

Fuzzy Parameterized Fuzzy Soft Expert Set

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Abstract. In 2011 Alkhazaleh and Salleh defined the concept of soft expert sets where the user can know the opinion of all the experts in one model and give an application of this concept in decision making problems. Also, they introduced the concept of the fuzzy soft expert set as a combination between the soft expert set and the fuzzy set. In this paper we introduce the concept of fuzzy parameterized fuzzy soft expert set as a generalization of fuzzy soft expert set by giving a membership value for each parameter in a set of parameters. We shall, also, define its basic operations, complement, union intersection, "AND" and "OR" and study their properties. Finally, we shall give an application of this concept in decision making problems.

Keywords: soft set, soft expert set, fuzzy soft expert set, fuzzy parameterized fuzzy soft expert set

1. INTRODUCTION

Most of the problems in engineering, medical science, economics, and environment etc, have various uncertainties. Molodtsov[1] initiated the concept of the soft set theory as a mathematical tool for dealing with such uncertainties. After Molodtsov's work, some operations and applications of soft sets were studied by Chen et al.[2], Maji et al.[3] and Maji et al.[4]. Also Maji et al.[5] have introduced the concept of fuzzy soft set, as a more general concept, and as a combination of fuzzy set and soft set where they studied its properties. Also Roy and Maji[6] used this theory to solve some decision making problems. Further, In 2010 Çağman et al.[12] introduced the concept of fuzzy parameterized fuzzy soft set (*fpfs*) and their operations. as well as *fpfs*-aggregation operator to form *fpfs*-decision making method which allows constructing more efficient decision processes. Alkhazaleh and Salleh[10] introduced the concept of soft expert set and fuzzy soft expert set, where the user can know the opinion of all experts in one model without any operations.

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in this paper we shall introduce the concept of fuzzy parameterized fuzzy soft expert set, which is more effective and useful as we shall see. It is a combination of fuzzy soft expert set and fuzzy parameterized fuzzy soft expert. We shall also define its basic operations, namely complement, union, intersection, and study their properties. Finally, we shall give an application of this concept in decision making problems.

2. PRELIMINARIES

In this section, we introduce some basic notions in soft set theory. Molodtsov [1] defined soft set over U in the following way. Let U be a universe set and E set of parameters, $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.1. [1] *consider the mapping*

$$F : A \rightarrow P(U).$$

Then any A pair (F, A) is called a soft set over U . In other words, a soft set over U is a parameterized family of subsets of the universe set U . So that; For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) .

Definition 2.2. [5] *Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U . Let, $A \subseteq E$, F be a mapping*

$$F : A \rightarrow I^U.$$

A pair (F, E) is called a fuzzy soft set over U .

Definition 2.3. [12] *Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function*

$$\mu_X : E \rightarrow [0, 1],$$

and let γ_X be a fuzzy set over U for all $x \in E$. Then an fpfs-set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping $\gamma_X : E \rightarrow F(U)$ such that

$$\gamma_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, γ_X is called a fuzzy approximate function of the fpfs-set Γ_X , and the value $\gamma_X(x)$ is a set called x -element of the fpfs-set for all $x \in E$. Thus, an fpfs-set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0, 1]\}.$$

Let U be a universe set, E a set of parameters, X a set of experts (agents), and O a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.4. [10] A pair (F, A) is called a soft expert set over U , where F is a mapping given by

$$F : A \rightarrow P(U)$$

and $P(U)$ denotes the power set of U .

Let U be a universe set, E a set of parameters, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.5. [11] A pair (F, A) is called a fuzzy soft expert set over U , where F is a mapping given by

$$F : A \rightarrow I^U$$

and I^U denotes all fuzzy subsets of U .

Definition 2.6. [11] For two fuzzy soft expert sets (F, A) and (G, B) over U , (F, A) is called a fuzzy soft expert subset of (G, B) if

1. $A \subseteq B$,
2. $\forall \varepsilon \in A, F(\varepsilon)$ is fuzzy subset of $G(\varepsilon)$.

The last relationship is denoted by $(F, A) \tilde{\subseteq} (G, B)$. In this case (G, B) is called a fuzzy soft expert superset of (F, A) .

Definition 2.7. [11] Two fuzzy soft expert sets over U say, (F, A) and (G, B) are said to be equal if (F, A) is a fuzzy soft expert subset of (G, B) and (G, B) is a fuzzy soft expert subset of (F, A) .

Definition 2.8. [11] An agree-fuzzy soft expert set $(F, A)_1$ over U is a fuzzy soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Definition 2.9. [11] A disagree-fuzzy soft expert set $(F, A)_0$ over U is a fuzzy soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

Definition 2.10. [11] The complement of a fuzzy soft expert set (F, A) is, denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, \lceil A)$ where $F^c : \lceil A \rightarrow P(U)$ is a mapping given by

$$F^c(\alpha) = c(F(\lceil \alpha)), \forall \alpha \in \lceil A,$$

where c denoted as fuzzy complement.

Definition 2.11. [11] The union of two fuzzy soft expert sets (F, A) and (G, B) over U , denoted by $(F, A) \tilde{\cup} (G, B)$, is the fuzzy soft expert set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ s(F(\varepsilon), G(\varepsilon)), & \text{if } \varepsilon \in A \cap B \end{cases}$$

where s is a s -norm.

