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APPENDICES

APPENDIX A: Internet Pricing Scheme solved by LINGO 11.0

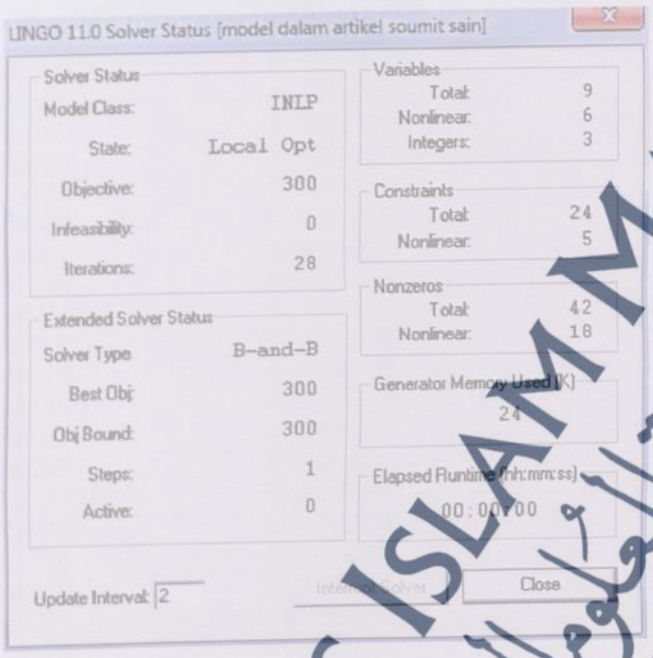
APPENDIX.A.1: Sain & Herper's Model (2003) Solved by LINGO 11.0

a. MINLP Model

```
max=p1*q1*x1+p2*q2*x2+p3*q3*x3;

d1*q1*x1<=C*a1;
d2*q2*x2<=C*a2;
d3*q3*x3<=C*a3;

d1*q1*x1+d2*q2*x2+d3*q3*x3<=C;

a1+a2+a3=1;

a1<=1;
a1>=0;
a2<=1;
a2>=0;
a3<=1;
a3>=0;

q1<=1;
q1>=m1;
q2<=1;
q2>=m2;
q3<=1;
q3>=m3;

x1<=n1;
x1>=0;
x2<=n2;
x2>=0;
x3<=n3;
x3>=0;

p1=3;
p2=45;
p3=15;

d1=60;
d2=750;
d3=330;

C=5000;

m1=0.8;
m2=0.6;
m3=0.5;

n1=10;
n2=10;
n3=10;

@GIN(x1);
```

```
@GIN(x2);
@GIN(x3);

end
```

b. Solver Status



c. Solution Report

```
Local optimal solution found.
  Objective value:                              300.0000
  Objective bound:                              300.0000
  Infeasibilities:                              0.000000
  Extended solver steps:                               1
  Total solver iterations:                            28


Variable           Value        Reduced Cost
P1                 3.000000     0.000000
Q1                 0.8000000    0.000000
X1                 0.000000     0.000000
P2                 45.00000     0.000000
Q2                 0.8333333    0.000000
X2                 8.000000     0.000000
P3                 15.00000     0.000000
Q3                 0.5000000    0.000000
X3                 0.000000     0.000000
D1                 60.00000     0.000000
C                  5000.000     0.000000
A1                 0.000000     0.000000
D2                 750.0000     0.000000
A2                 1.000000     0.000000
D3                 330.0000     0.000000
```

A3	0.000000	0.000000
M1	0.8000000	0.000000
M2	0.8000000	0.000000
M3	0.5000000	0.000000
N1	10.00000	0.000000
N2	10.00000	0.000000
N3	10.00000	0.000000

