CHAPTER V

PRICING SCHEMES FOR MULTIPLE CLASS QOS NETWORKS

5.1 Overview

The researches focused on pricing schemes for QoS networks based on Yang et al. (2003), Yang (2004), Yang et al. (2004) and Yang et al. (2005). They described the pricing scheme based on auction to allocate QoS and maximize ISP's revenue. The auction pricing scheme for differentiated service networks is actually scalability, efficiency and fairness in sharing resources. The solution for optimization problems goes from single bottleneck link in networks and then they are generalized into multiple bottleneck links using the heuristic method. In their study, they used single QoS parameter-bandwidth and formulated the pricing strategy for differentiated QoS networks. In their discussions, they had focused on the auction algorithm to find the optimal solution for differentiated service networks. Based on their idea, the attempt here is to improve and modify their mathematical formulation and combine it with mathematical formulation discussed by Byun & Chatterjee (2004).

The contributions are created by improving the mathematical formulations of Byun & Chatterjee (2004) and Yang (2004) to a simpler formulation in single and multi link links by taking into consideration the utility function, the base price as a fixed price or a variable, quality premium as fixed prices or variable, index performance, capacity in more than one link and also bandwidth required. The problem of an internet charging scheme is considered as MINLP to obtain optimal solutions by using the LINGO 11.0 (2011) software. In this part, the comparison of two models is conducted in which whether decision variable is to be fixed of user admission to the class or not. This study focuses to vary the quality premium parameters and see what decision can be made by ISP by choosing this parameter.

5.2 Model Formulation of a New Modified Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network

Optimization techniques are applied in solving the problem in this study. Like in Sain & Herpers (2003), this study also considers the optimization problem as MINLP that can be solved by using optimization tools such as LINGO 11.0. The problem of pricing the internet in multiservice networks is transformed into optimization models and are solved to get an optimal solution of pricing scheme models. This solution will help to interpret the current issues involving pricing, network sharing, base price, quality premium and QoS levels.

Assume that there is only one single network from source to destination since the pricing schemes is the main focus of the research not the routing schemes. Assume too that the routing schemes are already set up by the ISPs. As Yang (2004) pointed out, there are 2 parts to the utility function, namely the base price, which does not depend on resource consumption and cost, which depends on resource consumption. The utility function has characteristics such as marginal profit, which is the function of decreasing with increasing bandwidth. The objective of an ISP is to obtain maximum revenue subject to constraints based on the systems available resources.

The Original Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j to be fixed

Then, Original Model Formulation when setting up α_j to be Fixed will be as follows.

Max objective function (13) subject to Eq(14) To Eq (25).

where

Parameters when α_j and β_j are fixed:

α_j : Base price for class j

C : Total bandwidth

V_i : Minimum bandwidth required by user i

c_{ij} : Predetermined value of upper bound price sensitivity for user i at class j

Table 20 describes the parameter value of Original model proposed by Yang (2004).

TABLE 20: Parameter Values for The Original Model (Yang, 2004) of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j to be Fixed

Class j	j=1	j=2
α_j	0.2	0.3
Q	100	
V_i	6	5
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4

Decision variables when α_j and β_j fixed

Z_{ij} = $\begin{cases} 1, \text{if user } i \text{ is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\tilde{W}_{ij}$: Price sensitivity for user i in class j

The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j and β_j to be fixed

The new proposed formulation will be

$$\text{Max} \sum_j \sum_i ((\alpha_j Z_{ij} + \beta_j I_j) + W_j \log \frac{\tilde{X}_{ij}}{L_{m_j}}) \quad (75)$$

Where Eq (75) is the original maximized problem for Yang (2004) and subject to constraints (14)-(25) and the proposed additional constraints as follow.

$$\alpha_j + \beta_j I_j \ge \alpha_{j-1}\beta_{j-1}I_{j-1}, j > 1 \quad (76)$$

$$0 \le I_j \le d, j = 1, \ldots, m; d \in [0,1] \quad (77)$$

where

Parameters when α_j and β_j are fixed

α_j : Base price for class j

C : Total bandwidth

V_i : Minimum bandwidth required by user i

β_j : Quality premium of class j that has I_j service performance

c_{ij} : Predetermined value of upper bound price sensitivity for user i at class j

d_j : Maximum quality index value in class j

Table 21 represents the parameter values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j and β_j to be Fixed.

TABLE 21: Parameter Values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j and β_j to be Fixed

Class j	j=1	j=2
α_j	0.2	0.3
C	100	
V_i	6	5
β_j	0.01	0.02
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4
d_j	0.9	0.8

Decision variables when α_j and β_j fixed

Z_{ij} = $\begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\tilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

The model descriptions are as follows. The objective function (75) basically states that ISPs want to maximize profit by maximizing their utility function with base price α_j to be fixed to recover cost. Constraint (14) states that the total final bandwidth for all users cannot exceed the total bandwidth available. Constraint (15) states that bandwidth obtained by user i should exceed minimum bandwidth for class j if user i is admitted to class j or otherwise. Constraint (16) states that price sensitivity for class j should be less than the price sensitivity for user i in class j if user i is admitted to class j.

Constraint (17) states that the information about bandwidth obtained by client i for class j should exceed minimum bandwidth required by user i if user i is admitted to class j. Constraint (18) states that bandwidth obtained by user i in class j should exceed bandwidth assigned to each individual user in class j if a user i is admitted to class j. Constraint (19) states that bandwidth obtained by user i in class j should be greater than the availability of user i in class j and should be nonnegative (20). Non-negativity requirement occurs in price sensitivity (22) and minimum bandwidth for class j in (21). Constraint (23) shows that bandwidth obtained by user i in class j should not exceed bandwidth assigned to each individual user in class j. Constraint (24) states that the value of whether the user i is admitted to class j or not. Constraint (25) states the price sensitivity of user i in class j lies between the range of 0 and predetermined value (c) of price sensitivity for user i. Constraint (27) shows that the base price for class j lies between the fixed value, which (a and b) is lower bound and upper bound of predetermined base price, respectively. Constraint (28) shows that base price for j class is more than base price or j-1 class with j >1. Constraint (76) shows that the summation of price and quality premium to yield a perfect service for j class should exceed the ones in (j-1) class with j >1. Constraint (77) shows that the range of index quality should lie between 0 and 1 with the predetermined d value set up by ISPs.