Definition 2.12. [11] *The intersection of two fuzzy soft expert sets (F, A) and (G, B) over U , denoted by $(F, A) \tilde{\cap} (G, B)$, is the fuzzy soft expert set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,*

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ t(F(\varepsilon), G(\varepsilon)), & \text{if } \varepsilon \in A \cap B \end{cases}$$

where t is a t -norm.

Definition 2.13. [11] *If (F, A) and (G, B) are two fuzzy soft expert sets over U then " (F, A) AND (G, B) " denoted by $(F, A) \wedge (G, B)$ is defined by*

$$(F, A) \wedge (G, B) = (H, A \times B)$$

such that $H(\alpha, \beta) = t(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where t is a t -norm.

Definition 2.14. [11] *If (F, A) and (G, B) are two fuzzy soft expert sets over U then " (F, A) OR (G, B) " denoted by $(F, A) \vee (G, B)$ is defined by*

$$(F, A) \vee (G, B) = (H, A \times B)$$

such that $H(\alpha, \beta) = s(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where s is a s -norm.

3. FUZZY PARAMETERIZED FUZZY SOFT EXPERT SET

In this section, we introduce the definition of a Fuzzy parameterized fuzzy soft expert set and give the basic properties of this concept.

Let U be a universe set, E be a set of parameters, I^E denotes all fuzzy subsets of E , X be a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = \Psi \times X \times O$ and $A \subseteq Z$ where $\Psi \subset I^E$.

Definition 3.1. *Let F be mapping given by*

$$F_{\Psi} : A \rightarrow I^U$$

where I^U denotes the power set of U . pair $(F, A)_{\Psi}$ is called a fuzzy parameterized fuzzy soft expert set (FPFSES) over U .

Example 3.2. *Suppose that a A factory distributes its goods to three main areas of the country. Because of the distance between the supply and demand areas and the desire to satisfy customer demand, and in order to assure the availability of their product. The management of the factory has decided to establish out stores near their existing client bases. Now, let $U = \{u_1, u_2, u_3, u_4\}$ be a set of alternatives for stores location, $E = \{e_1, e_2, e_3\}$ be a set of decision parameters where e_i ($i = 1, 2, 3$) denotes the following parameters: "land or building costs related to the area", "transportation costs between the supply and demand areas" and "the on-going security of the area" respectively. Let $\Psi = \{\frac{0.5}{e_1}, \frac{0.7}{e_2}, \frac{0.9}{e_3}\}$ be a fuzzy subset of I^E and $X = \{m, n, r\}$ be a set of experts. which are important with degree 0.5, 0.7, 0.9, respectively. From those findings*

we can find the most suitable location for the establishment of the new factory stores. Now suppose that

$$\begin{aligned}
 F\left(\frac{0.5}{e_1}, m, 1\right) &= \left\{\frac{0.4}{u_1}, \frac{0.6}{u_2}, \frac{0.5}{u_3}, \frac{0.2}{u_4}\right\}, & F\left(\frac{0.5}{e_1}, n, 1\right) &= \left\{\frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.3}{u_4}\right\}, \\
 F\left(\frac{0.5}{e_1}, r, 1\right) &= \left\{\frac{0.3}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.3}{u_4}\right\}, & F\left(\frac{0.7}{e_2}, m, 1\right) &= \left\{\frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4}\right\}, \\
 F\left(\frac{0.7}{e_2}, n, 1\right) &= \left\{\frac{0.7}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4}\right\}, & F\left(\frac{0.7}{e_2}, r, 1\right) &= \left\{\frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4}\right\}, \\
 F\left(\frac{0.9}{e_3}, m, 1\right) &= \left\{\frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4}\right\}, & F\left(\frac{0.9}{e_3}, n, 1\right) &= \left\{\frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4}\right\}, \\
 F\left(\frac{0.9}{e_3}, r, 1\right) &= \left\{\frac{0.2}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4}\right\}, & F\left(\frac{0.5}{e_1}, m, 0\right) &= \left\{\frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.7}{u_4}\right\}, \\
 F\left(\frac{0.5}{e_1}, n, 0\right) &= \left\{\frac{0.4}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4}\right\}, & F\left(\frac{0.5}{e_1}, r, 0\right) &= \left\{\frac{0.5}{u_1}, \frac{0.1}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4}\right\}, \\
 F\left(\frac{0.7}{e_2}, m, 0\right) &= \left\{\frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4}\right\}, & F\left(\frac{0.7}{e_2}, n, 0\right) &= \left\{\frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4}\right\}, \\
 F\left(\frac{0.7}{e_2}, r, 0\right) &= \left\{\frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4}\right\}, & F\left(\frac{0.9}{e_3}, m, 0\right) &= \left\{\frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4}\right\}, \\
 F\left(\frac{0.9}{e_3}, n, 0\right) &= \left\{\frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.3}{u_4}\right\}, & F\left(\frac{0.9}{e_3}, r, 0\right) &= \left\{\frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.5}{u_3}, \frac{0.5}{u_4}\right\}.
 \end{aligned}$$