Row	Slack or Surplus	Dual Price
1	300.0000	1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.6000000E-01
6	0.000000	0.000000
7	1.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	1.000000	0.000000
11	1.000000	0.000000
12	0.000000	0.000000
13	0.2000000	0.000000
14	0.000000	0.000000
15	0.1666667	0.000000
16	0.3333333E-01	0.000000
17	0.5000000	0.000000
18	0.000000	0.000000
19	10.00000	0.000000
20	0.000000	-0.4800000
21	2.000000	0.000000
22	8.000000	0.000000
23	10.00000	0.000000
24	0.000000	-2.400000
25	0.000000	0.000000
26	0.000000	6.666667
27	0.000000	0.000000
28	0.000000	0.000000
29	0.000000	-0.4000000
30	0.000000	0.000000
31	0.000000	0.6000000E-01
32	0.000000	0.000000
33	0.000000	0.000000
34	0.000000	0.000000
35	0.000000	0.000000
36	0.000000	0.000000
37	0.000000	0.000000

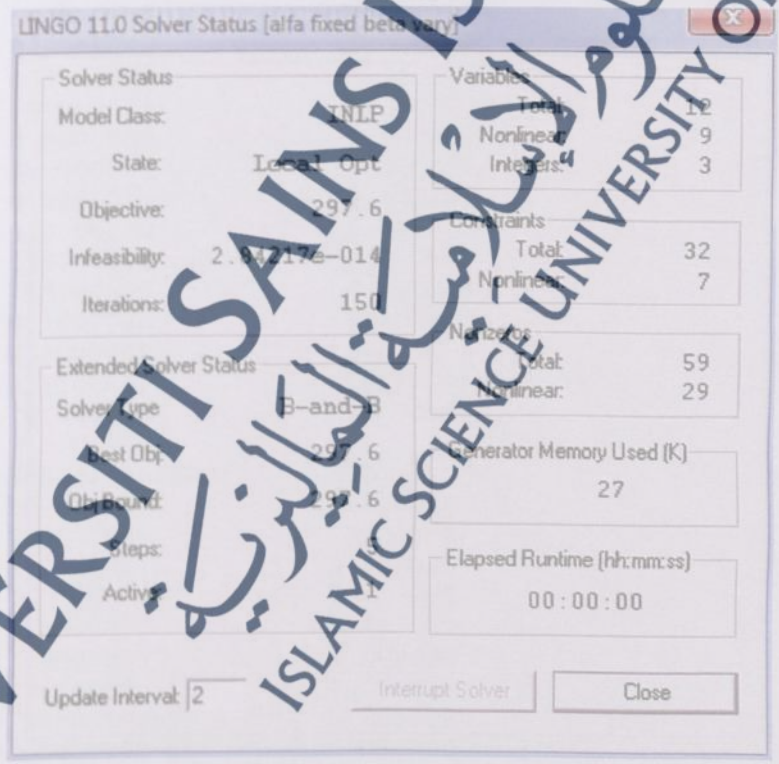
APPENDIX A. 2: A New Improved Model of Internet Pricing Scheme In Single Link Multi Service Networks when α_j is Fixed and β_j Varies Solved by LINGO 11.0

a. MINLP Model

```
!case multiple sain 2nd modification alfa fixed, beta vary;
max=(alf+bet1*I1)*p1*x1 + (alf+bet2*I2)*p2*x2+
(alf+bet3*I3)*p3*x3;


I1*d1*x1<=a1*C;
I2*d2*x2<=a2*C;
I3*d3*x3<=a3*C;

I1*d1*x1+I2*d2*x2+I3*d3*x3<=C;

a1+a2+a3=1;

a1<=1;
a1>=0;
a2<=1;
a2>=0;
a3<=1;
a3>=0;

I1<=1;
I1>=0.8;
I2<=1;
I2>=0.8;
I3<=1;
I3>=0.5;

bet2*I2>=bet1*I1;
bet3*I3>=bet2*I2;

bet1>=0.05;
bet1<=0.8;
bet2>=0.02;
bet2<=0.5;
bet3>=0.01;
bet3<=0.3;

x1<=10;
x1>=0;
x2<=10;
x2>=0;
x3<=10;
x3>=0;

p1=3;
p2=45;
p3=15;

d1=60;
d2=750;
d3=330;
```

```
m1=0.8;
m2=0.8;
m3=0.5;

n1=10;
n2=10;
n3=10;

u1=0.1;
u2=0.2;
u3=0.15;

alf=0.5;
bet=0.4;
C=5000;


@GIN(x1);
@GIN(x2);
@GIN(x3);
end
```

b. Solver Status



c. Solution Report

```
Local optimal solution found.
  Objective value:                              297.6000
  Objective bound:                              297.6000
  Infeasibilities:                          0.2842171E-13
  Extended solver steps:                               5
  Total solver iterations:                           150
```