The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j to be fixed and β_j varies

The proposed formulation is as follows.

Max objective function (75) subject to constraints (14)-(25), constraints (76)-(77) and additional constraints as follow.

$$\beta_j \le \beta_{j-1}, j > 1 \quad (78)$$

$$f \le \beta_i \le g, j = 1, \ldots, m; [f, g] \in [0,1] \quad (79)$$

where

Parameters when α_j is fixed and β_j varies

α_j : Base price for class j

C : Total bandwidth

V_i : Minimum bandwidth required by user i

c_{ij} : Predetermined value of upper bound price sensitivity for user i at class j

d_j : Maximum quality index value in class j

f_j : Minimum quality premium value for class j

g_j : Maximum quality premium value for class j

Table 22 explains the parameter values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j and β_j to be Fixed.

TABLE 22: Parameter Values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j to be Fixed and β_j Varies

Class j	j=1	j=2
α_j	0.2	0.3
C	100	
V_i	6	5
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4
d_j	0.9	0.8
f_j	0.01	0.02
g_j	0.04	0.03

Decision variables when α_j is fixed and β_j varies

Z_{ij} = $\begin{cases} 1, \text{ if user } i \text{ is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

$\widetilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

β_j : Quality premium for class j

Constraint (78) shows that the quality premium in class j should not exceed the quality premium in previous classes. Constraint (79) shows that the range of quality premium lies in [f, g] with $f \geq 0$, $g \geq 0$ and both are predetermined values set up by ISPs.

The Original Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j varies

The model formulation for model 2 original proposed by Yang (2004) is as follows.

Max objective function (26) subject to Eq(14) to Eq(25) and Eq(27) and Eq(28).

Where

Parameters:

C : Total bandwidth

V_i : Minimum bandwidth required by customer i

c_{ij} : upper bound value of user's i price sensitivity at class j

a_j : lower bound of base price in class j

b_j : upper bound of base price in class j

Table 23 explains the parameter values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j varies.

Table 23: Parameter Values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j Varies

Class j	**j=1**	**j=2**
C	100	
V_i	6	5
β_j	0.01	0.02
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4
a_j	0.1	0.2
b_j	0.4	0.3

Decision variables:

$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

X_{ij} : Final bandwidth obtained by user i for class j

L_{mj} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

α_j : Base price for class j

The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j varies

The proposed improved mathematical formulation will be as follows.

$$\text{Max} \sum_j \sum_i ((\alpha_j + \beta_j I_j) + W_j \log \frac{\tilde{X}_{ij}}{L_{m_j}}) Z_{ij} \tag{80}$$

subject to constraints (14)-(25), the improved constraints (76)-(77) and constraints (27)-(28).

where
Parameters when α_j varies and β_j is fixed

C	:	Total bandwidth
V_i	:	Minimum bandwidth required by user i
β_j	:	Quality premium of class j that has I_j service performance
c_{ij}	:	Predetermined value of upper bound price sensitivity for user i at class j
d_j	:	Maximum quality index value in class j
a_j	:	Minimum base price value set up by ISP for class j
b_j	:	Maximum base price value set up by ISP for class j

Table 24 explains the parameter values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j varies and β_j is fixed.

TABLE 24: Parameter Values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j Varies and β_j to be Fixed

Class j	j=1	j=2
C	100	
V_i	6	5
β_j	0.01	0.02
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4
d_j	0.9	0.8
a_j	0.1	0.2
b_j	0.4	0.3

Decision Variables when α_j vary and β_j is fixed

$Z_{ij} \; = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

$\widetilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

α_j : Base price for class j

The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j and β_j vary

The proposed improved mathematical programming will be as follows.

Maximum objective function (80) subject to constraints (14)-(25), constraints (72)-(73) and constraints (27)-(28).

where

Parameters when α_j and β_j vary

C : Total bandwidth

V_i : Minimum bandwidth required by user i

c_{ij} : Predetermined value of upper bound price sensitivity for user i at class j

d_j : Maximum quality index value in class j

f_j : Minimum quality premium value for class j

g_j : Maximum quality premium value for class j

a_j : Minimum base price value set up by ISP for class j

b_j : Maximum base price value set up by ISP for class j

Table 25 explains the parameter values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when setting up α_j and β_j vary.

TABLE 25: Parameter Values for The New Improved Model of Internet Pricing Scheme in Single Bottleneck Link QoS Network when Setting up α_j and β_j Vary

Class j	j=1	j=2
C		100
V_i	6	5
c_{ij}, i=1,2	c_{1j}=5, c_{2j}=4	c_{1j}=5, c_{2j}=4
d_j	0.9	0.8
f_j	0.1	0.2
g_j	0.4	0.3
a_j	0.1	0.2
b_j	0.4	0.3

Decision variables when α_j vary and β_j vary

$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

$\widetilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

α_j : Base price for class j

β_j : Quality premium for class j

The description of the objective function is as follows. Objective function (80) states that ISPs want to maximize profit by maximizing its utility function with the base price α_j to be a variable to enable ISPs to have market competition if there are chances.

5.3. Optimal Solutions for Internet Pricing Schemes in Single Bottleneck Link QoS Networks

In the next part, the results of the computation of models in multiple class QoS are shown by introducing two classes and two users, so j=2 and i=2. Table 26 explains the solver status for Internet Pricing Schemes in Single Bottleneck Link QoS Networks when α_j is fixed with by comparing three cases of original, modified with β fixed and β vary. Table 27 solver status for Internet Pricing Schemes in Single Bottleneck Link QoS Networks when α_j is varies of the original, modified when β is fixed and β varies.

