

CHAPTER IV
PRESENTATION OF RESULT

Having discuss the methodology and seen the program used for the solution of nonlinear Schrödinger equation, we listed below the result of the programming at all cases listed in the just concluded subsection. Details about the explanation of the results at each stage will be discussed in the next chapter.

Table 4.1

Partial output of *Listing 3.1*

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.83	1.069038	1.069038	0.000000
0.00	-0.7	-0.67	1.320728	1.320728	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.33	1.793259	1.793259	0.000000
0.00	-0.2	-0.17	1.945457	1.945457	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.17	1.945457	1.945457	0.000000
0.00	0.3	0.33	1.793259	1.793259	0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.7	0.67	1.320728	1.320728	0.000000
0.00	0.8	0.83	1.069038	1.069038	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840248	0.839949	0.000300
5.00	4.2	-0.83	1.069518	1.069038	0.000480
5.00	4.3	-0.67	1.321247	1.320728	0.000519
5.00	4.5	-0.50	1.573586	1.572895	0.000690
5.00	4.7	-0.33	1.793963	1.793259	0.000704
5.00	4.8	-0.17	1.946167	1.945457	0.000709
5.00	5.0	0.00	2.000489	2.000000	0.000489
5.00	5.2	0.17	1.945642	1.945457	0.000185
5.00	5.3	0.33	1.793063	1.793259	-0.000196
5.00	5.5	0.50	1.572403	1.572895	-0.000492
5.00	5.7	0.67	1.320050	1.320728	-0.000678
5.00	5.8	0.83	1.068315	1.069038	-0.000723
5.00	6.0	1.00	0.839278	0.839949	-0.000670
10.00	9.0	-1.00	0.840847	0.839949	0.000898
10.00	9.2	-0.83	1.070089	1.069038	0.001051

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Table 4.1 continued

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
10.00	9.3	-0.67	1.321921	1.320728	0.001193
10.00	9.5	-0.50	1.574220	1.572895	0.001325
10.00	9.7	-0.33	1.794482	1.793259	0.001223
10.00	9.8	-0.17	1.946537	1.945457	0.001080
10.00	10.0	0.00	2.000540	2.000000	0.000540
10.00	10.2	0.17	1.945497	1.945457	0.000039
10.00	10.3	0.33	1.792596	1.793259	-0.000663
10.00	10.5	0.50	1.571878	1.572895	-0.001017
10.00	10.7	0.67	1.319418	1.320728	-0.001310
10.00	10.8	0.83	1.067761	1.069038	-0.001277
10.00	11.0	1.00	0.838769	0.839949	-0.001179
15.00	14.0	-1.00	0.841347	0.839949	0.001399
15.00	14.2	-0.83	1.070764	1.069038	0.001726
15.00	14.3	-0.67	1.322506	1.320728	0.001777
15.00	14.5	-0.50	1.574779	1.572895	0.001884
15.00	14.7	-0.33	1.794960	1.793259	0.001701
15.00	14.8	-0.17	1.946854	1.945457	0.001396
15.00	15.0	0.00	2.000506	2.000000	0.000506
15.00	15.2	0.17	1.945269	1.945457	-0.000189
15.00	15.3	0.33	1.792144	1.793259	-0.001115
15.00	15.5	0.50	1.571445	1.572895	-0.001451
15.00	15.7	0.67	1.318778	1.320728	-0.001950
15.00	15.8	0.83	1.067193	1.069038	-0.001845
15.00	16.0	1.00	0.838186	0.839949	-0.001763
20.00	19.0	-1.00	0.842002	0.839949	0.002053
20.00	19.2	-0.83	1.071253	1.069038	0.002215
20.00	19.3	-0.67	1.323372	1.320728	0.002644
20.00	19.5	-0.50	1.575308	1.572895	0.002412
20.00	19.7	-0.33	1.795589	1.793259	0.002330
20.00	19.8	-0.17	1.946769	1.945457	0.001311
20.00	20.0	0.00	2.000604	2.000000	0.000604
20.00	20.2	0.17	1.944733	1.945457	-0.000724
20.00	20.3	0.33	1.791742	1.793259	-0.001517
20.00	20.5	0.50	1.570615	1.572895	-0.002280
20.00	20.7	0.67	1.318211	1.320728	-0.002518
20.00	20.8	0.83	1.066595	1.069038	-0.002442
20.00	21.0	1.00	0.837682	0.839949	-0.002267
25.00	24.0	-1.00	0.842444	0.839949	0.002496
25.00	24.2	-0.83	1.071739	1.069038	0.002702
25.00	24.3	-0.67	1.323967	1.320728	0.003239
25.00	24.5	-0.50	1.575750	1.572895	0.002855

(Continued)

Table 4.1 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
25.00	24.7	-0.33	1.796039	1.793259	0.002780
25.00	24.8	-0.17	1.947033	1.945457	0.001576
25.00	25.0	0.00	2.000700	2.000000	0.000700
25.00	25.2	0.17	1.944459	1.945457	-0.000998
25.00	25.3	0.33	1.791378	1.793259	-0.001882
25.00	25.5	0.50	1.569983	1.572895	-0.002913
25.00	25.7	0.67	1.317653	1.320728	-0.003076
25.00	25.8	0.83	1.065894	1.069038	-0.003143
25.00	26.0	1.00	0.837229	0.839949	-0.002720
30.00	29.0	-1.00	0.842983	0.839949	0.003034
30.00	29.2	-0.83	1.072539	1.069038	0.003501
30.00	29.3	-0.67	1.324548	1.320728	0.003820
30.00	29.5	-0.50	1.576632	1.572895	0.003737
30.00	29.7	-0.33	1.796451	1.793259	0.003192
30.00	29.8	-0.17	1.947483	1.945457	0.002026
30.00	30.0	0.00	2.000546	2.000000	0.000546
30.00	30.2	0.17	1.944225	1.945457	-0.001232
30.00	30.3	0.33	1.790692	1.793259	-0.002567
30.00	30.5	0.50	1.569256	1.572895	-0.003639
30.00	30.7	0.67	1.316945	1.320728	-0.003783
30.00	30.8	0.83	1.065236	1.069038	-0.003802
30.00	31.0	1.00	0.836712	0.839949	-0.003236

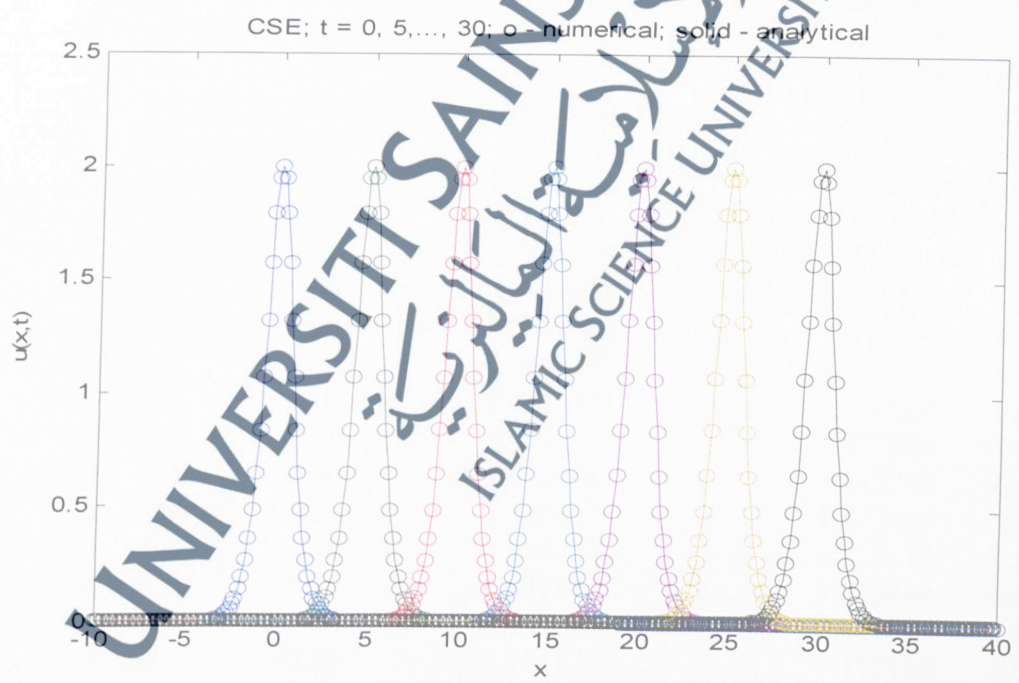


Figure 4.1 output of the main program pde_1_main for Listing 3.1

Table 4.2

Partial output of *Listing 3.2*

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.80	1.118110	1.118110	0.000000
0.00	-0.6	-0.60	1.423156	1.423156	0.000000
0.00	-0.4	-0.40	1.711278	1.711278	-0.000000
0.00	-0.2	-0.20	1.922086	1.922086	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.20	1.922086	1.922086	0.000000
0.00	0.4	0.40	1.711278	1.711278	0.000000
0.00	0.6	0.60	1.423156	1.423156	0.000000
0.00	0.8	0.80	1.118110	1.118110	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840692	0.839949	0.000743
5.00	4.2	-0.80	1.119083	1.118110	0.000973
5.00	4.4	-0.60	1.424426	1.423156	0.001271
5.00	4.6	-0.40	1.712740	1.711278	0.001463
5.00	4.8	-0.20	1.923553	1.922086	0.001467
5.00	5.0	0.00	2.000991	2.000000	0.000991
5.00	5.2	0.20	1.922272	1.922086	0.000186
5.00	5.4	0.40	1.710552	1.711278	-0.000725
5.00	5.6	0.60	1.421850	1.423156	-0.001305
5.00	5.8	0.80	1.116591	1.118110	-0.001520
5.00	6.0	1.00	0.838547	0.839949	-0.001402
10.00	9.0	-1.00	0.841776	0.839949	0.001827
10.00	9.2	-0.80	1.120379	1.118110	0.002269
10.00	9.4	-0.60	1.425802	1.423156	0.002647
10.00	9.6	-0.40	1.714002	1.711278	0.002724
10.00	9.8	-0.20	1.924345	1.922086	0.002260
10.00	10.0	0.00	2.001183	2.000000	0.001183
10.00	10.2	0.20	1.921711	1.922086	-0.000375
10.00	10.4	0.40	1.709649	1.711278	-0.001628
10.00	10.6	0.60	1.420661	1.423156	-0.002494
10.00	10.8	0.80	1.115437	1.118110	-0.002674
10.00	11.0	1.00	0.837441	0.839949	-0.002508
15.00	14.0	-1.00	0.842866	0.839949	0.002917
15.00	14.2	-0.80	1.121625	1.118110	0.003515
15.00	14.4	-0.60	1.427007	1.423156	0.003851
15.00	14.6	-0.40	1.715080	1.711278	0.003803
15.00	14.8	-0.20	1.924948	1.922086	0.002862

(Continued)