Then we can find the FPFSES (F, Z) as consisting of the following collection of approximations:

$$\begin{aligned}
 (F, Z)_\psi &= \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.6}{u_2}, \frac{0.5}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.3}{u_4} \right\} \right), \right. \\
 &\quad \left(\left(\frac{0.5}{e_1}, r, 1 \right), \left\{ \frac{0.3}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
 &\quad \left(\left(\frac{0.7}{e_2}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, r, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
 &\quad \left(\left(\frac{0.9}{e_3}, m, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_3}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
 &\quad \left(\left(\frac{0.9}{e_3}, r, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.7}{u_4} \right\} \right), \\
 &\quad \left(\left(\frac{0.5}{e_1}, n, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.1}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
 &\quad \left(\left(\frac{0.7}{e_2}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, n, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right),
 \end{aligned}$$

$$\left(\left(\frac{0.7}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_3}, m, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ \left(\left(\frac{0.9}{e_3}, n, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_3}, r, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.5}{u_3}, \frac{0.5}{u_4} \right\} \right) \Big\}.$$

Definition 3.3. For two FPFSES $(F, A)_\Psi$ and $(G, B)_\Upsilon$ over U , $(F, A)_\Psi$ is called a FPFSE subset of $(G, B)_\Upsilon$ if

1. $A \subseteq B$,
2. $\forall \varepsilon \in A, F_\Psi(\varepsilon)$ is fuzzy subset of $G_\Upsilon(\varepsilon)$.

Definition 3.4. Two FPFSES $(F, A)_\Psi$ and $(G, B)_\Upsilon$ over U , are said to be equal if $(F, A)_\Psi$ is a FPFSE subset of $(G, A)_\Upsilon$ and $(G, A)_\Upsilon$ is a FPFSE subset of $(F, A)_\Psi$.

Example 3.5. Consider Example 3.2. and Suppose that the management of factory takes the opinion of the experts once again.

Let $\Psi = \left\{ \frac{0.5}{e_1}, \frac{0.3}{e_2}, \frac{0.6}{e_3} \right\}$ be a fuzzy subset over E , and $\Upsilon = \left\{ \frac{0.6}{e_1}, \frac{0.3}{e_2}, \frac{0.8}{e_3} \right\}$ be another fuzzy subset over E . Then

$$A_\Psi = \left\{ \left(\frac{0.5}{e_1}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 0 \right), \left(\frac{0.5}{e_1}, n, 1 \right), \left(\frac{0.3}{e_2}, n, 1 \right), \left(\frac{0.3}{e_2}, r, 0 \right), \right. \\ \left. \left(\frac{0.6}{e_3}, n, 0 \right), \left(\frac{0.3}{e_2}, r, 1 \right), \left(\frac{0.6}{e_3}, r, 1 \right), \left(\frac{0.6}{e_3}, r, 0 \right) \right\}, \\ B_\Upsilon = \left\{ \left(\frac{0.6}{e_1}, m, 1 \right), \left(\frac{0.8}{e_3}, m, 0 \right), \left(\frac{0.6}{e_1}, n, 1 \right), \left(\frac{0.3}{e_2}, n, 1 \right), \left(\frac{0.3}{e_2}, r, 0 \right), \right. \\ \left. \left(\frac{0.8}{e_3}, r, 1 \right), \left(\frac{0.8}{e_3}, r, 0 \right) \right\}$$

Since Υ is a fuzzy subset of Ψ , clearly $B_\Upsilon \subset A_\Psi$. Let $(F, A)_\Psi$ and $(G, A)_\Upsilon$ be defined as follows:

$$(F, A)_\Psi = \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right), \right. \\ \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \\ \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ \left(\left(\frac{0.6}{e_3}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \\ \left. \left(\left(\frac{0.6}{e_3}, n, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right) \right\}, \\ (G, A)_\Upsilon = \left\{ \left(\left(\frac{0.6}{e_1}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_1}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right), \right.$$

$$\left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, m, 0 \right), \left\{ \frac{0.7}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.8}{e_3}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right) \}.$$

Therefore $(G, B)_{\Upsilon} \subseteq (F, A)_{\Psi}$.

Definition 3.6. An agree-FPFSES $(F, A)_{\Psi_1}$ over U is a FPFSE subset of $(F, A)_{\Psi}$ defined as follows:

$$(F, A)_{\Psi_1} = \{F_{\Psi_1}(\alpha) : \alpha \in \Psi \times X \times \{1\}\}.$$

Definition 3.7. A disagree-FPFSES $(F, A)_{\Psi_0}$ over U is a FPFSE subset of $(F, A)_{\Psi}$ defined as follows:

$$(F, A)_{\Psi_0} = \{F_{\Psi_0}(\alpha) : \alpha \in \Psi \times X \times \{0\}\}.$$

Example 3.8. Consider Example 3.2. Then the agree-FPFSES $(F, Z)_{\Psi_1}$ over U is

$$(F, Z)_{\Psi_1} = \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.6}{u_2}, \frac{0.5}{u_3}, \frac{0.2}{u_4} \right\} \right), \right.$$

$$\left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.5}{e_1}, r, 1 \right), \left\{ \frac{0.3}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.7}{e_2}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, r, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.5}{e_3}, m, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_3}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right),$$

$$\left. \left(\left(\frac{0.5}{e_3}, r, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right) \right\},$$

and the disagree-FPFSES $(F, Z)_{\Psi_0}$ over U is

$$(F, Z)_{\Psi_0} = \left\{ \left(\left(\frac{0.5}{e_1}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.7}{u_4} \right\} \right) \left(\left(\frac{0.5}{e_1}, n, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \right.$$

$$\left(\left(\frac{0.5}{e_1}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.1}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right),$$

$$\left(\left(\frac{0.7}{e_2}, n, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right),$$

$$\left. \left(\left(\frac{0.5}{e_3}, m, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_3}, n, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.3}{u_4} \right\} \right), \right.$$

$$\left(\left(\frac{0.5}{e_3}, r, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.5}{u_3}, \frac{0.5}{u_4} \right\} \right).$$