TABLE 26: Solver Status for Internet Pricing Schemes in Single Bottleneck Link QoS Networks when α_j is Fixed

Solver status	Original	Modified (β fixed)	Modified (β vary)
Model Class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Objective	156.981	157.031	157.117
Infeasibility	0	0	0
Iterations	32	32	32
Extended Solver state			
Solver type	B & B	B & B	B & B
Best Objective	156.981	157.031	157.117
Objective bound	156.981	157.031	157.117
Steps	0	0	0
Active	0	0	0
Update interval	2	2	2
GMU(K)	28	29	30
ER(sec)	0	0	0

In Table 26, the solver status for model 1 is obtained. The model class will be MINLP since in the model there is at least one nonlinear constraint. The models have one or more nonlinear constraints but the solver's local search procedure is unable to search for better optimal solutions, thus it terminates only in local optimum. For that reason, the solver will be able to find the objective value for each model, which is 156.981, 157.031 and 157.117, respectively. No constraints in the models are violated as the 0 infeasibility field shows. The number of iterations completed by LINGO's solver varies according to the models created. Like in our model 1, it obtained the same iterations, which were 32 iterations, to find the local optimal solution. LINGO employs branch and bound solver while dealing with integer constraints. The best possible objective value found so far in the model, turns out to be the same value as in the objective, with the highest value achieved by a modified model 1 (β vary).

At some point, the best objective and objective bound can be very close in value as the bound shows the limit on how far the solver can improve the objective. Since the solver type is branch and bound then the number of branches in the branch and bound tree is explained in steps taken by the extended solver which are 0 in all model 1.The number of active sub-problems remaining to be analyzed in model 1 is 2 as stated in the active field that indicates that there are 2 remaining open sub-problems to be evaluated until it reaches zero. GMU varies according to the model, which indicates that the memory generator is currently using the LINGO's memory allocation. Total time to generate and solve the models is all 0 as shown in ER =0 sec.

The interpretation for Table 27 is quite similar with Table 26. Model class for each model is MINLP, which is the local optimal found so far by the solver. Objective value reached so far is the highest value achieved by the modified model 2 (β vary). There are 0 or near 0 number of constraints violated as the infeasibility field shows. The Branch and bound solver type are applied in the models. The same value for the best objective and objective bound were obtained. There are 2 branches in the branch bound tree taken by the extended solver in each model. The number of active sub-problems that remain to be analyzed is 1 in model 2 original and 2 in model 2 modified (β fixed) and model 2 modified (β vary). The solver must then run until it reaches zero.

TABLE 27: Solver Status for Internet Pricing Schemes in Single Bottleneck Link QoS Networks when α_j Varies

Solver status	Original	Modified (β fixed)	Modified (β vary)
Model Class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Objective	79.0385	79.0635	79.0865
Infeasibility	0	1.4×10^{-14}	0
Iterations	60	78	82
Extended Solver state			
Solver type	B & B	B & B	B & B
Best Objective	79.0385	79.0635	79.0865
Objective bound	79.0385	79.0635	79.0865
Steps	2	2	2
Active	1	2	2
Update interval	2	2	2
GMU(K)	28	30	31
ER(sec)	0	0	0

In Table 28, user 1 is allowed to take class 1 and class 2 since the final price sensitivity for user 1 in class 1 ($\widetilde{W}_{11}$ and $\widetilde{W}_{12}$) is at least greater than or equal to price sensitivity for class 1 and/or class 2 (W_1 and W_2). That is why only user 1 is admitted to either class 1 or 2. Bandwidth obtained by user 1 in class 1 is 25 bps. It happens also for user 2 in class 1 or 2. Bandwidth obtained by user 1 in class 1 is at least more than or equal to bandwidth for class 1. Bandwidth obtained by user 1 in class 2 also faces the same condition as in class 1. From the objective value in each model, it is obvious that the modified model yields better solution compared to the original model proposed by (Yang, 2004).

TABLE 28: A Comparison of for Internet Pricing Schemes in Single Bottleneck Link QoS Networks when α_j is Fixed and when α_j Varies for Different Parameter Settings

Variable	Model 1 (α_j is Fixed)			Model 2 (α_j Varies)		
	Original	Modified (β fixed)	Modified (β vary)	Original	Modified (β fixed)	Modified (β vary)
α_1(\$/bps)	0.2fixed	0.2fixed	0.2fixed	0.3	0.3	0.288
α_2(\$/bps)	0.3fixed	0.3fixed	0.3fixed	0.3	0.3	0.3
β_1(\$)	-	0.01fixed	0.04	-	0.01fixed	0.04
β_2(\$)	-	0.02fixed	0.03	-	0.02fixed	0.03
I_1	-	0.9	0.9	-	0.9	0.9
I_2	-	0.8	0.8	-	0.8	0.8
W_1(bps)	5	5	5	5	5	5
W_2(bps)	5	5	5	5	5	5
$\widetilde{X}_{11}$(bps)	25	25	25	25.5	25.5	25.5
$\widetilde{X}_{21}$(bps)	25	25	25	24.5	24.5	24.5
$\widetilde{X}_{12}$(bps)	25	25	25	25.5	25.5	25.5
$\widetilde{X}_{22}$(bps)	25	25	25	24.5	24.5	24.5
L_{m1}(bps)	0.01	0.01	0.01	0.01	0.01	0.01
L_{m2}(bps)	0.01	0.01	0.01	0.01	0.01	0.01
Z_{11}	1	1	1	1	1	1
Z_{12}	0	0	0	0	0	0
Z_{21}	1	1	1	1	1	1
Z_{22}	0	0	0	0	0	0
$\widetilde{W}_{11}$	5	5	5	5	5	5
$\widetilde{W}_{12}$	5	5	5	5	5	5
$\widetilde{W}_{21}$	4	4	4	4	4	4
$\widetilde{W}_{22}$	4	4	4	4	4	4
X_1(bps)	25	25	25	25.5	25.5	25.5
X_2(bps)	25	25	25	25.5	25.5	25.5