Table 4.2 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
15.00	15.0	0.00	2.001288	2.000000	-0.001288
15.00	15.2	0.20	1.921148	1.922086	-0.000938
15.00	15.4	0.40	1.708711	1.711278	-0.002567
15.00	15.6	0.60	1.419372	1.423156	-0.003784
15.00	15.8	0.80	1.114223	1.118110	-0.003888
15.00	16.0	1.00	0.836325	0.839949	-0.003624
20.00	19.0	-1.00	0.843951	0.839949	0.004002
20.00	19.2	-0.80	1.123048	1.118110	0.004938
20.00	19.4	-0.60	1.428368	1.423156	0.005213
20.00	19.6	-0.40	1.716169	1.711278	0.004892
20.00	19.8	-0.20	1.925484	1.922086	0.003398
20.00	20.0	0.00	2.000912	2.000000	0.000912
20.00	20.2	0.20	1.920351	1.922086	-0.001735
20.00	20.4	0.40	1.707293	1.711278	-0.003985
20.00	20.6	0.60	1.418163	1.423156	-0.004992
20.00	20.8	0.80	1.112913	1.118110	-0.005197
20.00	21.0	1.00	0.835423	0.839949	-0.004525
25.00	24.0	-1.00	0.844906	0.839949	0.004957
25.00	24.2	-0.80	1.124041	1.118110	0.005931
25.00	24.4	-0.60	1.429421	1.423156	0.006265
25.00	24.6	-0.40	1.717158	1.711278	0.005881
25.00	24.8	-0.20	1.926096	1.922086	0.004010
25.00	25.0	0.00	2.001210	2.000000	0.001210
25.00	25.2	0.20	1.919751	1.922086	-0.002335
25.00	25.4	0.40	1.706370	1.711278	-0.004907
25.00	25.6	0.60	1.416704	1.423156	-0.006451
25.00	25.8	0.80	1.111702	1.118110	-0.006408
25.00	26.0	1.00	0.834255	0.839949	-0.005694
30.00	29.0	-1.00	0.846284	0.839949	0.006335
30.00	29.2	-0.80	1.125482	1.118110	0.007371
30.00	29.4	-0.60	1.431189	1.423156	0.008033
30.00	29.6	-0.40	1.718323	1.711278	0.007046
30.00	29.8	-0.20	1.926872	1.922086	0.004786
30.00	30.0	0.00	2.000883	2.000000	0.000883
30.00	30.2	0.20	1.919042	1.922086	-0.003044
30.00	30.4	0.40	1.704914	1.711278	-0.006364
30.00	30.6	0.60	1.415336	1.423156	-0.007820
30.00	30.8	0.80	1.110307	1.118110	-0.007803
30.00	31.0	1.00	0.833140	0.839949	-0.006808

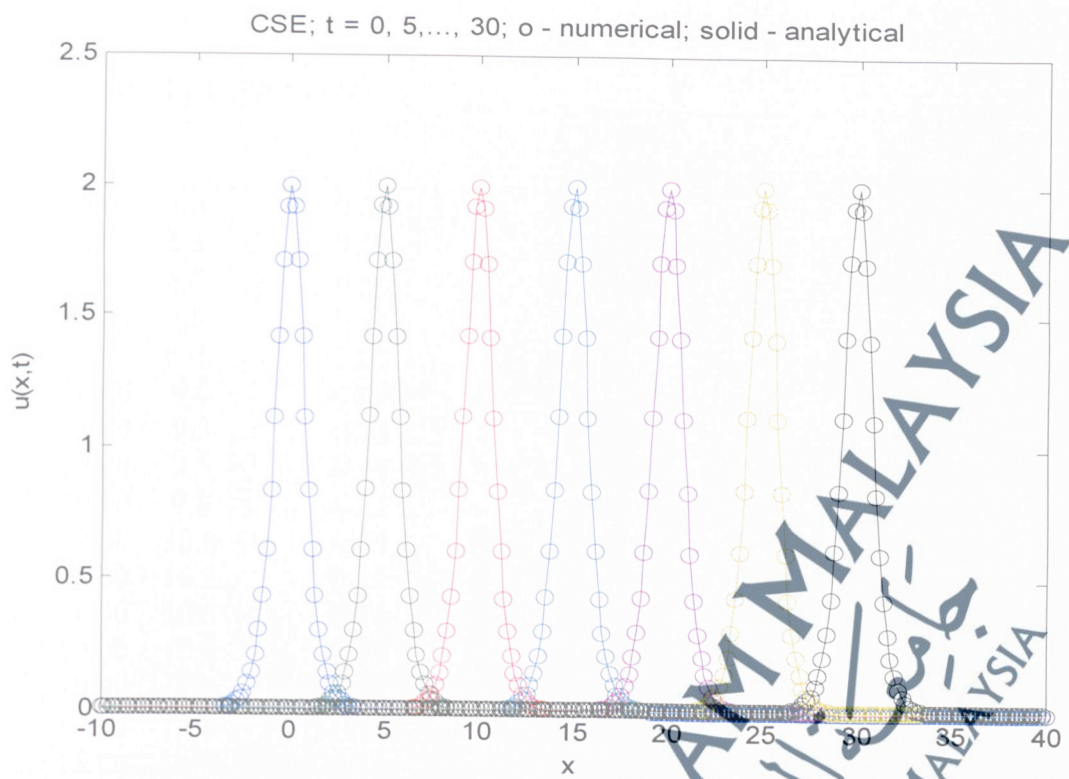


Figure 4.2 output of the main program pde_1_main for Listing 3.2

Table 4.3

Partial output of Listing 3.3

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.75	1.193172	1.193172	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.25	1.880030	1.880030	-0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.3	0.25	1.880030	1.880030	-0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.8	0.75	1.193172	1.193172	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.841744	0.839949	0.001796
5.00	4.3	-0.75	1.195789	1.193172	0.002617
5.00	4.5	-0.50	1.576253	1.572895	0.003357
5.00	4.8	-0.25	1.883654	1.880030	0.003624

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Table 4.3 (Continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
5.00	5.0	0.00	2.002365	2.000000	0.002365
5.00	5.3	0.25	1.879821	1.880030	-0.000208
5.00	5.5	0.50	1.570255	1.572895	-0.002641
5.00	5.8	0.75	1.189515	1.193172	-0.003657
5.00	6.0	1.00	0.836525	0.839949	-0.003423
10.00	9.0	-1.00	0.844269	0.839949	0.004320
10.00	9.3	-0.75	1.198933	1.193172	0.005762
10.00	9.5	-0.50	1.579617	1.572895	0.006722
10.00	9.8	-0.25	1.885974	1.880030	0.005944
10.00	10.0	0.00	2.002806	2.000000	0.002806
10.00	10.3	0.25	1.878295	1.880030	-0.001734
10.00	10.5	0.50	1.567573	1.572895	-0.005322
10.00	10.8	0.75	1.186637	1.193172	-0.006534
10.00	11.0	1.00	0.833979	0.839949	-0.005970
15.00	14.0	-1.00	0.846956	0.839949	0.007008
15.00	14.3	-0.75	1.202058	1.193172	0.008886
15.00	14.5	-0.50	1.582419	1.572895	0.009524
15.00	14.8	-0.25	1.887828	1.880030	0.007798
15.00	15.0	0.00	2.002901	2.000000	0.002901
15.00	15.3	0.25	1.876598	1.880030	-0.003432
15.00	15.5	0.50	1.564729	1.572895	-0.008166
15.00	15.8	0.75	1.183598	1.193172	-0.009574
15.00	16.0	1.00	0.831390	0.839949	-0.008559
20.00	19.0	-1.00	0.849810	0.839949	0.009862
20.00	19.3	-0.75	1.205252	1.193172	0.012081
20.00	19.5	-0.50	1.585295	1.572895	0.012399
20.00	19.8	-0.25	1.889356	1.880030	0.009326
20.00	20.0	0.00	2.002313	2.000000	0.002313
20.00	20.3	0.25	1.874205	1.880030	-0.005825
20.00	20.5	0.50	1.561507	1.572895	-0.011389
20.00	20.8	0.75	1.180561	1.193172	-0.012611
20.00	21.0	1.00	0.828994	0.839949	-0.010955
25.00	24.0	-1.00	0.852051	0.839949	0.012102
25.00	24.3	-0.75	1.207816	1.193172	0.014644
25.00	24.5	-0.50	1.587916	1.572895	0.015021
25.00	24.8	-0.25	1.891100	1.880030	0.011070
25.00	25.0	0.00	2.002618	2.000000	0.002618
25.00	25.3	0.25	1.872692	1.880030	-0.007338
25.00	25.5	0.50	1.558664	1.572895	-0.014232
25.00	25.8	0.75	1.177335	1.193172	-0.015837
25.00	26.0	1.00	0.826198	0.839949	-0.013751

(Continued)

Table 4.3 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
30.00	29.0	-1.00	0.855257	0.839949	0.015308
30.00	29.3	-0.75	1.211702	1.193172	0.018530
30.00	29.5	-0.50	1.591576	1.572895	0.018681
30.00	29.8	-0.25	1.893096	1.880030	0.013066
30.00	30.0	0.00	2.002153	2.000000	0.002153
30.00	30.3	0.25	1.870085	1.880030	-0.009945
30.00	30.5	0.50	1.555248	1.572895	-0.017647
30.00	30.8	0.75	1.174006	1.193172	-0.019165
30.00	31.0	1.00	0.823428	0.839949	-0.016521

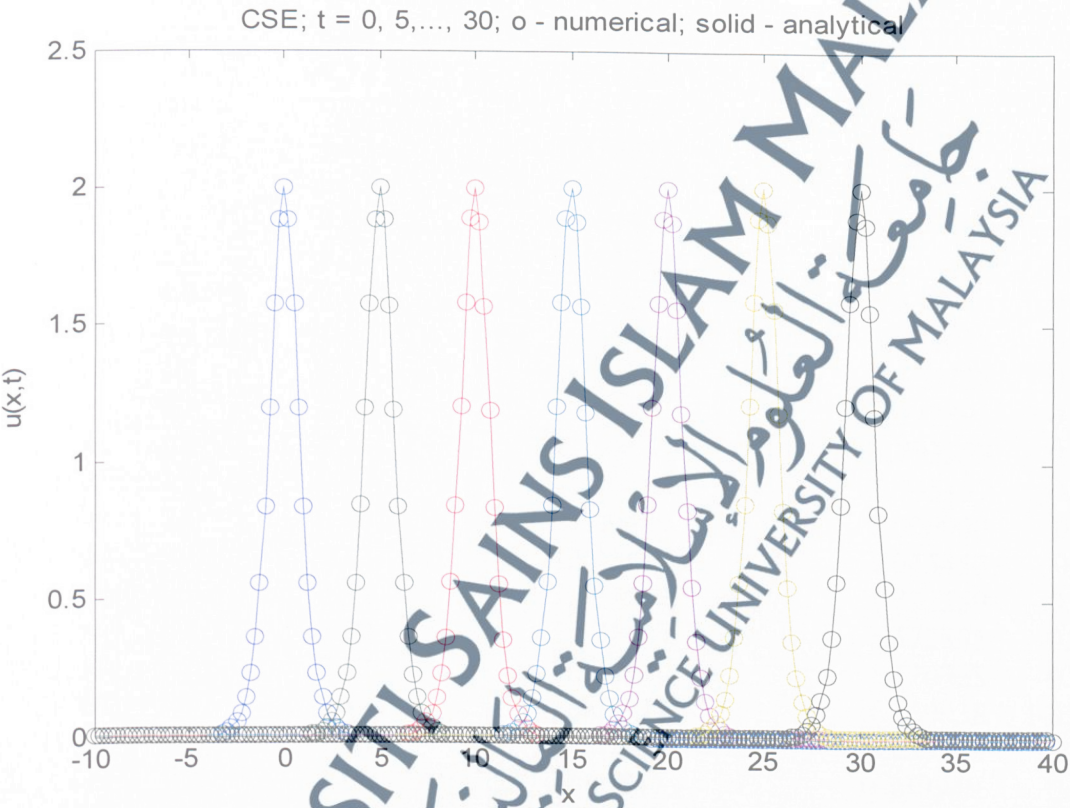


Figure 4.3 output of the main program pde_1_main for Listing 3.3

Table 4.4

Partial output of *Listing 3.4*

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.83	1.069038	1.069038	0.000000
0.00	-0.7	-0.67	1.320728	1.320728	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.33	1.793259	1.793259	0.000000
0.00	-0.2	-0.17	1.945457	1.945457	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.17	1.945457	1.945457	0.000000
0.00	0.3	0.33	1.793259	1.793259	0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.7	0.67	1.320728	1.320728	0.000000
0.00	0.8	0.83	1.069038	1.069038	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840248	0.839949	0.000300
5.00	4.2	-0.83	1.069518	1.069038	0.000480
5.00	4.3	-0.67	1.321247	1.320728	0.000519
5.00	4.5	-0.50	1.573586	1.572895	0.000690
5.00	4.7	-0.33	1.793963	1.793259	0.000704
5.00	4.8	-0.17	1.946167	1.945457	0.000709
5.00	5.0	0.00	2.000489	2.000000	0.000489
5.00	5.2	0.17	1.945642	1.945457	0.000185
5.00	5.3	0.33	1.793063	1.793259	-0.000196
5.00	5.5	0.50	1.572403	1.572895	-0.000492
5.00	5.7	0.67	1.320050	1.320728	-0.000678
5.00	5.8	0.83	1.068315	1.069038	-0.000723
5.00	6.0	1.00	0.839278	0.839949	-0.000670
10.00	9.0	-1.00	0.840847	0.839949	0.000898
10.00	9.2	-0.83	1.070089	1.069038	0.001051
10.00	9.3	-0.67	1.321921	1.320728	0.001193
10.00	9.5	-0.50	1.574220	1.572895	0.001325
10.00	9.7	-0.33	1.794482	1.793259	0.001223
10.00	9.8	-0.17	1.946537	1.945457	0.001080
10.00	10.0	0.00	2.000540	2.000000	0.000540
10.00	10.2	0.17	1.945497	1.945457	0.000039
10.00	10.3	0.33	1.792596	1.793259	-0.000663
10.00	10.5	0.50	1.571878	1.572895	-0.001017
10.00	10.7	0.67	1.319418	1.320728	-0.001310
10.00	10.8	0.83	1.067761	1.069038	-0.001277
10.00	11.0	1.00	0.838769	0.839949	-0.001179