Definition 3.9. The complement of a FPFSES $(F, A)_\Psi$ is, denoted by $(F, A)_\Psi^c$, is defined by $(F, A)_\Psi^c = (F^c, \lrcorner A)_\Psi$ where $F_\Psi^c : \lrcorner A \rightarrow I^U$ is the mapping given by

$$F_\Psi^c(\alpha) = c(F(\lrcorner \alpha)), \forall \alpha \in \lrcorner A,$$

, c denotes a fuzzy complement and $\lrcorner A \subset \{\Psi^c \times X \times O\}$.

Example 3.10. Consider Example 3.2. By using the basic fuzzy complement, we have

$$\begin{aligned} (F, Z)_\Psi^c = & \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.5}{u_3}, \frac{0.8}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \right. \\ & \left(\left(\frac{0.5}{e_1}, r, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.3}{u_2}, \frac{0.5}{u_3}, \frac{0.7}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.3}{u_1}, \frac{0.7}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.8}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left(\left(\frac{0.5}{e_3}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_3}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.8}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \\ & \left(\left(\frac{0.5}{e_3}, r, 1 \right), \left\{ \frac{0.8}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.3}{u_4} \right\} \right), \\ & \left(\left(\frac{0.5}{e_1}, n, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.9}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left(\left(\frac{0.3}{e_2}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, n, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.7}{u_2}, \frac{0.6}{u_3}, \frac{0.7}{u_4} \right\} \right), \\ & \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_3}, m, 0 \right), \left\{ \frac{0.8}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left. \left(\left(\frac{0.5}{e_3}, n, 0 \right), \left\{ \frac{0.7}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0.7}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_3}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.8}{u_2}, \frac{0.5}{u_3}, \frac{0.5}{u_4} \right\} \right) \right\}. \end{aligned}$$

Proposition 3.11. If $(F, A)_\Psi$ is a FPFSES over U , then

1. $((F, A)_\Psi^c)^c = (F, A)_\Psi$,
2. $(F, A)_{\Psi_1}^c = (F, A)_{\Psi_0}$,
3. $(F, A)_{\Psi_0}^c = (F, A)_{\Psi_1}$.

Proof. The proof is straightforward. □

4. UNION AND INTERSECTION

In this section, we introduce the definitions of union and intersection of FPFSESs, derive their properties, and give some examples.

Definition 4.1. The union of two FPFSEs $(F, A)_\Psi$ and $(G, B)_\Upsilon$ over U , denoted by $(F, A)_\Psi \tilde{\cup} (G, B)_\Upsilon$, is the FPFSE $(H, C)_\Omega$ such that $C = \{\Omega \times X \times O\}$ where $\Omega = \Psi \cup \Upsilon$ and $\forall \varepsilon \in C$,

$$H(\varepsilon)_\Omega = F(\varepsilon)_\Psi \cup G(\varepsilon)_\Upsilon$$

Example 4.2. Consider Example 3.2. Let

Let $\Psi = \{\frac{0.5}{e_1}, \frac{0.3}{e_2}, \frac{0.6}{e_3}\}$ be a fuzzy subset over E , $\Upsilon = \{\frac{0.6}{e_1}, \frac{0.3}{e_2}, \frac{0.8}{e_3}\}$ be another fuzzy subset over E .

$$A_\Psi = \left\{ \left(\frac{0.5}{e_1}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 0 \right), \left(\frac{0.5}{e_1}, n, 1 \right), \left(\frac{0.3}{e_2}, n, 1 \right), \left(\frac{0.3}{e_2}, r, 0 \right), \right. \\ \left. \left(\frac{0.6}{e_3}, n, 0 \right), \left(\frac{0.3}{e_2}, r, 1 \right), \left(\frac{0.6}{e_3}, r, 1 \right), \left(\frac{0.6}{e_3}, r, 0 \right) \right\},$$

$$B_\Upsilon = \left\{ \left(\frac{0.6}{e_1}, m, 1 \right), \left(\frac{0.8}{e_3}, m, 0 \right), \left(\frac{0.6}{e_1}, n, 1 \right), \left(\frac{0.3}{e_2}, n, 1 \right), \left(\frac{0.3}{e_2}, r, 0 \right), \right. \\ \left. \left(\frac{0.8}{e_3}, r, 1 \right), \left(\frac{0.8}{e_3}, r, 0 \right) \right\},$$

Suppose $(F, A)_\Psi$ and $(G, B)_\Upsilon$ are two FPFSEs over U such that

$$(F, A)_\Psi = \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right), \right. \\ \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \\ \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ \left(\left(\frac{0.6}{e_3}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \\ \left. \left(\left(\frac{0.6}{e_3}, n, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right) \right\},$$

$$(G, A)_\Upsilon = \left\{ \left(\left(\frac{0.6}{e_1}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_1}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right), \right. \\ \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, m, 0 \right), \left\{ \frac{0.7}{u_1}, \frac{0.4}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right) \\ \left. \left(\left(\frac{0.8}{e_3}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right) \right\}.$$