Variable	Value	Reduced Cost
ALF	0.5000000	0.000000
BET1	0.3750000	0.000000
I1	0.8000000	0.000000
P1	3.000000	0.000000
X1	4.000000	-2.400000
BET2	0.3750000	0.000000
I2	0.8000000	0.000000
P2	45.00000	0.000000
X2	8.000000	-36.00000
BET3	0.3000000	0.000000
I3	1.000000	0.000000
P3	15.00000	0.000000
X3	0.000000	-12.00000
D1	60.00000	0.000000
A1	0.3840000E-01	0.000000
C	5000.000	0.000000
D2	750.0000	0.000000
A2	0.9616000	0.000000
D3	330.0000	0.000000
A3	0.000000	0.000000
M1	0.8000000	0.000000
M2	0.8000000	0.000000
M3	0.5000000	0.000000
N1	10.00000	0.000000
N2	10.00000	0.000000
N3	10.00000	0.000000
U1	0.1000000	0.000000
U2	0.2000000	0.000000
U3	0.1500000	0.000000
BET	0.4000000	0.000000

Row	Slack or Surplus	Dual Price
1	297.6000	1.000000
2	0.000000	0.000000
3	8.000000	0.000000
4	0.000000	0.000000
5	8.000000	0.000000
6	0.000000	0.000000
7	0.9616000	0.000000
8	0.3840000E-01	0.000000
9	0.3840000E-01	0.000000
10	0.9616000	0.000000
11	1.000000	0.000000
12	0.000000	0.000000
13	0.2000000	0.000000
14	0.000000	0.000000
15	0.2000000	0.000000
16	0.000000	0.000000

17	0.000000	111.6000
18	0.5000000	0.000000
19	0.000000	-12.00000
20	0.000000	-372.0000
21	0.3250000	0.000000
22	0.4250000	0.000000
23	0.3550000	0.000000
24	0.1250000	0.000000
25	0.2900000	0.000000
26	0.000000	372.0001
27	6.000000	0.000000
28	4.000000	0.000000
29	2.000000	0.000000
30	8.000000	0.000000
31	10.00000	0.000000
32	0.000000	0.000000
33	0.000000	3.200000
34	0.000000	6.400000
35	0.000000	0.000000
36	0.000000	0.000000
37	0.000000	0.000000
38	0.000000	0.000000
39	0.000000	0.000000
40	0.000000	0.000000
41	0.000000	0.000000
42	0.000000	0.000000
43	0.000000	0.000000
44	0.000000	0.000000
45	0.000000	0.000000
46	0.000000	0.000000
47	0.000000	0.000000
48	0.000000	372.0000
49	0.000000	0.000000
50	0.000000	0.000000