5.4 The New Modified Model for Internet Pricing Schemes in Multiple Bottleneck Link Multiple QoS Networks

The idea basically generates from Yang (2004) and Byun and Chatterjee (2004) for single QoS networks. Assume that the routing schemes are already set up by the ISP. As Yang (2004) pointed out, there exist 2 parts of utility function namely, base price, which does not depend on resource consumption and cost, which depends on resource consumption. The utility function has characteristics such as marginal profit as a function of decreasing with increasing bandwidth. The objective of ISPs is to obtain maximum profit subject to constraints based on the system's available resources.

5.4.1 Model Formulation of A New modified model for Internet pricing Schemes in Multiple Bottleneck Link QoS Networks

Original Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks α_j is Fixed

The original model formulation for model 1 original in multi link multi QoS network is as follows.

Max objective function (29) subject to Eq(30) to Eq (41).

where

Parameters:

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network.

α_j : Base price of a network service j, since the internet's level of service quality cannot guaranteed so α could be equivalent to the price of the best-effort service in the current internet architecture.

$\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l.

$\widetilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

Table 29 explains the parameter values for original Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j is fixed.

TABLE 29: Parameter Values for Original Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j to be fixed

Link l=1		
Class j	j=1	j=2
C_l	100	
α_j	0.2	0.3
$\tilde{L}_{ij}^l$	$\tilde{L}_{1j}^1$=5, $\tilde{L}_{2j}^1$=6	$\tilde{L}_{1j}^1$=4, $\tilde{L}_{2j}^1$=7
$\widetilde{W}_{ij}^l$	$\widetilde{W}_{1j}^1$=8, $\widetilde{W}_{2j}^1$=7	$\widetilde{W}_{1j}^1$=9, $\widetilde{W}_{2j}^1$=10
Link l=2		
C_l	150	
α_j	0.2	0.3
$\tilde{L}_{ij}^l$	$\tilde{L}_{1j}^2$=5, $\tilde{L}_{2j}^2$=6	$\tilde{L}_{1j}^2$=4, $\tilde{L}_{2j}^2$=7
$\widetilde{W}_{ij}^l$	$\widetilde{W}_{1j}^2$=8, $\widetilde{W}_{2j}^2$=7	$\widetilde{W}_{1j}^2$=9, $\widetilde{W}_{2j}^2$=10

Decision variables are as follows.

$\tilde{X}_{ij}^l$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user i in class j.

$Z_{ij} = \begin{cases} 1, & i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class j.

L_j : Bandwidth for class j.

X_j : Bandwidth assigned to each individual user in class j.

The Improved Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks when α_j and β_j are Fixed

In this improved model formulation, we obtain the mathematical programming as follows.

$$\text{Max} \sum_{j=1}^{m} \sum_{i=1}^{n} ((\alpha_j Z_{ij} + \beta_j I_j) + W_j \log \frac{\tilde{X}_{ij}}{L_j}) \tag{81}$$

Subject to constraints (30)-(41) and an additional proposed constraint as follows:

$$0 \le I_j \le d; d \in [0,1] \tag{82}$$

where

Parameters when α_j and β_j are fixed

- C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network
- α_j : Base price of a network service j
- $\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l
- $\widetilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.
- β_j : Quality premium of service class j that has I_j service performance.
- d_j : Upper bound value of quality index for class j

Table 30 explains the parameter values for original Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j and β_j are fixed.

TABLE 30: Parameter Values for The New Improved Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j and β_j are fixed

Link l=1		
Class j	j=1	j=2
C_l	100	
α_j	0.2	0.3
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{1}$=5, $\tilde{L}_{2j}^{1}$=6	$\tilde{L}_{1j}^{1}$=4, $\tilde{L}_{2j}^{1}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{1}$=8, $\widetilde{W}_{2j}^{1}$=7	$\widetilde{W}_{1j}^{1}$=9, $\widetilde{W}_{2j}^{1}$=10
β_j	0.01	0.08
d_j	0.9	0.8
Link l=2		
C_l	150	
α_j	0.2	0.3
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{2}$=5, $\tilde{L}_{2j}^{2}$=6	$\tilde{L}_{1j}^{2}$=4, $\tilde{L}_{2j}^{2}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{2}$=8, $\widetilde{W}_{2j}^{2}$=7	$\widetilde{W}_{1j}^{2}$=9, $\widetilde{W}_{2j}^{2}$=10
β_j	0.01	0.08

Decision variables when α_j and β_j fixed

- $\tilde{X}_{ij}^{l}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$	:	Final bandwidth obtained by user i in class j.
Z_{ij}	=	$\begin{cases} 1, \text{if user } i \text{ is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$
W_j	:	Price sensitivity for class j.
L_j	:	Bandwidth for class j.
X_j	:	Bandwidth assigned to each individual user in class j.
I_j	:	Quality index of class j.