By using basic fuzzy union (maximum) we have $(F, A)_\Psi \tilde{\cup} (G, B)_\Upsilon = (H, C)_\Omega$ where

$$(H, C)_\Omega = \left\{ \left(\left(\frac{0.6}{e_1}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_1}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right), \right.$$

$$\left. \begin{aligned} & \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \\ & \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.6}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left(\left(\frac{0.8}{e_3}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.7}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, m, 0 \right), \left\{ \frac{0.7}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right), \\ & \left(\left(\frac{0.6}{e_3}, n, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, r, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.6}{u_4} \right\} \right) \end{aligned} \right\}.$$

Proposition 4.3. If $(F, A)_\Psi$, $(G, B)_\Upsilon$ and $(H, C)_\Omega$ are three FPFSEs over U , then

1. $(F, A)_\Psi \tilde{\cup} ((G, B)_\Upsilon \tilde{\cup} (H, C)_\Omega) = ((F, A)_\Psi \tilde{\cup} (G, B)_\Upsilon) \tilde{\cup} (H, C)_\Omega$,
2. $(F, A)_\Psi \tilde{\cup} (F, A)_\Psi = (F, A)_\Psi$.

Proof. The proof is straightforward. \square

Definition 4.4. The intersection of two FPFSEs $(F, A)_\Psi$ and $(G, B)_\Upsilon$ over U , denoted by $(F, A)_\Psi \tilde{\cap} (G, B)_\Upsilon$, is the FPFSES $(H, C)_\Omega$ such that $C = \{\Omega \times X \times O\}$ where $\Omega = \Psi \cap \Upsilon$ and $\forall \varepsilon \in C$,

$$H(\varepsilon)_\Omega = F(\varepsilon)_\Psi \cap G(\varepsilon)_\Upsilon$$

Example 4.5. Consider Example 4.2. By using basic fuzzy intersection (minimum) we have $(F, A)_\Psi \tilde{\cap} (G, B)_\Upsilon = (H, C)_\Omega$ where

$$\begin{aligned} (H, C)_\Omega = & \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_1}, n, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right), \right. \\ & \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.8}{e_3}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ & \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.5}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.6}{e_3}, m, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.3}{u_3}, \frac{0.2}{u_4} \right\} \right), \\ & \left. \left(\left(\frac{0.6}{e_3}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right) \right\}. \end{aligned}$$

Proposition 4.6. If $(F, A)_\Psi$, $(G, B)_\Upsilon$ and $(H, C)_\Omega$ are three FPFSEs over U , then

1. $(F, A)_\Psi \tilde{\cap} ((G, B)_\Upsilon \tilde{\cap} (H, C)_\Omega) = ((F, A)_\Psi \tilde{\cap} (G, B)_\Upsilon) \tilde{\cap} (H, C)_\Omega$,
2. $(F, A)_\Psi \tilde{\cap} (F, A)_\Psi = (F, A)_\Psi$.

Proof. The proof is straightforward. \square

Proposition 4.7. If $(F, A)_\Psi$, $(G, B)_\Upsilon$ and $(H, C)_\Omega$ are three FPFSEs over U , then

1. $(F, A)_{\Psi} \tilde{\cup} ((G, B)_{\Upsilon} \tilde{\cap} (H, C)_{\Omega}) = ((F, A)_{\Psi} \tilde{\cup} (G, B)_{\Upsilon}) \tilde{\cap} ((F, A)_{\Psi} \tilde{\cup} (H, C)_{\Omega})$,
2. $(F, A)_{\Psi} \tilde{\cap} ((G, B)_{\Upsilon} \tilde{\cup} (H, C)_{\Omega}) = ((F, A)_{\Psi} \tilde{\cap} (G, B)_{\Upsilon}) \tilde{\cup} ((F, A)_{\Psi} \tilde{\cap} (H, C)_{\Omega})$.

Proof. The proof is straightforward □

5. AND AND OR OPERATIONS

In this section, we introduce the definitions of AND and OR operations for FPFSEs, derive their properties, and give some examples.

Definition 5.1. *If $(F, A)_{\Psi}$ and $(G, B)_{\Upsilon}$ are two FPFSEs over U then " $(F, A)_{\Psi}$ AND $(G, B)_{\Upsilon}$ " denoted by $(F, A)_{\Psi} \wedge (G, B)_{\Upsilon}$ is defined by*

$$(F, A)_{\Psi} \wedge (G, B)_{\Upsilon} = (H, A \times B)_{\Omega}$$

such that $H(\alpha, \beta)_{\Omega} = F_{\Psi}(\alpha) \cap G_{\Upsilon}(\beta), \forall (\alpha, \beta) \in A \times B$, where $\Omega = \Psi \times \Upsilon$.