(Continued)

Table 4.4 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
15.00	14.0	-1.00	0.841347	0.839949	0.001399
15.00	14.2	-0.83	1.070764	1.069038	0.001726
15.00	14.3	-0.67	1.322506	1.320728	0.001777
15.00	14.5	-0.50	1.574779	1.572895	0.001884
15.00	14.7	-0.33	1.794960	1.793259	0.001701
15.00	14.8	-0.17	1.946854	1.945457	0.001396
15.00	15.0	0.00	2.000506	2.000000	0.000506
15.00	15.2	0.17	1.945269	1.945457	-0.000189
15.00	15.3	0.33	1.792144	1.793259	-0.001115
15.00	15.5	0.50	1.571445	1.572895	-0.001451
15.00	15.7	0.67	1.318778	1.320728	-0.001950
15.00	15.8	0.83	1.067193	1.069038	-0.001845
15.00	16.0	1.00	0.838186	0.839949	-0.001763
20.00	19.0	-1.00	0.842002	0.839949	0.002053
20.00	19.2	-0.83	1.071253	1.069038	0.002215
20.00	19.3	-0.67	1.323372	1.320728	0.002644
20.00	19.5	-0.50	1.575308	1.572895	0.002412
20.00	19.7	-0.33	1.795589	1.793259	0.002330
20.00	19.8	-0.17	1.946769	1.945457	0.001311
20.00	20.0	0.00	2.000604	2.000000	0.000604
20.00	20.2	0.17	1.944733	1.945457	-0.000724
20.00	20.3	0.33	1.791742	1.793259	-0.001517
20.00	20.5	0.50	1.570615	1.572895	-0.002280
20.00	20.7	0.67	1.318211	1.320728	-0.002518
20.00	20.8	0.83	1.066595	1.069038	-0.002442
20.00	21.0	1.00	0.837682	0.839949	-0.002267
25.00	24.0	-1.00	0.842444	0.839949	0.002496
25.00	24.2	-0.83	1.071739	1.069038	0.002702
25.00	24.3	-0.67	1.323967	1.320728	0.003239
25.00	24.5	-0.50	1.576750	1.572895	0.002855
25.00	24.7	-0.33	1.796039	1.793259	0.002780
25.00	24.8	-0.17	1.947033	1.945457	0.001576
25.00	25.0	0.00	2.000700	2.000000	0.000700
25.00	25.2	0.17	1.944459	1.945457	-0.000998
25.00	25.3	0.33	1.791378	1.793259	-0.001882
25.00	25.5	0.50	1.569983	1.572895	-0.002913
25.00	25.7	0.67	1.317653	1.320728	-0.003076
25.00	25.8	0.83	1.065894	1.069038	-0.003143
25.00	26.0	1.00	0.837229	0.839949	-0.002720
30.00	29.0	-1.00	0.842983	0.839949	0.003034
30.00	29.2	-0.83	1.072539	1.069038	0.003501
30.00	29.3	-0.67	1.324548	1.320728	0.003820

(Continued)

Table 4.4 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
30.00	29.5	-0.50	1.576632	1.572895	0.003737
30.00	29.7	-0.33	1.796451	1.793259	0.003192
30.00	29.8	-0.17	1.947483	1.945457	0.002026
30.00	30.0	0.00	2.000546	2.000000	0.000546
30.00	30.2	0.17	1.944225	1.945457	-0.001232
30.00	30.3	0.33	1.790692	1.793259	-0.002567
30.00	30.5	0.50	1.569256	1.572895	-0.003639
30.00	30.7	0.67	1.316945	1.320728	-0.003783
30.00	30.8	0.83	1.065236	1.069038	-0.003802
30.00	31.0	1.00	0.836712	0.839949	-0.003236

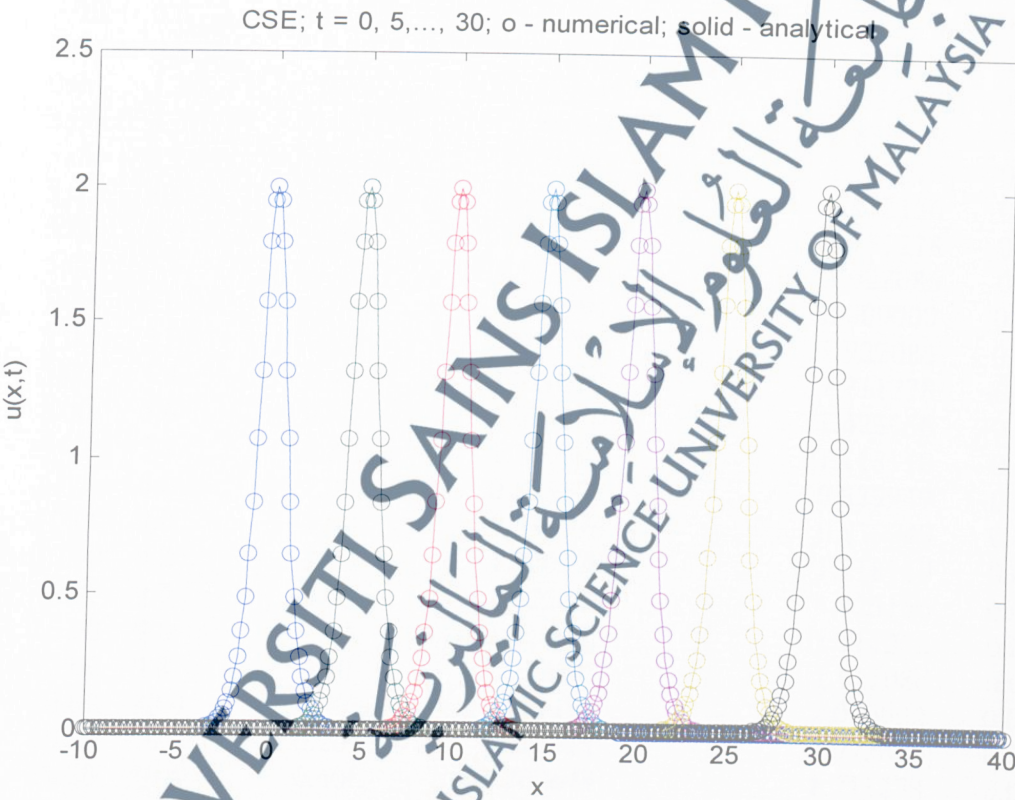


Figure 4.4 output of the main program pde_1_main for Listing 3.4

Table 4.5

Partial output of *Listing 3.5*

t	x	x - t	v ² +w ² numerical	v ² +w ² analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.80	1.118110	1.118110	0.000000
0.00	-0.6	-0.60	1.423156	1.423156	0.000000
0.00	-0.4	-0.40	1.711278	1.711278	-0.000000
0.00	-0.2	-0.20	1.922086	1.922086	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.20	1.922086	1.922086	0.000000
0.00	0.4	0.40	1.711278	1.711278	0.000000
0.00	0.6	0.60	1.423156	1.423156	0.000000
0.00	0.8	0.80	1.118110	1.118110	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840692	0.839949	0.000743
5.00	4.2	-0.80	1.119083	1.118110	0.000973
5.00	4.4	-0.60	1.424426	1.423156	0.001271
5.00	4.6	-0.40	1.712740	1.711278	0.001463
5.00	4.8	-0.20	1.923553	1.922086	0.001467
5.00	5.0	0.00	2.000991	2.000000	0.000991
5.00	5.2	0.20	1.922272	1.922086	0.000186
5.00	5.4	0.40	1.710552	1.711278	-0.000725
5.00	5.6	0.60	1.421850	1.423156	-0.001305
5.00	5.8	0.80	1.116591	1.118110	-0.001520
5.00	6.0	1.00	0.838547	0.839949	-0.001402
10.00	9.0	-1.00	0.841776	0.839949	0.001827
10.00	9.2	-0.80	1.120379	1.118110	0.002269
10.00	9.4	-0.60	1.425802	1.423156	0.002647
10.00	9.6	-0.40	1.714002	1.711278	0.002724
10.00	9.8	-0.20	1.924345	1.922086	0.002260
10.00	10.0	0.00	2.001183	2.000000	0.001183
10.00	10.2	0.20	1.921711	1.922086	-0.000375
10.00	10.4	0.40	1.709649	1.711278	-0.001628
10.00	10.6	0.60	1.420661	1.423156	-0.002494
10.00	10.8	0.80	1.115437	1.118110	-0.002674
10.00	11.0	1.00	0.837441	0.839949	-0.002508
15.00	14.0	-1.00	0.842866	0.839949	0.002917
15.00	14.2	-0.80	1.121625	1.118110	0.003515
15.00	14.4	-0.60	1.427007	1.423156	0.003851
15.00	14.6	-0.40	1.715080	1.711278	0.003803
15.00	14.8	-0.20	1.924948	1.922086	0.002862

(Continued)

Table 4.5 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
15.00	15.0	0.00	2.001288	2.000000	0.001288
15.00	15.2	0.20	1.921148	1.922086	-0.000938
15.00	15.4	0.40	1.708711	1.711278	-0.002567
15.00	15.6	0.60	1.419372	1.423156	-0.003784
15.00	15.8	0.80	1.114223	1.118110	-0.003888
15.00	16.0	1.00	0.836325	0.839949	-0.003624
20.00	19.0	-1.00	0.843951	0.839949	0.004002
20.00	19.2	-0.80	1.123048	1.118110	0.004938
20.00	19.4	-0.60	1.428368	1.423156	0.005213
20.00	19.6	-0.40	1.716169	1.711278	0.004892
20.00	19.8	-0.20	1.925484	1.922086	0.003398
20.00	20.0	0.00	2.000912	2.000000	0.000912
20.00	20.2	0.20	1.920351	1.922086	-0.001735
20.00	20.4	0.40	1.707293	1.711278	-0.003985
20.00	20.6	0.60	1.418163	1.423156	-0.004992
20.00	20.8	0.80	1.112913	1.118110	-0.005197
20.00	21.0	1.00	0.835423	0.839949	-0.004525
25.00	24.0	-1.00	0.844906	0.839949	0.004957
25.00	24.2	-0.80	1.124041	1.118110	0.005931
25.00	24.4	-0.60	1.429421	1.423156	0.006265
25.00	24.6	-0.40	1.717158	1.711278	0.005881
25.00	24.8	-0.20	1.926096	1.922086	0.004010
25.00	25.0	0.00	2.001210	2.000000	0.001210
25.00	25.2	0.20	1.919751	1.922086	-0.002335
25.00	25.4	0.40	1.706370	1.711278	-0.004907
25.00	25.6	0.60	1.416704	1.423156	-0.006451
25.00	25.8	0.80	1.111702	1.118110	-0.006408
25.00	26.0	1.00	0.834256	0.839949	-0.005694
30.00	29.0	-1.00	0.846284	0.839949	0.006335
30.00	29.2	-0.80	1.125482	1.118110	0.007371
30.00	29.4	-0.60	1.431189	1.423156	0.008033
30.00	29.6	-0.40	1.718323	1.711278	0.007046
30.00	29.8	-0.20	1.926872	1.922086	0.004786
30.00	30.0	0.00	2.000883	2.000000	0.000883
30.00	30.2	0.20	1.919042	1.922086	-0.003044
30.00	30.4	0.40	1.704914	1.711278	-0.006364
30.00	30.6	0.60	1.415336	1.423156	-0.007820
30.00	30.8	0.80	1.110307	1.118110	-0.007803
30.00	31.0	1.00	0.833140	0.839949	-0.006808