In objective function (81), ISPs want to maximize profits by maximizing the total sum of base price for bps of jth class and utility function as a function of decreasing and increasing bandwidth, which shows the user satisfaction when receiving a bandwidth $\hat{X}_{ij}$. Since the base price is already set up by ISPs then α_j should be fixed. ISPs want to maximize profit by maximizing the total sum of base price for bps of jth class and utility function as a function of decreasing and increasing bandwidth, which shows the user satisfaction when receiving a bandwidth $\hat{X}_{ij}$. Since the base price is already set up by ISPs, at least to avoid ISPs from making losses, thus ISPs have fixed α_j but ISPs have the choice to fix quality premium β_j or to make β_j as a variable. Constraint (30) explains that total bandwidth obtained by user i in each link cannot exceed the bandwidth available in each link. Constraint (31) tell us that bandwidth obtained by user i in class j on link l must equal to the final bandwidth obtained by user i in class j. Constraint (32) shows us that the final bandwidth obtained by user i in class j must exceed the minimum bandwidth for user i in class j on link l, if user i is admitted to class j. Final bandwidth obtained by user i in class j should exceed 1 if user i is admitted to class j and 0, otherwise, as stated in Constraint (33). Constraint (34) states that price sensitivity for class j cannot exceed price sensitivity for user i in class j on link l, if user i is admitted to class j.

Minimum bandwidth for class j should exceed the minimum bandwidth for user i in class j on link l, as stated in Constraint (35). Constraint (36) tells us that final bandwidth obtained by user i in class j should exceed the bandwidth assigned to each individual user in class j, if user i is admitted to class j. Constraint (37) informs us that the final bandwidth obtained by user i in class j cannot exceed the bandwidth assigned to each individual user in class j. Constraint (38), (39) and (40) are non-negativity constraints where final bandwidth obtained by user i in class j, minimum bandwidth for class j and price sensitivity for class j should be non-negative value. Constraint (41) states that admission decision for user i in class j, if 0 than it means that user i is admitted to class j, or else 0. Constraint (82) shows that the range of index quality should be $[0, d]$, $d\in[0,1]$ and this range is set up by ISPs.

The Improved Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks when α_j is Fixed and β_j varies

The new improved model formulation when α_j is fixed and β_j varies in internet pricing scheme of multi bottleneck link multi QoS network is as follows:

Maximum objective function (81) but subject to constraints (30)-(41), (82) and additional proposed constraints as follows:

$$\alpha_j + \beta_j I_j \geq \alpha_{j-1}\beta_{j-1}I_{j-1}, j > 1 \quad (83)$$

$$\beta_j \leq \beta_{j-1}, j > 1 \quad (84)$$

$$f \leq \beta_i \leq g, j = 1,\ldots,m; [f, g] \in [0,1] \quad (85)$$

where

Parameters when α_j is fixed and β_j varies

- C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network
- α_j : Base price of a network service j
- $\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l
- $\tilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.
- d_j : Upper bound value of quality index for class j
- f_j : Lower bound value of quality premium for class j
- g_j : Upper bound value of quality premium for class j

Table 31 explains the parameter values for new improved Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j is fixed and β_j varies.

TABLE 31: Parameter Values for The New Improved Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j is Fixed and β_j Varies

Link *l*=1		
Class *j*	***j*=1**	***j*=2**
C_l	100	
α_j	0.2	0.3
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{1}$=5, $\tilde{L}_{2j}^{1}$=6	$\tilde{L}_{1j}^{1}$=4, $\tilde{L}_{2j}^{1}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{1}$=8, $\widetilde{W}_{2j}^{1}$=7	$\widetilde{W}_{1j}^{1}$=9, $\widetilde{W}_{2j}^{1}$=10
β_j	0.01	0.08
d_j	0.9	0.8
f_j	0.01	0.02
g_j	0.04	0.03
Link *l*=2		
C_l	150	
α_j	0.2	0.3
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{2}$=5, $\tilde{L}_{2j}^{2}$=6	$\tilde{L}_{1j}^{2}$=4, $\tilde{L}_{2j}^{2}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{2}$=8, $\widetilde{W}_{2j}^{2}$=7	$\widetilde{W}_{1j}^{2}$=9, $\widetilde{W}_{2j}^{2}$=10
β_j	0.01	0.08
f_j	0.01	0.02
g_j	0.04	0.03

Decision variables when α_j is fixed and β_j varies.

$\tilde{X}_{ij}^{l}$: Bandwidth obtained by user *i* in class *j* on link *l*.

$\hat{X}_{ij}$: Final bandwidth obtained by user *i* in class *j*.

$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class *j*.

L_j : Bandwidth for class *j*.

X_j : Bandwidth assigned to each individual user in class *j*.

I_j : Quality index of class *j*.

β_j : Quality premium for class *j*

Constraint (83) shows that summation of base price and quality premium in class *j* should exceed the summation of base price and quality premium in previous classes (or in *j*-1). Constraint (84) shows that quality premium in class *j* should not exceed the quality premium in previous classes. Constraint (85) shows that the range of quality premium lies in $[f, g] \in [0, 1]$ and both are predetermined value set up by ISPs. Constraint (43) shows that base price for class j should exceed the base price for previous classes. Constraint (44) shows that the range of base price lies between $[a, b] \in [0, 1]$ where a and b are predetermined values set up by ISPs. Objective function (86) tells us that ISPs want to maximize profits by maximizing the total sum of varying base price for bps of jth class and utility function as a function of decreasing and increasing bandwidth, which shows the user satisfaction when receiving a bandwidth $\hat{X}_{ij}$. ISPs will vary the base price α_j to see whether in certain classes ISPs are able to meet market competition and ISPs have the choice to fix quality premium β_j with the purpose of enabling users to choose the class based on their budget or to make β_j as a variable with the objective to promote certain services in the network.

Original Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks when Setting α_j Varies

Mathematical formulation for the model that considers the base price as a variable is as follows.

Max objective function (42). Subject to constraint (30)-(41) and additional constraints (43) to constraint (44)

where

The parameters and the decision variables are as follows.

Parameters:

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network.

$\tilde{L}^l_{ij}$: Minimum bandwidth for user i in class j on link l.

$\tilde{W}^l_{ij}$: Price sensitivity for user i, which shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

a_j : Lower bound value for base price in class j

b_j : Upper bound value for base price in class j

Table 32 explains the parameter values for original Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j varies.