Example 5.2. *Consider Example 3.2. Let*

$$A_{\Psi} = \left\{ \left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.3}{e_3}, n, 0 \right) \right\} \text{ and}$$

$$B_{\Upsilon} = \left\{ \left(\frac{0.5}{e_1}, m, 1 \right), \left(\frac{0.5}{e_2}, n, 0 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right\}.$$

Suppose $(F, A)_{\Psi}$ and $(G, B)_{\Upsilon}$ are two fuzzy soft expert sets over U such that

$$(F, A)_{\Psi} = \left\{ \left(\left(\frac{0.7}{e_1}, m, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_1}, r, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right), \right. \\ \left. \left(\left(\frac{0.5}{e_2}, m, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{1}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_3}, n, 0 \right), \left\{ \frac{0.8}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right) \right\}$$

$$(G, B)_{\Upsilon} = \left\{ \left(\left(\frac{0.5}{e_1}, m, 1 \right), \left\{ \frac{0.3}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.6}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_2}, n, 0 \right), \left\{ \frac{0.8}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \right. \\ \left. \left(\left(\frac{0.6}{e_3}, m, 1 \right), \left\{ \frac{0.5}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right) \right\}.$$

By using basic fuzzy intersection (minimum) we have $(F, A)_{\Psi} \wedge (G, B)_{\Upsilon} = (H, A \times B)_{\Omega}$ where

$$(H, A \times B)_{\Omega} = \left\{ \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \right. \\ \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.7}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\ \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \right.$$

$$\begin{aligned}
& \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.3}{u_1}, \frac{0.3}{u_2}, \frac{0.5}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.3}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.4}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.3}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
& \left. \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right) \right\}.
\end{aligned}$$

Definition 5.3. If $(F, A)_\Psi$ and $(G, B)_\Upsilon$ are two FPFSEs over U then " $(F, A)_\Psi$ OR $(G, B)_\Upsilon$ " denoted by $(F, A)_\Psi \vee (G, B)_\Upsilon$ is defined by

$$(F, A)_\Psi \vee (G, B)_\Upsilon = (H, A \times B)_\Omega$$

such that $H(\alpha, \beta)_\Omega = F_\Psi(\alpha) \cup G_\Upsilon(\beta), \forall (\alpha, \beta) \in A \times B$, where $\Omega = \Psi \times \Upsilon$.

Example 5.4. Consider Example 5.2. By using basic fuzzy union (maximum) we have $(F, A)_\Psi \vee (G, B)_\Upsilon = (H, A \times B)_\Omega$ where

$$\begin{aligned}
(H, A \times B)_\Omega &= \left\{ \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.7}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.6}{u_4} \right\} \right), \right. \\
& \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.7}{e_1}, m, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.7}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\
& \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.7}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left. \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.5}{u_2}, \frac{0.7}{u_3}, \frac{0.6}{u_4} \right\} \right), \right\}
\end{aligned}$$

$$\left. \begin{aligned} & \left(\left(\left(\frac{0.7}{e_1}, r, 1 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.4}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.1}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.1}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.5}{e_2}, m, 0 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.5}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.1}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.5}{e_1}, m, 1 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.7}{u_2}, \frac{0.5}{u_3}, \frac{0.6}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.5}{e_2}, n, 0 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\ & \left(\left(\left(\frac{0.3}{e_3}, n, 0 \right), \left(\frac{0.6}{e_3}, m, 1 \right) \right), \left\{ \frac{0.8}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right) \end{aligned} \right\}.$$

6. AN APPLICATION OF FUZZY PARAMETERIZED FUZZY SOFT EXPERT SET IN DECISION MAKING

In this section, we present an application of Fuzzy parameterized fuzzy soft expert set theory in a decision making problem. Suppose that a restaurant chain is planning to invest in the establishment of a new branch in one of the suburbs of the city. They consulted some experts to complete an economic feasibility study of their selected area for them. The experts used the following criteria to determine their findings - their four alternatives are as follows: $U = \{u_1, u_2, u_3, u_4\}$,. There may be six parameters $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, to evaluate the restaurant site. For $i = 1, 2, 3, 4, 5, 6$ the parameters e_i ($i = 1, 2, 3, 4, 5, 6$) stand for "the number of similar restaurants that compete within the same selected area", "the proportion of people in the region that rely on restaurants for their daily meals", "structural changes needed to the selected site", "projected number of weekly/monthly diners", "accessibility of the restaurant, including ample parking facilities", "affordability of menu choices" which are important with degree 0.1, 0.3, 0.4, 0.5, 0.7, 0.9, respectively. That is, the subset of parameters is $Y = 0.1/e_1, 0.3/e_2, 0.4/e_3, 0.5/e_4, 0.7/e_5, 0.9/e_6$. et $X = \{m, n, r\}$ be a set of experts (Committee members).From those findings we can find the most suitable location for the establishment of the new restaurant in the chain.After a serious discussion the committee constructs the following Fuzzy parameterized fuzzy soft expert set

$$(F, Z) = \left\{ \left(\left(\frac{0.1}{e_1}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0.1}{u_4} \right\} \right), \left(\left(\frac{0.1}{e_1}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.2}{u_2}, \frac{0.4}{u_3}, \frac{1}{u_4} \right\} \right), \right.$$