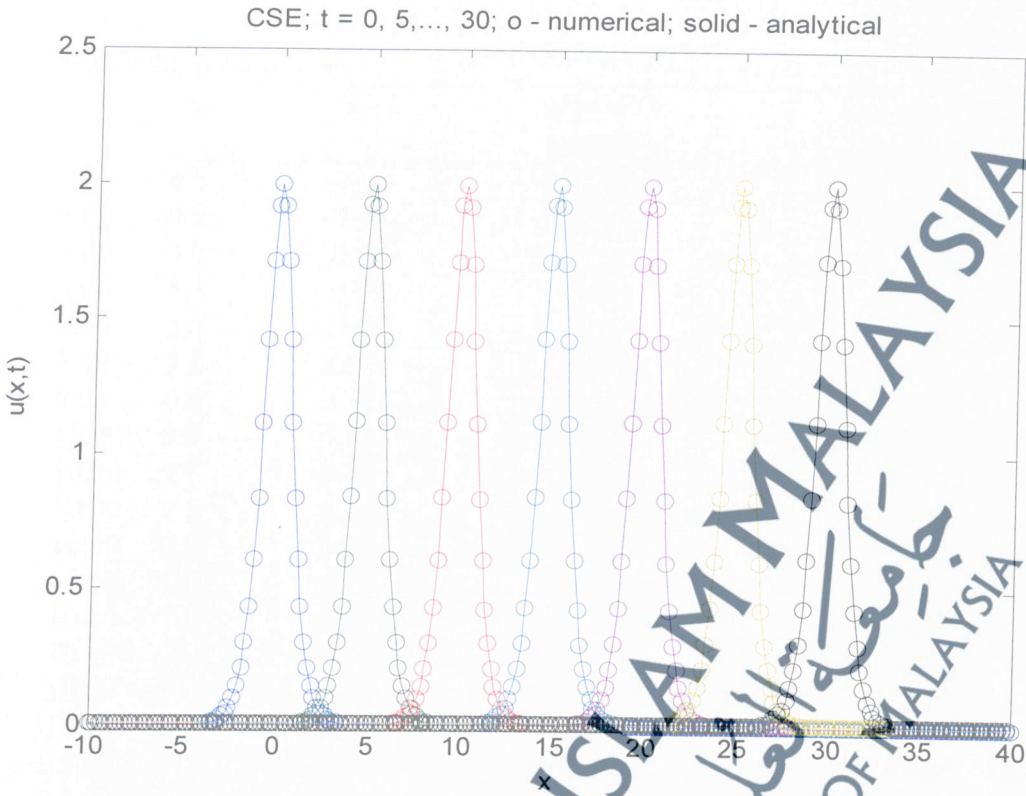


Figure 4.5 output of the main program pde_1_main for Listing 3.5

Table 4.6

Partial output of Listing 3.6

t	x	x - t	$v^2 + w^2$ numerical	$v^2 + w^2$ analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.75	1.193172	1.193172	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.25	1.880030	1.880030	-0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.3	0.25	1.880030	1.880030	-0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.8	0.75	1.193172	1.193172	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.841744	0.839949	0.001796
5.00	4.3	-0.75	1.195789	1.193172	0.002617

(Continued)

Table 4.6 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
5.00	4.5	-0.50	1.576253	1.572895	0.003357
5.00	4.8	-0.25	1.883654	1.880030	0.003624
5.00	5.0	0.00	2.002365	2.000000	0.002365
5.00	5.3	0.25	1.879821	1.880030	-0.000208
5.00	5.5	0.50	1.570255	1.572895	-0.002641
5.00	5.8	0.75	1.189515	1.193172	-0.003657
5.00	6.0	1.00	0.836525	0.839949	-0.003423
10.00	9.0	-1.00	0.844269	0.839949	0.004320
10.00	9.3	-0.75	1.198933	1.193172	0.005762
10.00	9.5	-0.50	1.579617	1.572895	0.006722
10.00	9.8	-0.25	1.885974	1.880030	0.005944
10.00	10.0	0.00	2.002806	2.000000	0.002806
10.00	10.3	0.25	1.878295	1.880030	-0.001734
10.00	10.5	0.50	1.567573	1.572895	-0.005322
10.00	10.8	0.75	1.186637	1.193172	-0.006534
10.00	11.0	1.00	0.833979	0.839949	-0.005970
15.00	14.0	-1.00	0.846956	0.839949	0.007008
15.00	14.3	-0.75	1.202058	1.193172	0.008886
15.00	14.5	-0.50	1.582419	1.572895	0.009524
15.00	14.8	-0.25	1.887828	1.880030	0.007798
15.00	15.0	0.00	2.002901	2.000000	0.002901
15.00	15.3	0.25	1.876598	1.880030	-0.003432
15.00	15.5	0.50	1.564729	1.572895	-0.008166
15.00	15.8	0.75	1.183598	1.193172	-0.009574
15.00	16.0	1.00	0.831390	0.839949	-0.008559
20.00	19.0	-1.00	0.849810	0.839949	0.009862
20.00	19.3	-0.75	1.205262	1.193172	0.012081
20.00	19.5	-0.50	1.585295	1.572895	0.012399
20.00	19.8	-0.25	1.889356	1.880030	0.009326
20.00	20.0	0.00	2.002313	2.000000	0.002313
20.00	20.3	0.25	1.874205	1.880030	-0.005825
20.00	20.5	0.50	1.561507	1.572895	-0.011389
20.00	20.8	0.75	1.180561	1.193172	-0.012611
20.00	21.0	1.00	0.828994	0.839949	-0.010955
25.00	24.0	-1.00	0.852051	0.839949	0.012102
25.00	24.3	-0.75	1.207816	1.193172	0.014644
25.00	24.5	-0.50	1.587916	1.572895	0.015021
25.00	24.8	-0.25	1.891100	1.880030	0.011070
25.00	25.0	0.00	2.002618	2.000000	0.002618
25.00	25.3	0.25	1.872692	1.880030	-0.007338
25.00	25.5	0.50	1.558664	1.572895	-0.014232

(Continued)

Table 4.6 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
25.00	25.8	0.75	1.177335	1.193172	-0.015837
25.00	26.0	1.00	0.826198	0.839949	-0.013751
30.00	29.0	-1.00	0.855257	0.839949	0.015308
30.00	29.3	-0.75	1.211702	1.193172	0.018530
30.00	29.5	-0.50	1.591576	1.572895	0.018681
30.00	29.8	-0.25	1.893096	1.880030	0.013066
30.00	30.0	0.00	2.002153	2.000000	0.002153
30.00	30.3	0.25	1.870085	1.880030	-0.009945
30.00	30.5	0.50	1.555248	1.572895	-0.017647
30.00	30.8	0.75	1.174006	1.193172	-0.019165
30.00	31.0	1.00	0.823428	0.839949	-0.016521

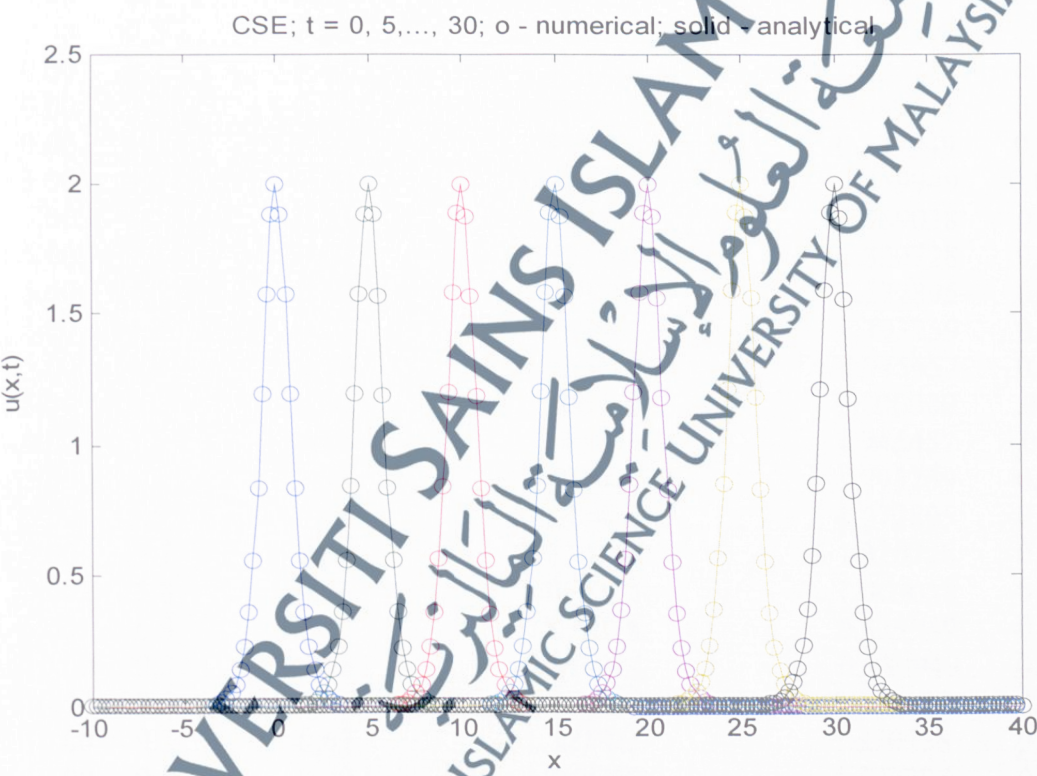


Figure 4.6 output of the main program pde_1_main for Listing 3.6

Table 4.7

Partial output of *Listing 3.7*

t	x	x - t	v ² +w ² numerical	v ² +w ² analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.83	1.069038	1.069038	0.000000
0.00	-0.7	-0.67	1.320728	1.320728	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.33	1.793259	1.793259	0.000000
0.00	-0.2	-0.17	1.945457	1.945457	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.17	1.945457	1.945457	0.000000
0.00	0.3	0.33	1.793259	1.793259	0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.7	0.67	1.320728	1.320728	0.000000
0.00	0.8	0.83	1.069038	1.069038	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840248	0.839949	0.000300
5.00	4.2	-0.83	1.069518	1.069038	0.000480
5.00	4.3	-0.67	1.321247	1.320728	0.000519
5.00	4.5	-0.50	1.573586	1.572895	0.000690
5.00	4.7	-0.33	1.793963	1.793259	0.000704
5.00	4.8	-0.17	1.946167	1.945457	0.000709
5.00	5.0	0.00	2.000489	2.000000	0.000489
5.00	5.2	0.17	1.945642	1.945457	0.000185
5.00	5.3	0.33	1.793063	1.793259	-0.000196
5.00	5.5	0.50	1.572403	1.572895	-0.000492
5.00	5.7	0.67	1.320050	1.320728	-0.000678
5.00	5.8	0.83	1.068315	1.069038	-0.000723
5.00	6.0	1.00	0.839278	0.839949	-0.000670
10.00	9.0	-1.00	0.840847	0.839949	0.000898
10.00	9.2	-0.83	1.070089	1.069038	0.001051
10.00	9.3	-0.67	1.321921	1.320728	0.001193
10.00	9.5	-0.50	1.574220	1.572895	0.001325
10.00	9.7	-0.33	1.794482	1.793259	0.001223
10.00	9.8	-0.17	1.946537	1.945457	0.001080
10.00	10.0	0.00	2.000540	2.000000	0.000540
10.00	10.2	0.17	1.945497	1.945457	0.000039
10.00	10.3	0.33	1.792596	1.793259	-0.000663
10.00	10.5	0.50	1.571878	1.572895	-0.001017
10.00	10.7	0.67	1.319418	1.320728	-0.001310
10.00	10.8	0.83	1.067761	1.069038	-0.001277
10.00	11.0	1.00	0.838769	0.839949	-0.001179