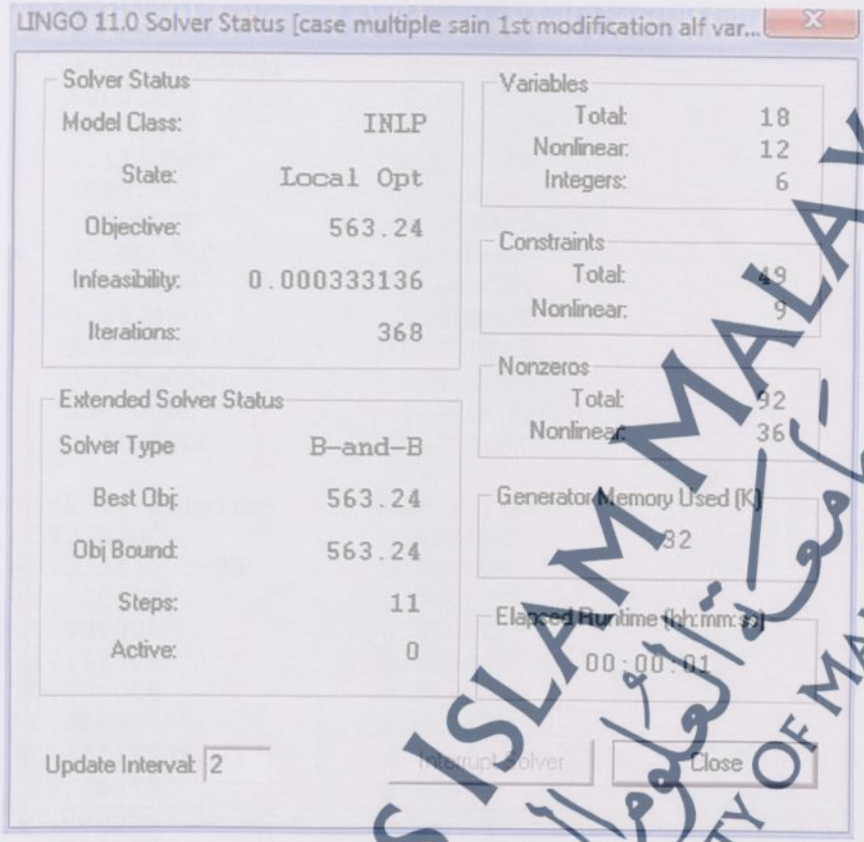
APPENDIX A.3: A New Improved Model of Internet Pricing Scheme in multi link multi service networks when α_j vary and β_j is fixed solved by LINGO 11.0

a. MINLP Model

```
!case multiple sain(modified)alf  vary beta fixed;
max=(alf1+bet*I1)*p11*x11 + (alf2+bet*I2)*p21*x21 +
(alf3+bet*I3)*p31*x31 +(alf1+bet*I1)*p12*x12 +
(alf2+bet*I2)*p22*x22 + (alf3+bet*I3)*p32*x32;

!eqn 2;
I1*d11*x11<=a11*C1;
I1*d12*x12<=a12*C2;
I2*d21*x21<=a21*C1;
I2*d22*x22<=a22*C2;
I3*d31*x31<=a31*C1;
I3*d32*x32<=a32*C2;

!eqn 3;
(I1*d11*x11)+(I2*d21*x21)+(I3*d31*x31)<=C1;
(I1*d12*x12)+(I2*d22*x22)+(I3*d32*x32)<=C2;

!eqn 4;
a11+a21+a31=1;
a12+a22+a32=1;

!eqn 5;
a11<=1;
a11>=0;
a21<=1;
a21>=0;
a31<=1;
a31>=0;

a12<=1;
a12>=0;
a22<=1;
a22>=0;
a32<=1;
a32>=0;

!eqn 6;
I1<=1;
I1>=0.8;
I2<=1;
I2>=0.6;
I3<=1;
I3>=0.6;

!eqn 7;
x11<=10;
x11>=0;
x21<=10;
x21>=0;
x31<=10;
x31>=0;

x12<=10;
```

```
x12>=0;
x22<=10;
x22>=0;
x32<=10;
x32>=0;

p11=3;
p21=45;
p31=15;

p12=6;
p22=56;
p32=24;


d11=60;
d21=750;
d31=330;

d12=60;
d22=750;
d32=330;


bet=0.01;

C1=6000;
C2=4000;

!eqn 8;
@GIN(x11);
@GIN(x21);
@GIN(x31);
@GIN(x12);
@GIN(x22);
@GIN(x32);


!eqn 10;
alf2+bet*I2>=alf1+bet*I1;
alf3+bet*I3>=alf2+bet*I2;

!eqn 12;
alf1>=0;
alf1<=0.5;
alf2>=0;
alf2<=0.7;
alf3>=0;
alf3<=0.6;

end
```

b. Solver Status



c. Solution Window

```
Local optimal solution found.
  Objective value:                              563.2400
  Objective bound:                              563.2400
  Infeasibilities:                          0.3331359E-03
  Extended solver steps:                               11
  Total solver iterations:                            368


Variable           Value        Reduced Cost
ALF1           0.5000000            0.000000
BET            0.1000000E-01        0.000000
I1             6.8833352            0.000000
P11            3.000000             0.000000
X11            3.000000            -1.333334
ALF2           0.5970000            0.000000
I2             0.8000000            0.000000
P21            45.00000             0.000000
X21            7.000000           -24.92500
ALF3           0.6000000            0.000000
I3             0.5000000            0.000000
P31            15.00000             0.000000
X31            10.00000           0.000000
P12            6.000000           0.000000
X12            10.00000           0.000000
P22            56.00000           0.000000
```