$$\begin{aligned}
& \left(\left(\frac{0.1}{e_1}, r, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.6}{u_2}, \frac{0.1}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, m, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.5}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\
& \left(\left(\frac{0.3}{e_2}, n, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.1}{u_2}, \frac{0.2}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.8}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
& \left(\left(\frac{0.4}{e_3}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.7}{u_4} \right\} \right), \left(\left(\frac{0.4}{e_3}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.2}{u_2}, \frac{0.7}{u_3}, \frac{0.7}{u_4} \right\} \right), \\
& \left(\left(\frac{0.4}{e_3}, r, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.7}{u_2}, \frac{0.6}{u_3}, \frac{0.7}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_4}, m, 1 \right), \left\{ \frac{0.4}{u_1}, \frac{0.2}{u_2}, \frac{0.3}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
& \left(\left(\frac{0.5}{e_4}, n, 1 \right), \left\{ \frac{0.3}{u_1}, \frac{0.2}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_4}, r, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.8}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\
& \left(\left(\frac{0.7}{e_5}, m, 1 \right), \left\{ \frac{0.1}{u_1}, \frac{0.3}{u_2}, \frac{0.2}{u_3}, \frac{0.5}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_5}, n, 1 \right), \left\{ \frac{0.6}{u_1}, \frac{0.3}{u_2}, \frac{0.6}{u_3}, \frac{0.5}{u_4} \right\} \right), \\
& \left(\left(\frac{0.7}{e_5}, r, 1 \right), \left\{ \frac{0.2}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_6}, m, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.5}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\frac{0.9}{e_6}, n, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.8}{u_2}, \frac{0.8}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_6}, r, 1 \right), \left\{ \frac{0.7}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.8}{u_4} \right\} \right), \\
& \left(\left(\frac{0.1}{e_1}, m, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.6}{u_2}, \frac{0.3}{u_3}, \frac{0}{u_4} \right\} \right), \left(\left(\frac{0.1}{e_1}, n, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.1}{u_2}, \frac{0.4}{u_3}, \frac{0.6}{u_4} \right\} \right), \\
& \left(\left(\frac{0.1}{e_1}, r, 0 \right), \left\{ \frac{0.7}{u_1}, \frac{0.8}{u_2}, \frac{0.6}{u_3}, \frac{0.4}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, m, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \\
& \left(\left(\frac{0.3}{e_2}, n, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.3}{e_2}, r, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.5}{u_3}, \frac{0.1}{u_4} \right\} \right), \\
& \left(\left(\frac{0.4}{e_3}, m, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.3}{u_2}, \frac{0.1}{u_3}, \frac{0.1}{u_4} \right\} \right), \left(\left(\frac{0.4}{e_3}, n, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.6}{u_2}, \frac{0.7}{u_3}, \frac{0}{u_4} \right\} \right), \\
& \left(\left(\frac{0.4}{e_3}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.7}{u_2}, \frac{0.4}{u_3}, \frac{0.1}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_4}, m, 0 \right), \left\{ \frac{0.3}{u_1}, \frac{0.4}{u_2}, \frac{0.5}{u_3}, \frac{0.2}{u_4} \right\} \right), \\
& \left(\left(\frac{0.5}{e_4}, n, 0 \right), \left\{ \frac{0.1}{u_1}, \frac{0.2}{u_2}, \frac{0.1}{u_3}, \frac{0}{u_4} \right\} \right), \left(\left(\frac{0.5}{e_4}, r, 0 \right), \left\{ \frac{0.1}{u_1}, \frac{0.2}{u_2}, \frac{0.3}{u_3}, \frac{0.1}{u_4} \right\} \right), \\
& \left(\left(\frac{0.7}{e_5}, m, 0 \right), \left\{ \frac{0.4}{u_1}, \frac{0.5}{u_2}, \frac{0.6}{u_3}, \frac{0.3}{u_4} \right\} \right), \left(\left(\frac{0.7}{e_5}, n, 0 \right), \left\{ \frac{0.6}{u_1}, \frac{0.7}{u_2}, \frac{0.8}{u_3}, \frac{0.9}{u_4} \right\} \right), \\
& \left(\left(\frac{0.7}{e_5}, r, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_6}, m, 0 \right), \left\{ \frac{0.1}{u_1}, \frac{0.4}{u_2}, \frac{0.5}{u_3}, \frac{0.7}{u_4} \right\} \right), \\
& \left(\left(\frac{0.9}{e_6}, n, 0 \right), \left\{ \frac{0.5}{u_1}, \frac{0.6}{u_2}, \frac{0.4}{u_3}, \frac{0.2}{u_4} \right\} \right), \left(\left(\frac{0.9}{e_6}, r, 0 \right), \left\{ \frac{0.2}{u_1}, \frac{0.4}{u_2}, \frac{0.5}{u_3}, \frac{0.1}{u_4} \right\} \right),
\end{aligned}$$

In Table1 and Table2 we present the agree-fuzzy parameterized fuzzy soft expert set and disagree-fuzzy parameterized fuzzy soft expert set respectively.