TABLE 32: Parameter Values for Original Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j Varies

Link l=1		
Class j	j=1	j=2
C_l	100	
$\tilde{L}^l_{ij}$	$\tilde{L}^1_{1j}$=5, $\tilde{L}^1_{2j}$=6	$\tilde{L}^1_{1j}$=4, $\tilde{L}^1_{2j}$=7
$\tilde{W}^l_{ij}$	$\tilde{W}^1_{1j}$=8, $\tilde{W}^1_{2j}$=7	$\tilde{W}^1_{1j}$=9, $\tilde{W}^1_{2j}$=10
a_j	0.1	0.2
b_j	0.4	0.3
Link l=2		
C_l	150	
$\tilde{L}^l_{ij}$	$\tilde{L}^2_{1j}$=5, $\tilde{L}^2_{2j}$=6	$\tilde{L}^2_{1j}$=4, $\tilde{L}^2_{2j}$=7
$\tilde{W}^l_{ij}$	$\tilde{W}^2_{1j}$=8, $\tilde{W}^2_{2j}$=7	$\tilde{W}^2_{1j}$=9, $\tilde{W}^2_{2j}$=10
a_j	0.1	0.2
b_j	0.4	0.3

Decision variables:

$\tilde{X}^l_{ij}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user i in class j.

$Z_{ij} = \begin{cases} 1, & i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class j.

L_j : Bandwidth for class j.

X_j : Bandwidth assigned to each individual user in class j.

The Improved Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks when Setting α_j Varies and β_j is fixed

In this improved model formulation, adopt the objective function (86) then the mathematical programming problem will be as follows.

$$\text{Max} \sum_{j=1}^{m}\sum_{i=1}^{n}((\alpha_j + \beta_j I_j) + W_j \log \frac{\tilde{X}_{ij}}{L_j}) Z_{ij} \tag{86}$$

Subject to constraints (30)-(41) and (43)-(44).

where

Parameters when α_j vary and β_j is fixed

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network

$\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l

$\widetilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

β_j : Quality premium of service class j that has I_j service performance.

d_j : Upper bound value of quality index for class j

a_j : Lower bound value of base price for class j

b_j : Upper bound value of base price for class j

Table 33 explains the parameter values for the improved Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j varies and β_j is fixed.

Decision variables when α_j vary and β_j is fixed

$\tilde{X}_{ij}^{l}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user i in class j.

$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class j.

L_j : Bandwidth for class j.

X_j : Bandwidth assigned to each individual user in class j.

I_j : Quality index of class j.

α_j : Base price for class j

TABLE 33: Parameter Values for Original Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j Varies and β_j is fixed

Link l=1		
Class j	**j=1**	**j=2**
C_l	100	
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{1}$=5, $\tilde{L}_{2j}^{1}$=6	$\tilde{L}_{1j}^{1}$=4, $\tilde{L}_{2j}^{1}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{1}$=8, $\widetilde{W}_{2j}^{1}$=7	$\widetilde{W}_{1j}^{1}$=9, $\widetilde{W}_{2j}^{1}$=10
β_j	0.01	0.02
d_j	0.9	0.8
a_j	0.1	0.2
b_j	0.4	0.3
Link l=2		
C_l	150	
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{2}$=5, $\tilde{L}_{2j}^{2}$=6	$\tilde{L}_{1j}^{2}$=4, $\tilde{L}_{2j}^{2}$=7
$\widetilde{W}_{ij}^{l}$	$\widetilde{W}_{1j}^{2}$=8, $\widetilde{W}_{2j}^{2}$=7	$\widetilde{W}_{1j}^{2}$=9, $\widetilde{W}_{2j}^{2}$=10
β_j	0.01	0.02
d_j	0.9	0.8
a_j	0.1	0.2
b_j	0.4	0.3

The Improved Model Formulation for Internet pricing Schemes in Multiple Bottleneck Link Multi QoS Networks when Setting α_j and β_j Vary

In this improved model formulation, adopt the objective function (86) then the mathematical programming problem will be as follows.

Maximum objective function (86) subject to constraints (30) to constraint (41), constraint (43) to constraint (44) and constraints (82) to constraint (85).

where

Parameters when α_j and β_j vary

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network

$\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l

$\widetilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

d_j : Upper bound value of quality index for class j

f_j : Lower bound value of quality premium for class j

g_j : Upper bound value of quality premium for class j

a_j : Lower bound value of base price for class j

b_j : Upper bound value of base price for class j

Table 34 explains the parameter values for the improved Model of Internet Pricing Scheme in multi bottleneck link QoS network when setting up α_j and β_j vary.

TABLE 34: Parameter Values for Original Model of Internet Pricing Scheme in Multi Bottleneck Link QoS Network when Setting up α_j and β_j Vary

Link l=1		
Class j	j=1	j=2
C_l	100	
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{1}$=5, $\tilde{L}_{2j}^{1}$=6	$\tilde{L}_{1j}^{1}$=4, $\tilde{L}_{2j}^{1}$=7
$\tilde{W}_{ij}^{l}$	$\tilde{W}_{1j}^{1}$=8, $\tilde{W}_{2j}^{1}$=7	$\tilde{W}_{1j}^{1}$=9, $\tilde{W}_{2j}^{1}$=10
β_j	0.01	0.02
d_j	0.9	0.8
a_j	0.1	0.2
b_j	0.4	0.3
Link l=2		
C_l	150	
$\tilde{L}_{ij}^{l}$	$\tilde{L}_{1j}^{2}$=5, $\tilde{L}_{2j}^{2}$=6	$\tilde{L}_{1j}^{2}$=4, $\tilde{L}_{2j}^{2}$=7
$\tilde{W}_{ij}^{l}$	$\tilde{W}_{1j}^{2}$=8, $\tilde{W}_{2j}^{2}$=7	$\tilde{W}_{1j}^{2}$=9, $\tilde{W}_{2j}^{2}$=10
β_j	0.01	0.02
d_j	0.9	0.8
a_j	0.1	0.2
b_j	0.4	0.3

Decision variables when α_j and β_j vary

$\tilde{X}_{ij}^{l}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user i in class j.

$$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$$

W_j : Price sensitivity for class j.