X22	3.000000	-33.88000
P32	24.00000	0.000000
X32	10.00000	0.000000
D11	60.00000	0.000000
A11	0.2500000E-01	0.000000
C1	6000.000	0.000000
D12	60.00000	0.000000
A12	0.1250003	0.000000
C2	4000.000	0.000000
D21	750.0000	0.000000
A21	0.7000000	0.000000
D22	750.0000	0.000000
A22	0.4624997	0.000000
D31	330.0000	0.000000
A31	0.2750000	0.000000
D32	330.0000	0.000000
A32	0.4125000	0.000000

Row	Slack or Surplus	Dual Price
1	563.2400	1.000000
2	-0.3331359E-03	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	49.99889	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	-0.3331359E-03	0.3833332E-02
9	49.99889	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	0.9750000	0.000000
13	0.2500000E-01	0.000000
14	0.3000000	0.000000
15	0.7000000	0.000000
16	0.7250000	0.000000
17	0.2750000	0.000000
18	0.8749997	0.000000
19	0.1250003	0.000000
20	0.5375003	0.000000
21	0.4624997	0.000000
22	0.5875000	0.000000
23	0.4125000	0.000000
24	0.1666648	0.000000
25	0.3333518E-01	0.000000
26	0.2000000	0.000000
27	0.000000	-26.12500
28	0.5000000	0.000000
29	0.000000	8.919995
30	7.000000	0.000000
31	3.000000	0.000000
32	3.000000	0.000000
33	7.000000	0.000000
34	0.000000	8.442500
35	10.00000	0.000000
36	0.000000	3.050000
37	10.00000	0.000000
38	7.000000	0.000000
39	3.000000	0.000000
40	0.000000	14.52000

41	10.00000	0.000000
42	0.000000	1.525000
43	0.000000	4.235000
44	0.000000	6.050000
45	0.000000	5.083334
46	0.000000	1.815000
47	0.000000	6.050000
48	0.000000	-0.2004470E-01
49	0.000000	-0.3192801E-01
50	0.000000	-0.2962801E-01
51	0.000000	0.000000
52	0.000000	0.000000
53	0.000000	0.000000
54	0.000000	494.0001
55	0.000000	-0.6628021E-02
56	0.000000	0.000000
57	0.9666665E-01	0.000000
58	0.000000	-483.0001
59	0.5000000	0.000000
60	0.000000	69.00000
61	0.5970000	0.000000
62	0.1030000	0.000000
63	0.6000000	0.000000
64	0.000000	873.0001

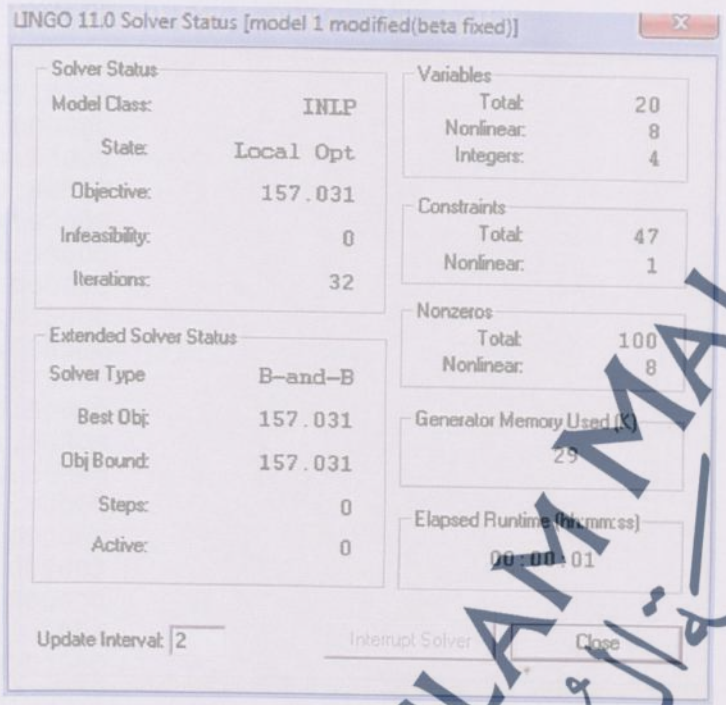
APPENDIX A.4: A New Improved Model of Internet Pricing Scheme in single link multi QoS Networks when α_j and β_j are fixed solving by LINGO 11.0