Table 1: Agree-fuzzy parameterized fuzzy soft expert set

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)$	0.4	0.6	0.7	0.1
$\left(\frac{0.3}{e_2}, m\right)$	0.6	0.5	0.8	0.9
$\left(\frac{0.4}{e_3}, m\right)$	0.4	0.5	0.4	0.7
$\left(\frac{0.5}{e_4}, m\right)$	0.4	0.2	0.3	0.5
$\left(\frac{0.7}{e_5}, m\right)$	0.1	0.3	0.2	0.5
$\left(\frac{0.9}{e_6}, m\right)$	0.7	0.5	0.4	0.6
$\left(\frac{0.1}{e_1}, n\right)$	0.6	0.2	0.4	1
$\left(\frac{0.3}{e_2}, n\right)$	0.2	0.1	0.2	0.5
$\left(\frac{0.4}{e_3}, n\right)$	0.6	0.2	0.7	0.7
$\left(\frac{0.5}{e_4}, n\right)$	0.3	0.2	0.4	0.6
$\left(\frac{0.7}{e_5}, n\right)$	0.6	0.3	0.6	0.5
$\left(\frac{0.9}{e_6}, n\right)$	0.7	0.8	0.8	0.4
$\left(\frac{0.1}{e_1}, r\right)$	0.2	0.6	0.1	0.2
$\left(\frac{0.3}{e_2}, r\right)$	0.2	0.4	0.8	0.5
$\left(\frac{0.4}{e_3}, r\right)$	0.6	0.7	0.6	0.7
$\left(\frac{0.5}{e_4}, r\right)$	0.7	0.8	0.8	0.9
$\left(\frac{0.7}{e_5}, r\right)$	0.2	0.6	0.4	0.4
$\left(\frac{0.9}{e_6}, r\right)$	0.7	0.4	0.4	0.8
$c_j = \sum_{x \in X} \sum_{i=1}^6 (u_{ij})(\mu_E(e_i))$	$c_1 = 4.28$	$c_2 = 4.03$	$c_3 = 4.37$	$c_4 = 5.28$

Table 2: Disagree-fuzzy parameterized fuzzy soft expert

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)$	0.2	0.6	0.3	0
$\left(\frac{0.3}{e_2}, m\right)$	0.3	0.4	0.4	0.2
$\left(\frac{0.4}{e_3}, m\right)$	0.2	0.3	0.1	0.1

Continued on next page

Table 2 – continued from previous page

U	u_1	u_2	u_3	u_4
$\left(\frac{0.5}{e_4}, m\right)$	0.3	0.4	0.5	0.2
$\left(\frac{0.7}{e_5}, m\right)$	0.5	0.6	0.4	0.2
$\left(\frac{0.9}{e_6}, m\right)$	0.4	0.5	0.6	0.3
$\left(\frac{0.1}{e_1}, n\right)$	0.3	0.1	0.4	0.6
$\left(\frac{0.3}{e_2}, n\right)$	0.5	0.6	0.4	0.2
$\left(\frac{0.4}{e_3}, n\right)$	0.3	0.6	0.7	0
$\left(\frac{0.5}{e_4}, n\right)$	0.1	0.2	0.1	0
$\left(\frac{0.7}{e_5}, n\right)$	0.6	0.7	0.8	0.9
$\left(\frac{0.9}{e_6}, n\right)$	0.5	0.6	0.4	0.2
$\left(\frac{0.1}{e_1}, r\right)$	0.7	0.8	0.6	0.4
$\left(\frac{0.3}{e_2}, r\right)$	0.2	0.4	0.5	0.1
$\left(\frac{0.4}{e_3}, r\right)$	0.5	0.7	0.4	0.1
$\left(\frac{0.5}{e_4}, r\right)$	0.1	0.2	0.3	0.1
$\left(\frac{0.7}{e_5}, r\right)$	0.5	0.6	0.4	0.2
$\left(\frac{0.9}{e_6}, r\right)$	0.2	0.4	0.5	0.1
$k_j = \sum_{x \in X} \sum_{i=1}^6 (u_{ij})(\mu_E(e_i))$	$k_1 = 3.18$	$k_2 = 3.56$	$k_3 = 4.09$	$k_4 = 2.6$

The following algorithm may be followed by the restaurant chain to fill the location.

1. Input the fuzzy parameterized fuzzy soft expert set (F, Z) .
2. Find an agree-fuzzy parameterized fuzzy soft expert set and a disagree-fuzzy parameterized fuzzy soft expert set.
3. Find $c_j = \sum_{x \in X} \sum_{i=1}^6 (u_{ij})(\mu_E(e_i))$ for fuzzy parameterized fuzzy soft expert set.
4. Find $k_j = \sum_{x \in X} \sum_{i=1}^6 (u_{ij})(\mu_E(e_i))$ fuzzy parameterized fuzzy soft expert set.
5. Find $s_j = c_j - k_j$.
6. Find m , for which $s_m = \max s_j$. Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the restaurant chain using its option.

Remark 6.1. *The previous algorithm given by Alkhazaleh and Salleh [11] and we generalized it to be compatible with our work.*

Now we use this algorithm to find the best choice for the restaurant chain to fill the location.

Table 3: $s_j = c_j - k_j$

$c_j = \sum_{x \in X} \sum_i (u_{ij})(\mu_E(e_i))$	$k_j = \sum_{x \in X} \sum_i (u_{ij})(\mu_E(e_i))$	$s_j = c_j - k_j$
$c_1 = 4.24$	$k_1 = 3.18$	$s_1 = 1.1$
$c_2 = 4.03$	$k_2 = 3.56$	$s_2 = 0.47$
$c_3 = 4.37$	$k_3 = 4.09$	$s_3 = 0.28$
$c_4 = 5.28$	$k_4 = 2.06$	$s_4 = 3.22$

Then $\max s_j = s_4$, so the committee will choose location 4 for the restaurant chain.

7. CONCLUSIONS

In this work we have introduced the concept of fuzzy parameterized fuzzy soft expert set and studied some of its properties. The complement, union and intersection operations have been defined on the fuzzy parameterized fuzzy soft expert set. An application of this theory is given in solving a decision making problem.

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