5
n_i	10	10	10
p_{il}	6	56	24
S	3		
d_i	60	750	330
β	0.01		
y_i	0	0	0
z_i	0.5	0.7	0.6

Decision variables when α vary and β is fixed

- a_{il} : Reserved total capacity share for service i in link l
- I_i : Quality index of the ith service class
- x_{il} : Number of users applying service i in link l
- α_i : Base price for service i

The description of the objective function is as follows. ISPs want to obtain maximum revenue by setting up the prices chargeable for a base price, quality premium and QoS level to recover cost and to enable users to choose services based on their preferences as stated in objective function (74).

4.4.2 Optimal Solution of a Newly Improved Model on Pricing Scheme of Multiple Link Multiservice Network

Table 24 describes the status of the model formulation in LINGO. In all cases the state of formulations are local optimal. The maximum profit is obtained when ISPs set up the value of α as decision variables and β as a fixed value. GMU shows how much memory is used for generating a model. The highest GMU is in the case where α and β are decision variables, which need an optimal solution from the solver 462 iterations.

The total time taken thus far to generate and solve the model for all cases is 0 seconds. Steps show that the amount of branches in the branch and bound tree. In the case where α is the decision variable and β is the fixed value, the amount of branches reach a maximum number of 16.

TABLE 17: Solver Status of Model Formulation on Pricing Schemes of Multiple Link Multiservice Networks using LINGO 11.0

	α, β fixed	**α fixed β vary**	**α, β vary**	**α vary β fixed**
Solver Status				
Model Class	MINLP	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal	Local optimal
Objective	477.4	667.2	750.6	563.2
Infeasibility	0.3×10^{-3}	0.1×10^{-15}	0	0.3×10^{-3}
Iterations	259	375	462	368
Extended Solver status				
Solver Type	Branch and Bound	Branch and Bound	Branch and Bound	Branch and Bound
Best Objective	477.4	667.2	750.6	563.2
Objective bound	477.4	667.2	750.6	563.2
Steps	9	8	11	11
Update interval	2	2	2	2
Active	0	0	0	0
GMU(K)	30	31	33	32
ER(sec)	0	0	0	0

Table 18 and 19 depict the optimal solutions for the formulation in each link. Various results were obtained in each case. In Table 18, the ISPs intend to fix the value of α and β to recover cost and enable the users to choose the services leading to the capability of service 3 to promote the highest number of users in link 1 and also service 1 and 3 in link 2. Highest QoS level achieved by service 1 and total network shared is 83%.

TABLE 18: Solution of Internet Pricing Schemes of Multiservice Networks in Link 1

Variable	α, β fixed	α fixed β vary	α, β vary	α vary β fixed
α_1	-	-	0.26	0.5
α_2	-	-	0.53	0.59
α_3	-	-	0.6	0.6
β_1	-	0.375	0.8	-
β_2	-	0.375	0.45	-
β_3	-	0.3	0.3	-
I_1	0.83	0.8	0.8	0.83
I_2	0.8	0.8	0.8	0.8
I_3	0.5	1	1	0.5
a_{11}	0.025	0	0	0.025
a_{21}	0.7	1	1	0.7
a_{31}	0.275	0	0	0.275
x_{11}	3	0	0	3
x_{21}	7	10	10	7
x_{31}	10	0	0	10

In Table 18 and Table 19, service 1 in link 1 and 2 achieved a QoS level of 83% was offered whereby 3 users applied the service in link 1 and 10 users applied the service in link 2 with 2.5% network is reserved in link 1 and 12% network is reserved in link 2.

In service 2, a QoS level of 80% was offered with 7 users applying for the service with 70% of the network reserved in link 1 and 3 users apply the service with 46% network is reserved in link 2. In service 3, a QoS level of 50% was offered in the network with 10 users choosing the service in link 1 and 10 users in link 2 with 27.5% and 40% network reserved in link 1 and link 2 respectively.

TABLE 19: Solution of Pricing Schemes of Multiservice Networks in Link 2

Variable	α, β fixed	α fixed β vary	α, β vary	α vary β fixed
α_1	-	-	0.26	0.5
α_2	-	-	0.53	0.59
α_3	-	-	0.6	0.6
β_1	-	0.375	0.8	-
β_2	-	0.375	0.45	-
β_3	-	0.3	0.3	-
I_1	0.83	0.8	0.8	0.83
I_2	0.8	0.8	0.8	0.8
I_3	0.5	1	1	0.5
a_{12}	0.12	0.1	0.1	0.12
a_{22}	0.46	0.9	0.9	0.46
a_{32}	0.4	0	0	0.4
x_{12}	10	8	8	10
x_{22}	3	6	6	3
x_{32}	10	0	0	10

4.4.3 Analysis of a Newly Improved Model on Pricing Schemes of Multiple Link Multiservice Networks

Based on the above results, better solutions were obtained in the modified formulation by obtaining maximum revenue by assigning α and β variables to allow ISPs to face market competition and promote certain services. In these solutions, ISPs can promote service 2 in link 1 since ISPs obtained the highest number of users applying the service in the network. ISPs can also promote service 1 and 2 in link 2 since the first and second highest number of users applying the service are in that link.

In fact, all modified model formulations can be adopted by ISPs depending on the ISP's objective whether to recover the cost (with the base priced fixed) but users can choose the class of their preferences that fits their budget limitations (with quality premium is fixed) or to vary the base price to be able to compete in market competition and encourage the users to choose the class of their preferences and budget (with quality premium to be fixed).

4.5 Summary

In this chapter, several models were studied to show the connection between pricing, QoS level, number of users applying the services, capacity allocation and total network share. But due to assumptions, the results is assumed as a theoretical point of view as a static model that is unlikely difficult to adapt to dynamic situation.

The paper by Sain & Herpers (2003) can be upgraded by adopting our new approach using other tools. Slightly increasing profits were obtained for several solutions proposed. In addition, Human resource allocations were saved by only applying few users to apply the service and can save the operating cost by only promote one service rather than two services. Our solutions show better profits with less idle time and number of users applied the services.

It is shown that by considering new parameters, more decision variables and constraints, better revenue maximization were obtained. The cases shown above basically are ISP strategy to vary its preference to achieve their goals. ISP is able to adopt the cases to suit their goals. But again, as stated by Byun & Chatterjee (2004) and Sain & Herpers (2003) since it is more theoretical point of view and assumptions, limitations of our results is to only static result in data changes, and cost preference is just based on our discrete data. Further research should address more generalization of the model to also consider numerous services offered or generalization of more services.