a. MINLP Model

```
max=((alf1*Z11+bet1*I1)+W1*@log(X11/Lm1))+((alf1*Z21+bet1*I1)+W1*@
log(X21/Lm1))+((alf2*Z12+bet2*I2)+W2*@log(X12/Lm2))+((alf2*Z22+bet
2*I2)+W2*@log(X22/Lm2));

!eqn 2;
X11+X12+X21+X22<=Q;

!eqn 3;
X11>=Lm1-(1-Z11);
X21>=Lm1-(1-Z21);
X12>=Lm2-(1-Z12);
X22>=Lm2-(1-Z22);

!eqn 4;
W1<=W11+(1-Z11);
W1<=W21+(1-Z21);
W2<=W12+(1-Z12);
W2<=W22+(1-Z22);

!eqn 5;
X11>=V1-(1-Z11);
X21>=V2-(1-Z21);
X12>=V1-(1-Z12);
X22>=V2-(1-Z22);

!eqn 6;
X11>=X1-(1-Z11);
X21>=X1-(1-Z21);
X12>=X2-(1-Z12);
X22>=X2-(1-Z22);

!eqn 7;
X11>=Z11;
X21>=Z21;
X12>=Z12;
X22>=Z22;

!eqn 8;
X11>=0;
X21>=0;
X12>=0;
X22>=0;

!eqn 9;
Lm1>=0.01;
Lm2>=0.01;

!eqn 10;
W1>=0;
W2>=0;
```

```
!eqn 11;
X11<=X1;
X21<=X1;
X12<=X2;
X22<=X2;

!eqn 12;
@BIN(Z11);
@BIN(Z21);
@BIN(Z12);
@BIN(Z22);

!eqn 15;
W11>=0;
W12>=0;
W21>=0;
W22>=0;

W11<=5;
W12<=5;
W21<=4;
W22<=4;

alf1=0.2;
alf2=0.3;

bet1=0.01;
bet2=0.02;

Q=100;

V1=6;
V2=5;

!eqn 16;
alf2+bet2*I2>=alf1+bet1*I1;

!eqn 17;
I1<=0.9;
I1>=0;
I2<=0.8;
I2>=0;
End
```

b. Solver Status



c. Solution Report

```
Local optimal solution found.
  Objective value:                              157.0309
  Objective bound:                              157.0309
  Infeasibilities:                              0.000000
  Extended solver steps:                               0
  Total solver iterations:                            32

Variable           Value
ALF1         0.2000000
Z11          1.000000
BET1         0.1000000E-01
I1           0.9000000
W1           5.000000
X11          25.00000
LM1          0.1000000E-01
Z21          0.000000
X21          25.00000
ALF2         0.8000000
Z12          1.000000
BET2         0.2000000E-01
I2           0.8000000
W2           5.000000
X12          25.00000
LM2          0.1000000E-01
Z22          0.000000
X22          25.00000
Q            100.0000
W11          5.000000
W21          4.000000
W12          5.000000
W22          4.000000
V1           6.000000
```

V2	5.000000
X1	25.00000
X2	25.00000

Row	Slack or Surplus
1	157.0309
2	0.000000
3	24.99000
4	25.99000
5	24.99000
6	25.99000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	19.00000
12	21.00000
13	19.00000
14	21.00000
15	0.000000
16	1.000000
17	0.000000
18	1.000000
19	24.00000
20	25.00000
21	24.00000
22	25.00000
23	25.00000
24	25.00000
25	25.00000
26	25.00000
27	0.000000
28	0.000000
29	5.000000
30	5.000000
31	0.000000
32	0.000000
33	0.000000
34	0.000000
35	5.000000
36	5.000000
37	4.000000
38	4.000000
39	0.000000
40	0.000000
41	0.000000
42	0.000000
43	0.000000
44	0.000000
45	0.000000
46	0.000000
47	0.000000
48	0.000000
49	0.000000
50	0.1070000
51	0.000000
52	0.9000000
53	0.000000
54	0.8000000