(Continued)

Table 4.7 (continued)

t	x	x - t	v ² +w ² numerical	v ² +w ² analytical	error
15.00	14.0	-1.00	0.841347	0.839949	0.001399
15.00	14.2	-0.83	1.070764	1.069038	0.001726
15.00	14.3	-0.67	1.322506	1.320728	0.001777
15.00	14.5	-0.50	1.574779	1.572895	0.001884
15.00	14.7	-0.33	1.794960	1.793259	0.001701
15.00	14.8	-0.17	1.946854	1.945457	0.001396
15.00	15.0	0.00	2.000506	2.000000	0.000506
15.00	15.2	0.17	1.945269	1.945457	-0.000189
15.00	15.3	0.33	1.792144	1.793259	-0.001115
15.00	15.5	0.50	1.571445	1.572895	-0.001451
15.00	15.7	0.67	1.318778	1.320728	-0.001950
15.00	15.8	0.83	1.067193	1.069038	-0.001845
15.00	16.0	1.00	0.838186	0.839949	-0.001763
20.00	19.0	-1.00	0.842002	0.839949	0.002053
20.00	19.2	-0.83	1.071253	1.069038	0.002215
20.00	19.3	-0.67	1.323372	1.320728	0.002644
20.00	19.5	-0.50	1.575308	1.572895	0.002412
20.00	19.7	-0.33	1.795589	1.793259	0.002330
20.00	19.8	-0.17	1.946769	1.945457	0.001311
20.00	20.0	0.00	2.000604	2.000000	0.000604
20.00	20.2	0.17	1.944733	1.945457	-0.000724
20.00	20.3	0.33	1.791742	1.793259	-0.001517
20.00	20.5	0.50	1.570615	1.572895	-0.002280
20.00	20.7	0.67	1.318211	1.320728	-0.002518
20.00	20.8	0.83	1.066595	1.069038	-0.002442
20.00	21.0	1.00	0.837682	0.839949	-0.002267
25.00	24.0	-1.00	0.842444	0.839949	0.002496
25.00	24.2	-0.83	1.071739	1.069038	0.002702
25.00	24.3	-0.67	1.323967	1.320728	0.003239
25.00	24.5	-0.50	1.575750	1.572895	0.002855
25.00	24.7	-0.33	1.796039	1.793259	0.002780
25.00	24.8	-0.17	1.947033	1.945457	0.001576
25.00	25.0	0.00	2.000700	2.000000	0.000700
25.00	25.2	0.17	1.944459	1.945457	-0.000998
25.00	25.3	0.33	1.791378	1.793259	-0.001882
25.00	25.5	0.50	1.569983	1.572895	-0.002913
25.00	25.7	0.67	1.317653	1.320728	-0.003076
25.00	25.8	0.83	1.065894	1.069038	-0.003143
25.00	26.0	1.00	0.837229	0.839949	-0.002720
30.00	29.0	-1.00	0.842983	0.839949	0.003034

(Continued)

Table 4.7 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
30.00	29.2	-0.83	1.072539	1.069038	0.003501
30.00	29.3	-0.67	1.324548	1.320728	0.003820
30.00	29.5	-0.50	1.576632	1.572895	0.003737
30.00	29.7	-0.33	1.796451	1.793259	0.003192
30.00	29.8	-0.17	1.947483	1.945457	0.002026
30.00	30.0	0.00	2.000546	2.000000	0.000546
30.00	30.2	0.17	1.944225	1.945457	-0.001232
30.00	30.3	0.33	1.790692	1.793259	-0.002567
30.00	30.5	0.50	1.569256	1.572895	-0.003639
30.00	30.7	0.67	1.316945	1.320728	-0.003783
30.00	30.8	0.83	1.065236	1.069038	-0.003802
30.00	31.0	1.00	0.836712	0.839949	-0.003236

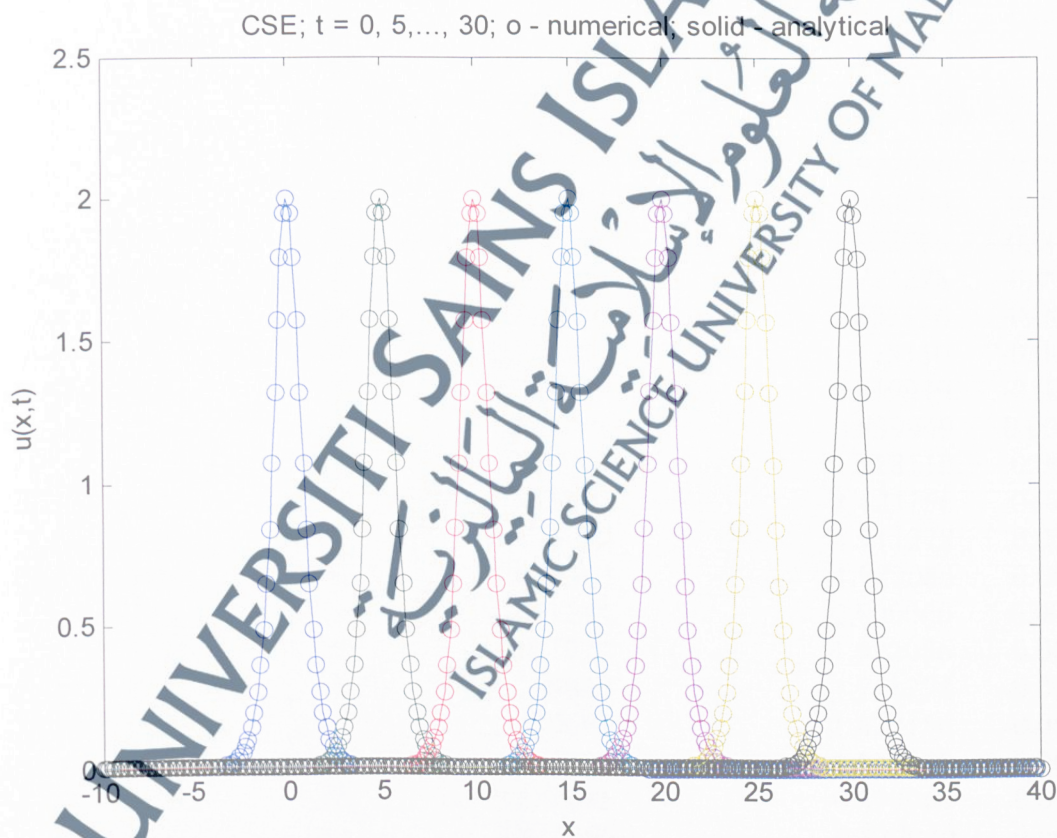


Figure 4.7 output of the main program pde_1_main for Listing 3.7

Table 4.8

Partial output of *Listing 3.8*

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.80	1.118110	1.118110	0.000000
0.00	-0.6	-0.60	1.423156	1.423156	0.000000
0.00	-0.4	-0.40	1.711278	1.711278	-0.000000
0.00	-0.2	-0.20	1.922086	1.922086	0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.2	0.20	1.922086	1.922086	0.000000
0.00	0.4	0.40	1.711278	1.711278	0.000000
0.00	0.6	0.60	1.423156	1.423156	0.000000
0.00	0.8	0.80	1.118110	1.118110	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.840692	0.839949	0.000743
5.00	4.2	-0.80	1.119083	1.118110	0.000973
5.00	4.4	-0.60	1.424426	1.423156	0.001271
5.00	4.6	-0.40	1.712740	1.711278	0.001463
5.00	4.8	-0.20	1.923553	1.922086	0.001467
5.00	5.0	0.00	2.000991	2.000000	0.000991
5.00	5.2	0.20	1.922272	1.922086	0.000186
5.00	5.4	0.40	1.710552	1.711278	-0.000725
5.00	5.6	0.60	1.421850	1.423156	-0.001305
5.00	5.8	0.80	1.116591	1.118110	-0.001520
5.00	6.0	1.00	0.838547	0.839949	-0.001402
10.00	9.0	-1.00	0.841776	0.839949	0.001827
10.00	9.2	-0.80	1.120379	1.118110	0.002269
10.00	9.4	-0.60	1.425802	1.423156	0.002647
10.00	9.6	-0.40	1.714002	1.711278	0.002724
10.00	9.8	-0.20	1.924345	1.922086	0.002260
10.00	10.0	0.00	2.001183	2.000000	0.001183
10.00	10.2	0.20	1.921711	1.922086	-0.000375
10.00	10.4	0.40	1.709649	1.711278	-0.001628
10.00	10.6	0.60	1.420661	1.423156	-0.002494
10.00	10.8	0.80	1.115437	1.118110	-0.002674
10.00	11.0	1.00	0.837441	0.839949	-0.002508
15.00	14.0	-1.00	0.842866	0.839949	0.002917
15.00	14.2	-0.80	1.121625	1.118110	0.003515
15.00	14.4	-0.60	1.427007	1.423156	0.003851
15.00	14.6	-0.40	1.715080	1.711278	0.003803
15.00	14.8	-0.20	1.924948	1.922086	0.002862

(Continued)

Table 4.8 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
15.00	15.0	0.00	2.001288	2.000000	0.001288
15.00	15.2	0.20	1.921148	1.922086	-0.000938
15.00	15.4	0.40	1.708711	1.711278	-0.002567
15.00	15.6	0.60	1.419372	1.423156	-0.003784
15.00	15.8	0.80	1.114223	1.118110	-0.003888
15.00	16.0	1.00	0.836325	0.839949	-0.003624
20.00	19.0	-1.00	0.843951	0.839949	0.004002
20.00	19.2	-0.80	1.123048	1.118110	0.004938
20.00	19.4	-0.60	1.428368	1.423156	0.005213
20.00	19.6	-0.40	1.716169	1.711278	0.004892
20.00	19.8	-0.20	1.925484	1.922086	0.003398
20.00	20.0	0.00	2.000912	2.000000	0.000912
20.00	20.2	0.20	1.920351	1.922086	-0.001735
20.00	20.4	0.40	1.707293	1.711278	-0.003985
20.00	20.6	0.60	1.418163	1.423156	-0.004992
20.00	20.8	0.80	1.112913	1.118110	-0.005197
20.00	21.0	1.00	0.835423	0.839949	-0.004525
25.00	24.0	-1.00	0.844906	0.839949	0.004957
25.00	24.2	-0.80	1.124041	1.118110	0.005931
25.00	24.4	-0.60	1.429421	1.423156	0.006265
25.00	24.6	-0.40	1.717158	1.711278	0.005881
25.00	24.8	-0.20	1.926096	1.922086	0.004010
25.00	25.0	0.00	2.001210	2.000000	0.001210
25.00	25.2	0.20	1.919751	1.922086	-0.002335
25.00	25.4	0.40	1.706370	1.711278	-0.004907
25.00	25.6	0.60	1.416704	1.423156	-0.006451
25.00	25.8	0.80	1.111702	1.118110	-0.006408
25.00	26.0	1.00	0.834255	0.839949	-0.005694
30.00	29.0	-1.00	0.846284	0.839949	0.006335
30.00	29.2	-0.80	1.125482	1.118110	0.007371
30.00	29.4	-0.60	1.431189	1.423156	0.008033
30.00	29.6	-0.40	1.718323	1.711278	0.007046
30.00	29.8	-0.20	1.926872	1.922086	0.004786
30.00	30.0	0.00	2.000883	2.000000	0.000883
30.00	30.2	0.20	1.919042	1.922086	-0.003044
30.00	30.4	0.40	1.704914	1.711278	-0.006364
30.00	30.6	0.60	1.415336	1.423156	-0.007820
30.00	30.8	0.80	1.110307	1.118110	-0.007803
30.00	31.0	1.00	0.833140	0.839949	-0.006808