L_j : Bandwidth for class j.

X_j : Bandwidth assigned to each individual user in class j.

I_j : Quality index of class j.

α_j : Base price for class j

β_j : Quality premium for class j

5.4.2 Optimal Solution of The New Modified Model of Internet Pricing Scheme in Multiple Bottleneck Link QoS Networks when Setting up α_j to be fixed and when Setting up α_j to varies

Table 35 shows the solver status of model 1. Model class for each model is MINLP, with status of current solutions being all locally optimal. Infeasibility shows us the amount of constraints violated and it is 0 for each model. The solver type used is the Branch and Bound solver and the active field lists 2 sub-problems remaining to be evaluated for each model. The solver must run until that number reaches 0. GMU shows us the amount of LINGO's memory allocation currently being used by the model generator. The highest amount of memory allocation is in Model 1 modified (β vary) which is 37K. ER explains about total time used to generate and solve the model maybe affected by the number of other applications running in our system.

For the improved models, the solver only needs 0 sec time to generate and solve those models. The best objective shows us that the objective value of the best solution in each model which is modified (β vary) that obtained the highest value of 46.67. The theoretical bound on the objective is also the same value as the best objective for each model. ESS depends on the solver that is running. Since all models have solver type Branch and Bound so ESS=0 shows us that there are 0 branches in the branch and bound tree. Lastly, Total solver iterations show the number of solver iterations for each model. In Model 1 Original there are 30 iterations to solve the model.

TABLE 35: Solver Status for Internet Pricing Schemes in Multiple Bottleneck Link Multiple Class QoS Network when Setting up α_j to be Fixed

Solver status	**Original**	**Modified (β fixed)**	**Modified (β vary)**
Model Class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Infeasibility	0	0	0
Extended Solver state			
Solver type	B & B	B & B	B & B
Active	0	0	0
Update interval	2	2	2
GMU(K)	34	36	37
ER(sec)	0	0	0
Best Objective	46.55	46.6	46.67
Objective bound	46.55	46.6	46.67
ESS	0	0	0
TSI	30	30	32

Table 36 shows the solver status of Model 2. The model class for each model falls under the MINLP, with the status of current solutions being all locally optimal. Infeasibility is 0 for each model. The solver type used is the Branch and Bound solver and the active field lists 2 sub-problems remaining to be evaluated for each model until that number reaches 0. The highest GMU for memory allocation is in Model 2 modified (β fixed) and Model 2 modified (β vary) which is 37K. ER for each model is 0 sec time to generate and solve those models. The best objective is Model 2 modified (β vary) that obtained the highest value of 0.97. The theoretical bound on the objective is also the same value as the best objective for each model. ESS=0 shows us that there is 0 of branches in the branch and bound tree. Lastly, for total solver iterations in Model 2 original, there are 5 iterations in solving the model.

TABLE 36: Solver Status for Internet Pricing Schemes in Multiple Bottleneck Link Multiple Class QoS Network when Setting up α_j Varies

Solver status	**Original**	**Modified (β fixed)**	**Modified (β vary)**
Model Class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Infeasibility	0	0	0.5×10^{-16}
Extended Solver state			
Solver type	B & B	B & B	B & B
Active	0	0	0
Update interval	2	2	2
GMU(K)	35	37	37
ER(sec)	0	0	0
Best Objective	0.9	0.93	0.97
Objective bound	0.9	0.93	0.97
ESS	0	0	0
TSI	5	5	5

Table 37 shows us that for 3 cases of model 1, only user 1 and user 2 are admitted to class 1 and class 2, respectively. Bandwidth obtained by user 1 of class 1 on link 1 and on link 2 ($\tilde{X}_{11}^1$ and $\tilde{X}_{11}^2$) is 22.2 bps. Since user 1 is admitted to class 1, then price sensitivity for class 1 equals to price sensitivity of user 1 in class 1 on link 1 and user 1 in class 1 on link 2 which is $W_1 = \tilde{W}_{11}^1 = \tilde{W}_{11}^2 = 8$. Thus, user 2 is admitted to class 2 since the price sensitivity of class 2 equals to price sensitivity of user 2 in class 2 on link 1 and user 2 in class 2 on link 2 which is $W_2 = \tilde{W}_{22}^1 = \tilde{W}_{22}^2 = 10$ with $\tilde{X}_{22}^1 = \tilde{X}_{22}^2 = 27.8$ bps. The final bandwidth obtained by user 1 in class 1 ($\hat{X}_{11}$) and user 2 in class 2 ($\hat{X}_{22}$) are 22.2 bps and 27.8 bps, respectively. Minimum bandwidth for class 1 and class 2 (L_1 and L_2) are 7 bps. Lastly, bandwidth assigned to each individual user in class 1(X_1) is 22.2 bps and in class 2(X_2) is 27.8 bps. So, $X_1 = \tilde{X}_{11}^1 = \tilde{X}_{11}^2 = 22.2$ bps and $X_2 = \tilde{X}_{22}^1 = \tilde{X}_{22}^2 = 27.8$ bps.

The quality index of each class is obtained which is $I_1=0.9$, and $I_2=0.8$. The same quality index result was also found with added results of quality premium in class 1 (β_1) of \$0.04 and in class 2 (β_2) of \$0.03. From the models it can be seen that model 1 modified (β vary) yields slightly better results of \$46.67. So if ISPs apply the option to recover cost (by fixing α) and also to promote certain services in the networks (by varying β), ISPs will gain more profit.