APPENDIX A.5: A New Improved Model of Internet Pricing Scheme in Multi Link Multi QoS Networks when α_j and β_j Vary Solved by LINGO 11.0

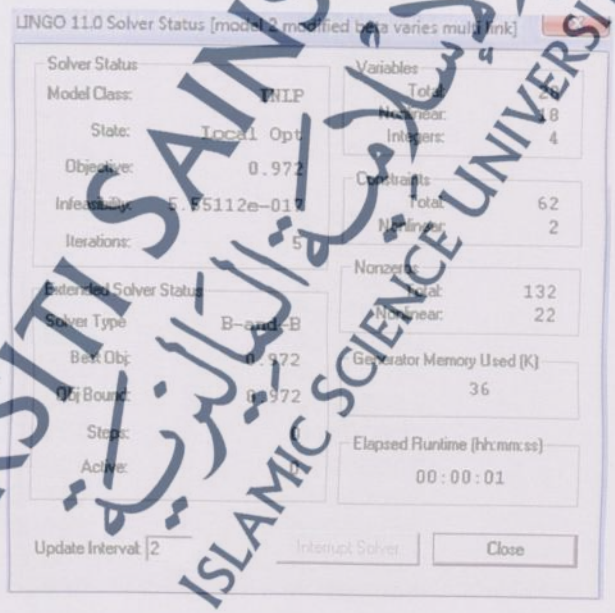
a. MINLP Model

```
!model 2 modified beta varies;
max=((alf1+bet1*I1)+W1*@log(X11/L1))*Z11+((alf1+bet1*I1)+W1*@log(X
21/L1))*Z21+((alf2+bet2*I2)+W2*@log(X12/L2))*Z12+((alf2+bet2*I2)+W
2*@log(X22/L2))*Z22;

!eqn 2;
X111+X211+X121+X221<=C1;
X112+X212+X122+X222<=C2;

C1=100;
C2=150;

!EQN 3;
X111=X11;
X211=X21;
X121=X12;
X221=X22;

X112=X11;
X212=X21;
X122=X12;
X222=X22;

!eqn 4;
X11>=L111-(1-Z11);
X21>=L211-(1-Z21);
X12>=L121-(1-Z12);
X22>=L221-(1-Z22);

X11>=L112-(1-Z11);
X21>=L212-(1-Z21);
X12>=L122-(1-Z12);
X22>=L222-(1-Z22);

!eqn 5;
X11>=Z11;
X21>=Z21;
X12>=Z12;
X22>=Z22;

!EQN 6;
W1<=W111+(1-Z11);
W1<=W211+(1-Z21);
W2<=W121+(1-Z12);
W2<=W222+(1-Z22);

!EQN 7;
L1>=L111+(1-Z11);
L1>=L211+(1-Z21);
L2>=L122+(1-Z12);
L2>=L222+(1-Z22);
```

```
!eqn 8;
X11>=X1-(1-Z11);
X21>=X1-(1-Z21);
X12>=X2-(1-Z12);
X22>=X2-(1-Z22);

!EQN 9;
X11<=X1;
X21<=X1;
X12<=X2;
X22<=X2;

!eqn 10;
X11>=0;
X21>=0;
X12>=0;
X22>=0;

!eqn 11;
L1>=0;
L2>=0;

!eqn 12;
W1>=0;
W2>=0;

!eqn 13;
@BIN(Z11);
@BIN(Z21);
@BIN(Z12);
@BIN(Z22);



L111=5;
L211=6;
L121=4;
L221=7;

L112=5;
L212=6;
L122=4;
L222=7;

W111=8;
W211=7;
W121=9;
W221=10;

W112=8;
W212=7;
W122=9;
W222=10;

!eqn 14;
I1<=0.9;
I1>=0;
```

```
I2<=0.8;
I2>=0;


! alfj+betj*Ij>=alf(j-1)+bet(j-1)*I(j-1) (eqn 15);
alf2+bet2*I2>=alf1+bet1*I1;

!eqn 16;
bet2<=bet1;

!eqn 17;
bet1>=0.01;
bet1<=0.04;
bet2>=0.02;
bet2<=0.03;

!EQN 18;
alf2>=alf1;

!EQN 19;
alf1>=0.1;
alf1<=0.4;
alf2>=0.2;
alf2<=0.3;
```