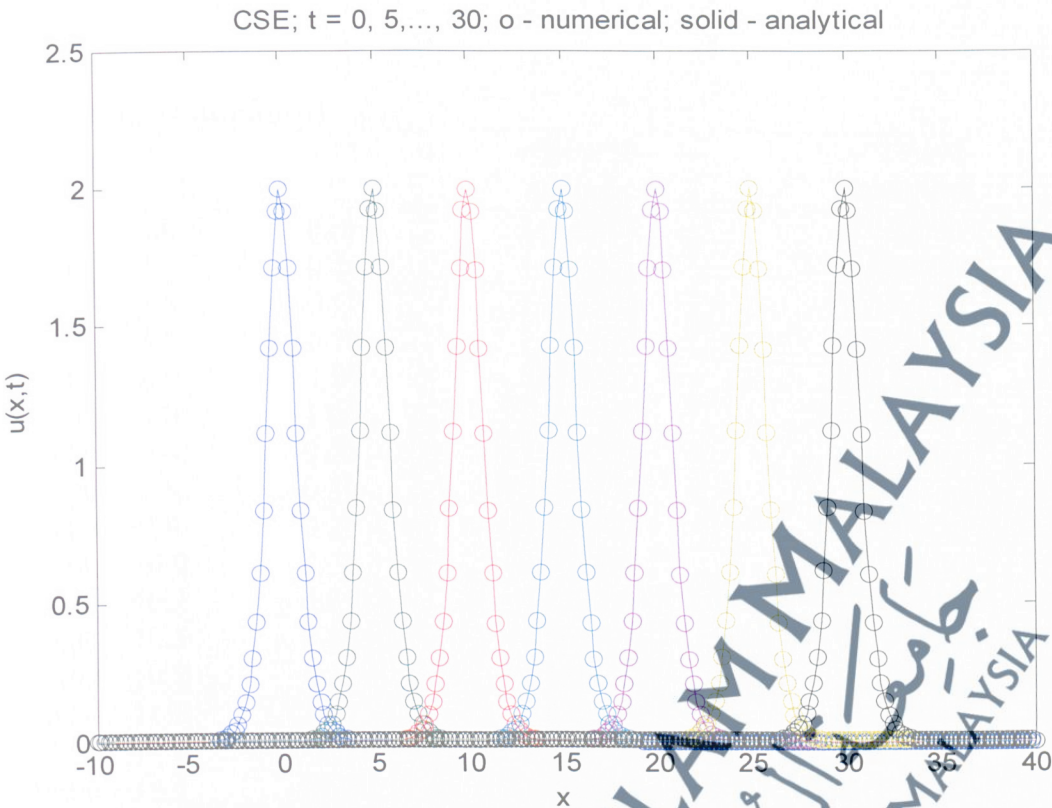


Figure 4.8 output of the main program pde_1_main for Listing 3.8

Table 4.9

Partial output of Listing 3.9

t	x	x - t	$v^2 + w^2$ numerical	$v^2 + w^2$ analytical	error
0.00	-1.0	-1.00	0.839949	0.839949	0.000000
0.00	-0.8	-0.75	1.193172	1.193172	0.000000
0.00	-0.5	-0.50	1.572895	1.572895	0.000000
0.00	-0.3	-0.25	1.880030	1.880030	-0.000000
0.00	0.0	0.00	2.000000	2.000000	0.000000
0.00	0.3	0.25	1.880030	1.880030	-0.000000
0.00	0.5	0.50	1.572895	1.572895	0.000000
0.00	0.8	0.75	1.193172	1.193172	0.000000
0.00	1.0	1.00	0.839949	0.839949	0.000000
5.00	4.0	-1.00	0.841744	0.839949	0.001796
5.00	4.3	-0.75	1.195789	1.193172	0.002617
5.00	4.5	-0.50	1.576253	1.572895	0.003357
5.00	4.8	-0.25	1.883654	1.880030	0.003624

(Continued)

Table 4.9 (Continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
5.00	5.0	0.00	2.002365	2.000000	0.002365
5.00	5.3	0.25	1.879821	1.880030	-0.000208
5.00	5.5	0.50	1.570255	1.572895	-0.002641
5.00	5.8	0.75	1.189515	1.193172	-0.003657
5.00	6.0	1.00	0.836525	0.839949	-0.003423
10.00	9.0	-1.00	0.844269	0.839949	0.004320
10.00	9.3	-0.75	1.198933	1.193172	0.005762
10.00	9.5	-0.50	1.579617	1.572895	0.006722
10.00	9.8	-0.25	1.885974	1.880030	0.005944
10.00	10.0	0.00	2.002806	2.000000	0.002806
10.00	10.3	0.25	1.878295	1.880030	-0.001734
10.00	10.5	0.50	1.567573	1.572895	-0.005322
10.00	10.8	0.75	1.186637	1.193172	-0.006534
10.00	11.0	1.00	0.833979	0.839949	-0.005970
15.00	14.0	-1.00	0.846956	0.839949	0.007008
15.00	14.3	-0.75	1.202058	1.193172	0.008886
15.00	14.5	-0.50	1.582419	1.572895	0.009524
15.00	14.8	-0.25	1.887828	1.880030	0.007798
15.00	15.0	0.00	2.002901	2.000000	0.002901
15.00	15.3	0.25	1.876598	1.880030	-0.003432
15.00	15.5	0.50	1.564729	1.572895	-0.008166
15.00	15.8	0.75	1.183598	1.193172	-0.009574
15.00	16.0	1.00	0.831390	0.839949	-0.008559
20.00	19.0	-1.00	0.849810	0.839949	0.009862
20.00	19.3	-0.75	1.205252	1.193172	0.012081
20.00	19.5	-0.50	1.585295	1.572895	0.012399
20.00	19.8	-0.25	1.889356	1.880030	0.009326
20.00	20.0	0.00	2.002313	2.000000	0.002313
20.00	20.3	0.25	1.874205	1.880030	-0.005825
20.00	20.5	0.50	1.561507	1.572895	-0.011389
20.00	20.8	0.75	1.180561	1.193172	-0.012611
20.00	21.0	1.00	0.828994	0.839949	-0.010955
25.00	24.0	-1.00	0.852051	0.839949	0.012102
25.00	24.3	-0.75	1.207816	1.193172	0.014644
25.00	24.5	-0.50	1.587916	1.572895	0.015021
25.00	24.8	-0.25	1.891100	1.880030	0.011070
25.00	25.0	0.00	2.002618	2.000000	0.002618
25.00	25.3	0.25	1.872692	1.880030	-0.007338
25.00	25.5	0.50	1.558664	1.572895	-0.014232
25.00	25.8	0.75	1.177335	1.193172	-0.015837
25.00	26.0	1.00	0.826198	0.839949	-0.013751

(Continued)

Table 4.9 (continued)

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
30.00	29.0	-1.00	0.855257	0.839949	0.015308
30.00	29.3	-0.75	1.211702	1.193172	0.018530
30.00	29.5	-0.50	1.591576	1.572895	0.018681
30.00	29.8	-0.25	1.893096	1.880030	0.013066
30.00	30.0	0.00	2.002153	2.000000	0.002153
30.00	30.3	0.25	1.870085	1.880030	-0.009945
30.00	30.5	0.50	1.555248	1.572895	-0.017647
30.00	30.8	0.75	1.174006	1.193172	-0.019165
30.00	31.0	1.00	0.823428	0.839949	-0.016521

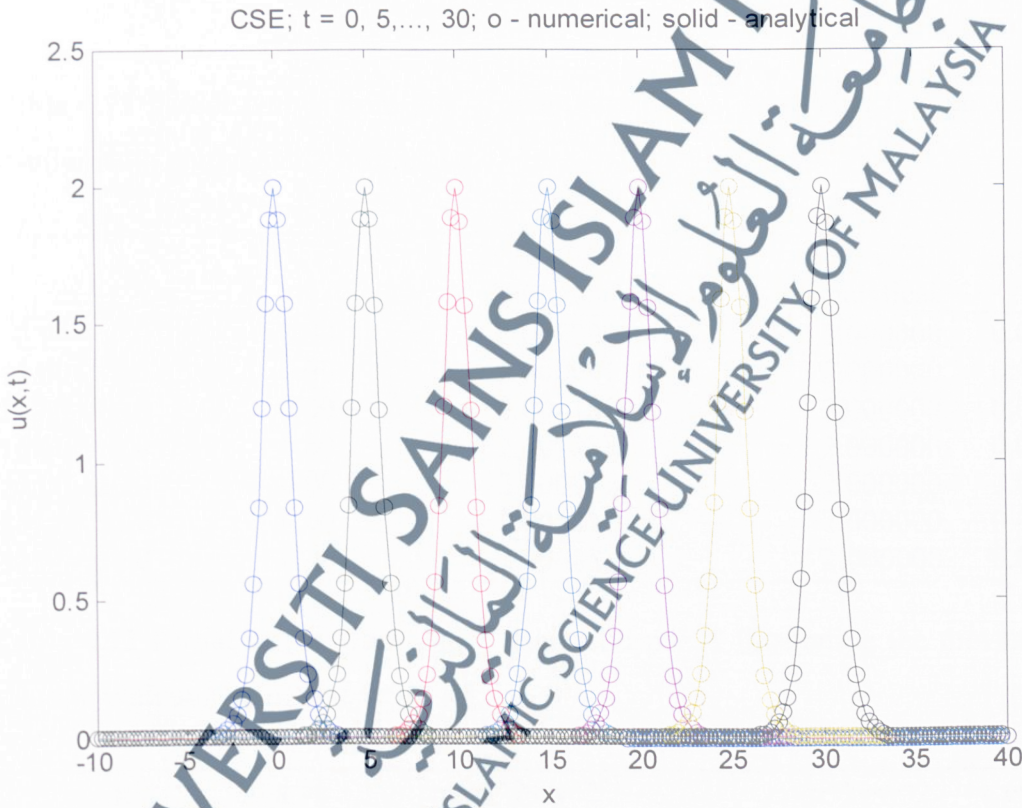


Figure 4.9 output of the main program pde_1_main for Listing 3.9

Table 4.10 Partial output from pde_1_main and pde_1 illustrating the movement of solution with unit velocity *Listing 3.1*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000489	2.000000	0.000489
10.00	10.0	0.00	2.000540	2.000000	0.000540
15.00	15.0	0.00	2.000506	2.000000	0.000506
20.00	20.0	0.00	2.000604	2.000000	0.000604
25.00	25.0	0.00	2.000700	2.000000	0.000700
30.00	30.0	0.00	2.000546	2.000000	0.000546

Table 4.11 Partial output from pde_1_main and pde_1 illustrating the movement of solution with unit velocity *Listing 3.2*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000991	2.000000	0.000991
10.00	10.0	0.00	2.001183	2.000000	0.001183
15.00	15.0	0.00	2.001288	2.000000	0.001288
20.00	20.0	0.00	2.000912	2.000000	0.000912
25.00	25.0	0.00	2.001210	2.000000	0.001210
30.00	30.0	0.00	2.000883	2.000000	0.000883