TABLE 37: Solution for Internet Pricing Schemes in Multiple Bottleneck Link Multiple Class QoS Network when Setting up α_j to be Fixed

	Original	**Modified (β fixed)**	**Modified (β vary)**
α_1	0.2	0.2	0.2
Z_{11}	1	1	1
W_1	8	8	8
$\hat{X}_{11}$	22.2	22.2	22.2
L_1	7	7	7
Z_{21}	0	0	0
$\hat{X}_{21}$	22.2	22.2	22.2
α_2	0.3 fixed	0.3 fixed	0.3 fixed
Z_{12}	0	0	0
W_2	10	10	10
$\hat{X}_{12}$	27.8	27.8	27.8
L_2	7	7	7
Z_{22}	1	1	1
$\hat{X}_{22}$	27.8	27.8	27.8
$\tilde{X}^{1}_{11}$	22.2	22.2	22.2
$\tilde{X}^{1}_{21}$	22.2	22.2	22.2
$\tilde{X}^{1}_{12}$	27.8	27.8	27.8
$\tilde{X}^{1}_{22}$	27.8	27.8	27.8
$\tilde{X}^{2}_{11}$	22.2	22.2	22.2
$\tilde{X}^{2}_{21}$	22.2	22.2	22.2
$\tilde{X}^{1}_{11}$	27.8	27.8	27.8
$\tilde{X}^{2}_{22}$	27.8	27.8	27.8
X_1	22.2	22.2	22.2
X_2	27.8	27.8	27.8
β_1	-	0.01	0.04
β_2	-	0.02	0.03
I_1	-	0.9	0.9
I_2	-	0.8	0.8

The results in Table 38 illustrates that for 3 cases of model 2, user 1 and user 2 are admitted in class 1, user 2 is admitted to class 2 with bandwidth obtained by user 1 in class 1 on link 1 ($\tilde{X}_{11}^{1}$ and $\tilde{X}_{11}^{2}$) is 6 bps, $\tilde{X}_{21}^{1}$= $\tilde{X}_{21}^{2}$=6 bps and $\tilde{X}_{22}^{1}$ = $\tilde{X}_{22}^{2}$ = 7 bps. So IT can be se that the bandwidth assigned to each individual user in class 1 equals to the final bandwidth obtained by user 1 and user 2 in class 1 and equals to the bandwidth obtained by user 1 and 2 in class 1 on link 1 and 2; or $X_1 = \hat{X}_{11} = \hat{X}_{21} = \tilde{X}_{11}^{1} = \tilde{X}_{11}^{2} = \tilde{X}_{21}^{1} = \tilde{X}_{21}^{2} = 6$ bps and $X_2 = \hat{X}_{22} = \tilde{X}_{22}^{2} = 7$ bps . Minimum bandwidth for class 1 and class 2 (L_1 and L_2) are 6 bps and 7 bps respectively. Next, the price sensitivity for class 1 and class 2 (W_1 and W_2) are 0. Since ISPs would choose to be able to compete in market competition, ISPs vary the base price in class 1 and class 2 of \$0.3/bps for all models 2.

In model 2 modified (β fixed) and model 2 modified (β vary), the quality index can be obtained to show that QoS level I_1=0.9 and I_2 = 0.8. In addition, the premium quality in class 1 and class 2 (β_1 and β_2) are of \$0.04 and \$0.03, respectively. From the models, observe that modified (β vary) model of model 2 formulation yields slightly better results of \$0.97. So, if ISPs would like to have market competition (by varying α) and also to promote certain services in the networks (by varying β), ISPs are able to gain more profit.

TABLE 38: Solution for Internet Pricing Schemes in Multiple Bottleneck Link Multiple Class QoS Network when Setting up α_j Varies

	Original	Modified (β fixed)	Modified (β vary)
α_1	0.3	0.3	0.29
Z_{11}	1	1	1
W_1	0	0	0
$\hat{X}_{11}$	6	6	6
L_1	6	6	6
Z_{21}	1	1	1
$\hat{X}_{21}$	6	6	6
α_2	0.3	0.3	0.3
Z_{12}	0	0	0
W_2	0	0	0
$\hat{X}_{12}$	7	7	7
L_2	7	7	7
Z_{22}	1	1	1
$\hat{X}_{22}$	7	7	7
$\tilde{X}^1_{11}$	6	6	6
$\tilde{X}^1_{21}$	6	6	6
$\tilde{X}^1_{12}$	7	7	7
$\tilde{X}^1_{22}$	7	7	7
$\tilde{X}^2_{11}$	6	6	6
$\tilde{X}^2_{21}$	6	6	6
$\tilde{X}^1_{11}$	7	7	7
$\tilde{X}^2_{22}$	7	7	7
X_1	6	6	6
X_2	7	7	7
β_1	-	0.01	0.04
β_2	-	0.02	0.03
I_1	-	0.9	0.9
I_2	-	0.8	0.8

5.4.3 Analysis of Pricing Schemes in Multiple Bottleneck Link QoS Networks

The model shows the connection between bandwidth required, bandwidth obtained and QoS level by giving the assumptions and data; we can find the optimal solution with profit maximization. ISPs have the choice to either adopt modified model 1 or modified model 2, according their priorities. If ISPs choose to recover cost while promoting certain services, then model 1 modified (β fixed) will be the best model to adopt. Again, if ISPs would like to have market competition when there is a chance to do that while promoting certain services then model 2 modified (β vary) will be a good model to apply. Overall, our proposed models show slightly better results than previous research models.

5.5 Summary

Chapter V explained the internet pricing scheme in a multiple class QoS networks. The model formulations began from schemes in single bottleneck link network and extended to model schemes in multiple bottleneck link networks. The solutions show that the improved models exhibited better results compared to original results proposed by past research. The results also have contributions, especially to ISPs, in order to achieve its goal of maximizing revenue. Further research should address issues on generalization of users and classes applying each model so that a more realistic network situation can be described.