b. Solver Status



c. Solution Report

```
Local optimal solution found.
  Objective value:                              0.9720000
  Objective bound:                              0.9720000
  Infeasibilities:                              0.5551115E-16
```

Extended solver steps: 0
Total solver iterations: 5

Variable	Value	Reduced Cost
ALF1	0.2880000	0.000000
BET1	0.4000000E-01	0.000000
I1	0.9000000	0.000000
W1	0.000000	0.000000
X11	6.000000	0.000000
L1	6.000000	0.000000
Z11	1.000000	0.000000
X21	6.000000	0.000000
Z21	1.000000	-0.3240000
ALF2	0.3000000	0.000000
BET2	0.3000000E-01	0.000000
I2	0.8000000	0.000000
W2	0.000000	0.000000
X12	7.000000	0.000000
L2	7.000000	0.000000
Z12	0.000000	-0.3240000
X22	7.000000	0.000000
Z22	1.000000	-0.3240000
X111	6.000000	0.000000
X211	6.000000	0.000000
X121	7.000000	0.000000
X221	7.000000	0.000000
C1	100.0000	0.000000
X112	6.000000	0.000000
X212	6.000000	0.000000
X122	7.000000	0.000000
X222	7.000000	0.000000
C2	150.0000	0.000000
L111	5.000000	0.000000
L211	6.000000	0.000000
L121	4.000000	0.000000
L221	7.000000	0.000000
L112	5.000000	0.000000
L212	6.000000	0.000000
L122	4.000000	0.000000
L222	7.000000	0.000000
W111	8.000000	0.000000
W211	7.000000	0.000000
W121	9.000000	0.000000
W222	10.00000	0.000000
X1	6.000000	0.000000
X2	7.000000	0.000000
W221	10.00000	0.000000
W112	8.000000	0.000000
W212	7.000000	0.000000
W122	9.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.9720000	1.000000
2	74.00000	0.000000
3	124.0000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000

7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	1.000000	0.000000
15	0.000000	0.000000
16	4.000000	0.000000
17	0.000000	0.000000
18	1.000000	0.000000
19	0.000000	0.000000
20	4.000000	0.000000
21	0.000000	0.000000
22	5.000000	0.000000
23	5.000000	0.000000
24	7.000000	0.000000
25	6.000000	0.000000
26	8.000000	0.000000
27	7.000000	0.000000
28	10.00000	0.000000
29	10.00000	0.000000
30	1.000000	0.000000
31	0.000000	0.000000
32	2.000000	0.000000
33	0.000000	0.000000
34	0.000000	-0.3240000
35	0.000000	0.000000
36	1.000000	0.000000
37	0.000000	0.000000
38	0.000000	0.3240000
39	0.000000	0.000000
40	0.000000	0.000000
41	0.000000	0.000000
42	6.000000	0.000000
43	6.000000	0.000000
44	7.000000	0.000000
45	7.000000	0.000000
46	6.000000	0.000000
47	7.000000	0.000000
48	0.000000	0.000000
49	0.000000	0.000000
50	0.000000	0.000000
51	0.000000	0.000000
52	0.000000	0.000000
53	0.000000	0.000000
54	0.000000	0.000000
55	0.000000	0.000000
56	0.000000	0.000000
57	0.000000	0.000000
58	0.000000	0.000000
59	0.000000	0.000000
60	0.000000	0.000000
61	0.000000	0.000000
62	0.000000	0.000000
63	0.000000	0.000000
64	0.000000	0.000000
65	0.000000	0.000000

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