Table 4.12 Partial output from pde_1_main and pde_1 illustrating the movement of solution with unit velocity *Listing 3.3*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.002365	2.000000	0.002365
10.00	10.0	0.00	2.002806	2.000000	0.002806
15.00	15.0	0.00	2.002901	2.000000	0.002901
20.00	20.0	0.00	2.002313	2.000000	0.002313
25.00	25.0	0.00	2.002618	2.000000	0.002618
30.00	30.0	0.00	2.002153	2.000000	0.002153

Table 4.13 Partial output from `pde_1_main` and `pde_1` illustrating the movement of solution with unit velocity *Listing 3.4*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000489	2.000000	0.000489
10.00	10.0	0.00	2.000540	2.000000	0.000540
15.00	15.0	0.00	2.000506	2.000000	0.000506
20.00	20.0	0.00	2.000604	2.000000	0.000604
25.00	25.0	0.00	2.000700	2.000000	0.000700
30.00	30.0	0.00	2.000546	2.000000	0.000546

Table 4.14 Partial output from `pde_1_main` and `pde_1` illustrating the movement of solution with unit velocity *Listing 3.5*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000991	2.000000	0.000991
10.00	10.0	0.00	2.001183	2.000000	0.001183
15.00	15.0	0.00	2.001288	2.000000	0.001288
20.00	20.0	0.00	2.000912	2.000000	0.000912
25.00	25.0	0.00	2.001210	2.000000	0.001210
30.00	30.0	0.00	2.000883	2.000000	0.000883

Table 4.15 Partial output from `pde_1_main` and `pde_1` illustrating the movement of solution with unit velocity *Listing 3.6*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.002365	2.000000	0.002365
10.00	10.0	0.00	2.002806	2.000000	0.002806
15.00	15.0	0.00	2.002901	2.000000	0.002901
20.00	20.0	0.00	2.002313	2.000000	0.002313
25.00	25.0	0.00	2.002618	2.000000	0.002618
30.00	30.0	0.00	2.002153	2.000000	0.002153

Table 4.16 Partial output from `pde_1_main` and `pde_1` illustrating the movement of solution with unit velocity *Listing 3.7*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000489	2.000000	0.000489
10.00	10.0	0.00	2.000540	2.000000	0.000540
15.00	15.0	0.00	2.000506	2.000000	0.000506
20.00	20.0	0.00	2.000604	2.000000	0.000604
25.00	25.0	0.00	2.000700	2.000000	0.000700
30.00	30.0	0.00	2.000546	2.000000	0.000546

Table 4.17 Partial output from pde_1_main and pde_1 illustrating the movement of solution with unit velocity *Listing 3.8*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.000991	2.000000	0.000991
10.00	10.0	0.00	2.001183	2.000000	0.001183
15.00	15.0	0.00	2.001288	2.000000	0.001288
20.00	20.0	0.00	2.000912	2.000000	0.000912
25.00	25.0	0.00	2.001210	2.000000	0.001210
30.00	30.0	0.00	2.000883	2.000000	0.000883

Table 4.18 Partial output from pde_1_main and pde_1 illustrating the movement of solution with unit velocity *Listing 3.9*.

t	x	x - t	v^2+w^2 numerical	v^2+w^2 analytical	error
0.00	0.0	0.00	2.000000	2.000000	0.000000
5.00	5.0	0.00	2.002365	2.000000	0.002365
10.00	10.0	0.00	2.002806	2.000000	0.002806
15.00	15.0	0.00	2.002901	2.000000	0.002901
20.00	20.0	0.00	2.002313	2.000000	0.002313
25.00	25.0	0.00	2.002618	2.000000	0.002618
30.00	30.0	0.00	2.002153	2.000000	0.002153

Table 4.19 Partial output from pde_1_main and pde_1 illustrating invariants I1,I2 and I3 of Equations (13.5.9a)-(13.5.9c) in Listing 3.1-Listing 3.3.

		Listing 3.1	Listing 3.2	Listing 3.3
(t = 0)	I1	6.7303	6.7303	6.7303
	I2	1.9998	1.9997	1.9992
	I3	-13.8865	-13.8880	-13.8920
(t = 5)	I1	6.7248	6.7252	6.7265
	I2	1.9998	1.9997	1.9992
	I3	-13.8580	-13.8635	-13.8790
(t = 10)	I1	6.7076	6.7089	6.7125
	I2	1.9998	1.9997	1.9992
	I3	-13.7648	-13.7750	-13.8040
(t = 15)	I1	6.6789	6.6814	6.6882
	I2	1.9998	1.9997	1.9992
	I3	-13.6089	-13.6262	-13.6715
(t = 20)	I1	6.6385	6.6427	6.6536
	I2	1.9998	1.9997	1.9992
	I3	-13.3899	-13.4157	-13.4821
(t = 25)	I1	6.5870	6.5935	6.6096
	I2	1.9998	1.9997	1.9992
	I3	-13.1140	-13.1521	-13.2478
(t = 30)	I1	6.5249	6.5337	6.5565
	I2	1.9998	1.9997	1.9992
	I3	-12.7830	-12.8329	-12.9622

Table 4.20 Partial output from pde_1_main and pde_1 illustrating invariants I1,I2 and I3 of Equations (13.5.9a)-(13.5.9c) in Listing 3.4-Listing 3.6.

		Listing 3.4	Listing 3.5	Listing 3.6
(t = 0)	I1	6.7303	6.7303	6.7303
	I2	1.9998	1.9997	1.9992
	I3	-13.8865	-13.8880	-13.8920

(t = 5)	I1	6.7248	6.7252	6.7265
	I2	1.9998	1.9997	1.9992
	I3	-13.8580	-13.8635	-13.8790
(t = 10)	I1	6.7076	6.7089	6.7125
	I2	1.9998	1.9997	1.9992
	I3	-13.7648	-13.7750	-13.8040
(t = 15)	I1	6.6789	6.6814	6.6882
	I2	1.9998	1.9997	1.9992
	I3	-13.6089	-13.6262	-13.6715
(t = 20)	I1	6.6385	6.6427	6.6536
	I2	1.9998	1.9997	1.9992
	I3	-13.3899	-13.4157	-13.4821
(t = 25)	I1	6.5870	6.5935	6.6096
	I2	1.9998	1.9997	1.9992
	I3	-13.1140	-13.1521	-13.2478
(t = 30)	I1	6.5249	6.5337	6.5565
	I2	1.9998	1.9997	1.9992
	I3	-12.7830	-12.8329	-12.9622

Table 4.21 Partial output from pde_1_main and pde_1 illustrating invariants I1,I2 and I3 of Equations (13.5.9a)-(13.5.9c) in Listing 3.7-Listing 3.9.

		Listing 3.7	Listing 3.8	Listing 3.9
(t = 0)	I1	6.7303	6.7303	6.7303
	I2	1.9998	1.9997	1.9992
	I3	-13.8865	-13.8880	-13.8920
(t = 5)	I1	6.7248	6.7252	6.7265
	I2	1.9998	1.9997	1.9992
	I3	-13.8580	-13.8635	-13.8790
(t = 10)	I1	6.7076	6.7089	6.7125
	I2	1.9998	1.9997	1.9992
	I3	-13.7648	-13.7750	-13.8040
(t = 15)	I1	6.6789	6.6814	6.6882
	I2	1.9998	1.9997	1.9992

	I3	-13.6089	-13.6262	-13.6715
(t = 20)	I1	6.6385	6.6427	6.6536
	I2	1.9998	1.9997	1.9992
	I3	-13.3899	-13.4157	-13.4821
(t = 25)	I1	6.5870	6.5935	6.6096
	I2	1.9998	1.9997	1.9992
	I3	-13.1140	-13.1521	-13.2478
(t = 30)	I1	6.5249	6.5337	6.5565
	I2	1.9998	1.9997	1.9992
	I3	-12.7830	-12.8329	-12.9622

In the subsequent tables is a summary of the CPU time and accuracy as a result of changing the discretization size and fixing the step size or vise verse. Discussion about all the tables will follow in the next chapter.

Table 4.22 Table showing the CPU time and accuracy when Δx is being *varied* and Δt is *fixed*.

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}
201	$0 \leq t \leq 30$	13.85	2.9×10^{-3}

Table 4.23a Table showing the CPU time and accuracy when Δx is *held constant* and Δt is being *varied*.

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 35$	74.90	7.0×10^{-4}
301	$0 \leq t \leq 32$	51.68	7.0×10^{-4}
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}

Table 4.23b Table showing the CPU time and accuracy when Δx is *reduced and held constant* and Δt is being *varied*.

Δx	Δt	CPU Time (sec)
Accuracy		
251	$0 \leq t \leq 35$	32.87
251	$0 \leq t \leq 32$	28.11
251	$0 \leq t \leq 30$	28.07

Table 4.23c Table showing the CPU time and accuracy when Δx is *reduced and held constant* and Δt is being *varied*.

Δx	Δt	CPU Time (sec)
Accuracy		
201	$0 \leq t \leq 35$	16.57
201	$0 \leq t \leq 32$	14.18
201	$0 \leq t \leq 30$	13.83

Table 4.24a Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (increased)* and Δt is *fixed*.

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}

Table 4.24b Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (increased)* and Δt is *fixed*.

Δx	Δt	CPU Time (sec)	Accuracy
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}

Table 4.24c Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* again and Δt is *fixed*.

Δx	Δt	CPU Time (sec)	Accuracy
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}

Table 4.25a Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *fixed* and Δt is *varied (increased)*

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 35$	74.90	7.0×10^{-4}
301	$0 \leq t \leq 35$	74.90	7.0×10^{-4}
301	$0 \leq t \leq 35$	74.90	7.0×10^{-4}

Table 4.25b Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *fixed* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 32$	51.68	7.0×10^{-4}
301	$0 \leq t \leq 32$	51.68	7.0×10^{-4}
301	$0 \leq t \leq 32$	51.68	7.0×10^{-4}

Table 4.25c Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *fixed* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}
301	$0 \leq t \leq 30$	51.22	7.0×10^{-4}

Table 4.26a Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (increased)*

Δx	Δt	CPU Time (sec)	Accuracy
251	$0 \leq t \leq 35$	32.87	1.3×10^{-3}
251	$0 \leq t \leq 35$	32.87	1.3×10^{-3}
251	$0 \leq t \leq 35$	32.87	1.3×10^{-3}

Table 4.26b Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
251	$0 \leq t \leq 32$	28.11	1.3×10^{-3}
251	$0 \leq t \leq 32$	28.11	1.3×10^{-3}
251	$0 \leq t \leq 32$	28.11	1.3×10^{-3}

Table 4.26c Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}
251	$0 \leq t \leq 30$	28.07	1.3×10^{-3}

Table 4.27a Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
201	$0 \leq t \leq 35$	16.57	2.9×10^{-3}
201	$0 \leq t \leq 35$	16.57	2.9×10^{-3}
201	$0 \leq t \leq 35$	16.57	2.9×10^{-3}

Table 4.27b Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
201	$0 \leq t \leq 32$	14.18	2.9×10^{-3}
201	$0 \leq t \leq 32$	14.18	2.9×10^{-3}
201	$0 \leq t \leq 32$	14.18	2.9×10^{-3}

Table 4.27c Table showing the behavior of the accuracy when CPU time is *fixed* Δx is *varied (reduced)* and Δt is *varied (reduced)*

Δx	Δt	CPU Time (sec)	Accuracy
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}
201	$0 \leq t \leq 30$	13.83	2.9×10^{-3}

CHAPTER V

DISCUSSION OF THE RESULTS

5.1 General discussion about the program

We have seen in the previous chapter the detail about the program (from *Listing 3.1* to *Listing 3.9*) and we also shown the tabulated results, resulting from program. In this subsection, we are going to give account on each step of the listed program above. More so, we shall pick only one of the program (*Listing 3.1*) since space may not allow us to explain all the program one after the other, but instead, when we explain one program then it applies to all because only minor changes distinguished it with the remaining program. Here we go with the brief explanation

1. After specifying some *global* variables, the spatial domain is defined as $-10 \leq x \leq 40$, and the MOL grid has 301 (or 251 or 201 as the case may be) points.

```
% Clear previous files
clear all
clc
```

```
%
% Parameters shared with the ODE routine
global ncall x xl xu n
```

```
%
% Boundaries, number of grid points
xl=-10.0;
xu= 40.0;
n=301;
```

Since $u(x,t) = v(x,t) + i\omega(x,t)$, *pde_1_maain* of *Listing 3.1* will actually integrate $2 \times 301 = 602$ ODEs (301 for $v(x,t)$ and 301 for $\omega(x,t)$).

2. Routine *initial_1* (discussed subsequently) is called to define IC (13.5.8) over $-10 \leq x \leq 40$. The t interval is then defined as $0 \leq t \leq 30$ (as the case may be) with a solution output at $t=0, 5 \dots 30$.

```

%
% Initial condition
t0=0.0;
u0=init_1(t0);
%
% Independent variable for ODE integration
tf=30.0;
tout=(t0:5.0:tf)';
nout=7;
ncall=0;

```

3. A call to either nonstiff integrator ode45 (for mf=1) or stiff integrator ode15s (for mf=2) were used to integrate $n = 2(301)$ (or as the case may be). For this problem, ode45 requires more calls to the ODE routine pde_1 than ode15s which is usual. However, the additional computation required by ode15s that is, the sparse matrix integrator, requires additional time, so the two integrators are not very different in their overall computational efficiency. In other words, a stiff integrator does not always lead to greater computational efficiency (depending on the characteristics of the problem) (Schiesser & Griffith, 2009). We shall see in the subsequent discussion the output from ode45.

```

%
% ODE integration
mf=1;
reftol=1.0e-06; abstol=1.0e-06;
options=odeset('RelTol',reftol,'AbsTol',abstol);
%
% Explicit (nonstiff) integration
if(mf==1)[t,u]=ode45(@pde_1,tout,u0,options); end
%
% Implicit (sparse stiff) integration
if(mf==2)
S=jpattern_num;
pause
options=odeset(options,'JPattern',S);
[t,u]=ode15s(@pde_1,tout,u0,options);
end

```

4. It is known fact that Matlab integrators such as ode45 integrate only a single vector of ODEs for this reason (and this case), the solution vector u returned from ode45 is then divided into two parts, $v(x,t)$ and $w(x,t)$ of $u(x,t) = v(x,t) + i w(x,t)$.

```
%
% One vector to two vectors
for it=1:nout
for i=1:n
v(it,i)=u(it,i);
w(it,i)=u(it,i+n);
end
end
```

5. We then computed the analytical solution of (13.5.7a) by function ua (to be discussed ahead) and the differences between analytical and numerical solution is computed also and then displayed only selected portions of the differences.

```
%
% Analytical solution and difference between the numerical and
% analytical solutions
for it=1:nout
fprintf(' t x x - t v^2+w^2 v^2+w^2 err\n');
fprintf(' num anal \n')
for i=1:n
v2w2_anal(it,i)=ua(t(it),x(i));
v2w2(it,i)=u(it,i)^2+u(it,i+n)^2;
err(it,i)=v2w2(it,i)-v2w2_anal(it,i);
%
% Output in the neighborhood of the soliton peak
if(abs(x(i)-t(it))<=1.0)
fprintf('%6.2f%8.1f%8.2f%15.6f%15.6f%15.6f\n',...
t(it),x(i),x(i)-t(it),v2w2(it,i),v2w2_anal(it,i),err(it,i));
end
end
```

We displayed some selected portions because v and w are computed over either 301 or 251 or 201 as the case may be, so if we are to display the whole result will be very lengthy as such we displayed only neighborhood of the peak of the

solution. This is achieved by monitoring the argument $x-t$ in equations (13.5.7a) and (13.5.7b). The solution has maximum (peak) value at $x=t$ and drop off the neighborhood of this peak, which in the preceding code is the interval defined by $|x-t| \leq 1$. From the output we can see that the traveling wave of equations (13.5.7a) and (13.5.7b) is clear (discussed subsequently).

6. By calls to Simpson's rule (we will discuss this subsequently) that implement numerical integration, the three invariants were evaluated.

```
%
% Calculate and display three invariants
ui=u(it,:);
uint=simp(xl,xu,n,ui);
fprintf('\n Invariants at t = %5.2f',t(it));
fprintf('\n I1 = %10.4f', uint(1));
fprintf('\n I2 = %10.4f', uint(2));
fprintf('\n I3 = %10.4f\n\n',uint(3));
fprintf('\n');
end
fprintf(' ncall = %4d\n\n',ncall);
```

7. We then plotted the numerical and analytical solutions

```
%
% Plot numerical and analytical solutions
figure(1)
plot(x,v2w2,'o',x,v2w2_anal,'-')
xlabel('x')
ylabel('u(x,t)')
title('CSE; t = 0, 5,..., 30; o - numerical; solid - analytical')
print -deps -r300 pde.eps; print -dps -r300 pde.ps; print -dpng -r300 pde.png
```

Having explain the program and how it resulted to the output of the results in the **Table 4.1 to 4.9**, we can also observe the following points from the output shown in **Figure 4.1 to 4.9**

1. The peak (maximum) values plotted indicated that, the movement of the soliton is from left to right. See Table 4.10 – 4.18 for the summary of the peak movement at $t = 0, 5, \dots, 30$ which corresponds to $x = 0, 5, \dots, 30$. In this we can make the following analysis;

- In **Tables 4.10, 4.13 and 4.16**, the maximum error in the peak amplitude is the same (i.e 0.000700) and therefore,

$$\frac{0.000700}{2} \times 100 = 0.0350\% \text{ of the exact value of } 2$$

- In **Table 4.11, 4.14 and 4.17**, the maximum error in the peak amplitude is also the same (i.e 0.001288), which is

$$\frac{0.001288}{2} \times 100 = 0.0644\% \text{ of the exact value of } 2$$

- And lastly, the **Tables 4.12, 4.15 and 4.18** all have the same maximum error in the peak amplitude (i.e. 0.02901), which is

$$\frac{0.02901}{2} \times 100 = 1.4505\% \text{ of the exact value of } 2$$

Maybe the question one may ask is why we have the same output in the three programs. Well! the answer (as we will elaborate further in the subsequent discussion) is that, though we have the same output, but CPU time is not the same in all the cases.

2. The three invariants which has to do with the degree of accuracy of MOL were also short listed in the **Table 4.19 – 4.21** and their maximum variations can be shown thus

- *Listing 3.1, 3.4 and 3.7* have the same invariants and therefore their maximum variation can given as;

$$\frac{[(-13.8865 - (-12.7830))]}{(-13.8865)} \times 100 = 7.9\%$$

This variation reflects the efficiency of MOL and how it is being affected when increment in grids point is made (the high the grid point the more the variation). Recall that in *Listing 3.1, 3.4 and 3.7* the number of grid points are 301 in all the three cases.

- *Listing 3.2, 3.5 and 3.8* have the same invariants and therefore their maximum variation can given as;

$$\frac{[(-13.8880 - (-12.8329))]}{(-13.8880)} \times 100 = 7.6\%$$

As the number of grids points are reduced to 251, the accuracy tends to increase. Possibly, if not because of limited accuracy of Simpson's rule integration in simp the accuracy may have been more than this (Schiesser & Griffith, 2009)

- *Listing 3.3, 3.6 and 3.9* all have the same invariants and therefore their maximum variation can given as;

$$\frac{[(-13.8920 - (-12.9622))]}{(-13.8920)} \times 100 = 6.7\%$$

And lastly as we can see the maximum variation now is more appreciative because the grids point were lowed to 201.

3. Nevertheless, there is very good agreement between the numerical and analytical solutions as been vindicated by the error differences in **Table 4.1-4.9** and **Table 4.10-4.18** and with also the plotted output in the **Figure 4.1-4.9**

5.2 Discretization size versus step size varying

In the previous section we have learn what the program is aimed to achieve; the detail about the output of the program including efficiency and limitation of MOL. In this subsection we will discuss the second and the last aspect of the program output; the behavior of the computing time when the discretization size and step size were varied so also the concept of the accuracy concurrently. Here we go!

We start with **Table 4.22**, where we first try to reduced and maintain the step size ($0 \leq t \leq 30$) and varied the number of grids ($n = 300, 250$ and 200 respectively). At small Δx ($n = 300$) there is more accuracy of the result (7.0×10^{-4}), even though the CPU time is high (51.22secs.). When the discretization size was increased (number of grids were reduced to $n = 250$), the accuracy of the result tend to reduce also but the CPU time reduced to about 48% of the first computing time. As Δx was finally increase (number of grids reduced to $n = 200$), the accuracy drastically reduced to 2.9×10^{-3}

As for **Table 4.23a**, where we fixed Δx at $n = 301$, the accuracy was impressive, as it tend to maintain its highest accuracy (7.0×10^{-4}) at each level of changing (varying) the step size. Additionally again, there exist a wide range of time consumption from when the step size was at its highest level (for this problem) ($0 \leq t \leq 35$) to when both it is at both $0 \leq t \leq 32$ and $0 \leq t \leq 30$ by about 40% - down.

Coincidentally again from **Table 4.23b** as we can observe that when the discretization size was increased and maintained (from number of grids $n = 301$ to $n = 251$) while varying the step size, the accuracy of the result remained average and same (1.3×10^{-3}) in all the three cases but the CPU time varies from when the step size was $0 \leq t \leq 35$ to $0 \leq t \leq 30$ with a small difference.

Moreover, in the **Table 4.23c** where number of grids were finally reduced to 201 and with step size ($0 \leq t \leq 35, 0 \leq t \leq 32$ and $0 \leq t \leq 30$), the computing time was very fast

in all the three cases (16.57, 14.18 and 13.83 respectively), although the accuracy of the result was very low (2.9×10^{-3}).

In the last twelve tables of chapter four, we tried to see the behavior of accuracy of our results if we fix the CPU time and vary the discretization size and step size. We therefore started with **Table 4.24a** where we fixed the CPU time at *51.22seconds*, we set Δt to be fixed and at $0 \leq t \leq 30$ and varying the discretization size Δx , while running the program we observed that, the discretization size had to attain certain level (number of grids $n = 301$) only for the accuracy to be 7.0×10^{-4} which was quite good. **Table 4.24b** also we did the same thing as above – fixed the CPU time to *28.07seconds*, Δt was also fixed at $0 \leq t \leq 30$ and while we vary Δx we then observed that, Δx had to reach $n = 251$ before the accuracy to be 1.3×10^{-3} . As for the **Table 4.24c**, we set respectively the CPU time and Δt to be *13.83seconds* and $0 \leq t \leq 30$, and we noticed the accuracy to be 2.9×10^{-3} when the Δx attain $n = 201$.

In **Table 4.25a** shows the result where the discretization size and CPU time were set fixed at ($n = 301$) and *74.90seconds* respectively, while running the program, the accuracy was 7.0×10^{-4} when Δt attain $0 \leq t \leq 35$. **Table 4.25b** and **4.45c** gave the same accuracy (7.0×10^{-4}) when we set the CPU time and discretization size as fixed (ie *51.68seconds*, $n = 310$ and *51.22second* $n = 301$ respectively).

In **Table 4.26a** to **4.26c** we decided to set CPU time (*32.87*, *28.11* and *28.09seconds* respectively) and the discretization size as fix ($n = 251$ in all the three cases), while varying the step size, the accuracy were the same in all the three cases (1.3×10^{-3}).

And last but not least in **Table 4.27a** to **4.27**, the CPU time and discretization size were set fixed (*16.57*, *14.18* and *13.83* seconds respectively and $n = 201$ in all the three cases), while running the program, the accuracy was the same and quite low (2.9×10^{-3}).