CHAPTER II
LITERATURE REVIEW

2.1 Overview

This chapter introduces some reviews of the schemes concerning the economic model of internet pricing. Other researches focus on the continuous pricing scheme by modeling into optimization problem solved numerically or analytically. The basic terminology dealing with QoS is also explained. The model of internet pricing in multi service and multi QoS networks are also described. A brief explanation about LINGO 11.0 is also included.

2.2 Review of Pricing Scheme

Karsten et al. (2000) discussed the charging fee for packet-switched network communications. The packet-switched network framework is formed for multiservice networks. Nowadays, internet is only applicable for best-effort service single class and prices are based on a flat fee. Due to an increasing demand for differentiation in transmission service quality, the requirement for different services is crucial. Past researches mainly showed us that extra pricing occurs when congestion occurs, which eventually causes degradation of the QoS. The three stages proposed by MacKie-Mason et al. (1996) consists of the use of feedback and user adaptation, use of feedback of closed-loop and one kind of variation of closed loop forms.

A scheme named congestion avoidance was also proposed by Jacobson (1988) and scheme of smart market was proposed by Henderson et al. (2001) and Kelly et al. (1998). Ros & Tuffin (2004) described the scheme based on subscription fees as not up-to-date since only incentives are used to support the utilization, hence, if demand exceeds the installed capacity and congestion occurs, the consumers have a tendency to pay more to avoid congestion. However, quality service to clients is a priority. Current internet deals with the type of QoS requirements such as interactive video, experiencing small delays and packet loss but video-on-demand tolerates higher delays and jitters but lower packet loss compared to interactive videos.

Charging methods for packet networks involves resource reservation, priorities between packets, auctions for bandwidth and pricing based on transfer rate. PMP is also discussed here. The network is divided into sub-networks and a fixed fraction of the capacity of each link is allocated to each sub-network. The tariff is different for each sub-network, so congestion can be reduced in the most expensive sub-network. There is no QoS guarantee, but this method is simple and easy to implement.

Description of the PMP is as follows. First, partition a network into separate networks or classes that have a fixed fraction of capacity from the entire network. There is no guarantee of QoS by charging different charges for different classes but it is assumed that the most expensive class would get better QoS since it is less congested. The advantage of PMP according to Odlyzko (1998) is that it allows dispensing with complex and non-scalable mechanisms, is simpler and cheaper with the current model of the internet and improves QoS experience. PMP looks convenient for the ISP since it is easier to implement in the ISP's current network but even PMP does not guarantee QoS; although users would appreciate it since total charge is linear in accordance to volume of data, and hence it is predictable. A drawback for a given network capacity is that the revenues may be lower in networks implementing PMP since the adopted bandwidth sharing policy among sub-networks can be reduced by having more efficient scheduling mechanisms. Future research is trying to do the same analysis for round robin schemes to handle revenue maximization and investigate the case of strict QoS requirements.

Wu & Banker (2010) and Wu et al. (2002) examined the optimal pricing scheme for information-monopoly service providers who provide only one kind of information service. Unique characteristics of information services are that with enough capacity, the marginal cost of providing service is negligible (almost zero or zero). Flat rate fee pricing becomes attractive while traditional nonlinear pricing theory suggests that optimal pricing strategy is usage-based and advocates the optimality of a two-part tariff pricing. All those researches on pricing schemes assume that there is a relatively high marginal cost of production. Many information service providers have struggled to find the best way to price their services and bill their customers and this is reflected in a variety of pricing scheme offered by different information service providers.

They basically propose to determine which among the three pricing schemes used in practice such as flat fee pricing, usage-based pricing and two-part tariff pricing, is the most popular. Analysis suggests that under conditions of zero marginal and monitoring cost, the flat fee and two-part tariff pricing always reaches the same profit level and is better than usage based pricing. When sum of the marginal cost and monitoring cost is below a certain value, flat fee pricing is the optimal scheme, regardless of how big or how small the monitoring cost is when customers are homogeneous or heterogeneous and there is a marginal willingness to pay. When the monitoring cost is zero, the two-part tariff becomes one of the optimal pricing schemes.

O'Donnell & Sethu (2003) elaborated about DiffServ, which allows up to 64 service classes in QoS allocations. A priori buffers and bandwidth reservations can reach QoS guarantee in IntServ but it depends on RSVP, as the protocol for setting up the reservation. DiffServ can reach scalability by collecting packets of data to become smaller service classes. Zhu & Boutaba (2008) described 2 main groups in network architecture, which is the elastic group such as emails and text file transfers where user satisfaction slowly increases with the bandwidth assigned; and the inelastic group such as video conferencing and multimedia application that needs a certain bandwidth to operate and to provide a better QoS.

Bailey & McKnight (1995) defined internet pricing into 3 main categories:

i. First, is the flat rate where the fee to connect must be paid by the user but no payment charged for each bit sent.
ii. Secondly, is usage-sensitive where the user pays a part of the internet connection bill and a part of each bit sent and/or received. Sending and/or receiving the consecutive bit are a marginal cost that will be non-zero during certain periods.
iii. Lastly is the transaction-based rate, which is also a marginal cost of sending and/or receiving other bit that is non-zero but also has additional pricing features determined by transaction characteristics and not by bites.

He et al. (2012) also defined internet pricing according to cost analysis. The categories are flat pricing, where ISPs use one price to charge users based on a specified time and users have equal speed access and equal price. The second category is based on usage pricing, where the pricing scheme charges the amount of traffic uploaded and downloaded. Congestion pricing occurs when the network does not reflect the individual traffic. This pricing scheme sets the price that represents the approximate use of real time network resources. They also categorize internet pricing according to service type. First is the best-effort service pricing. This scheme is normally provided because ISPs do not implement QoS control mechanisms. ISPs just attempt to adapt the basic pricing to influence use of resources. Second is QoS guaranteed service pricing that consists of three pricing strategies. Simple priority-based service scheme shows a connection between differentiation of QoS and the efficiency of resource usage by differentiating the traffic using characteristics of application. IntServ is based on end-to-end Resource Reservation Protocol for each flow that can guarantee QoS efficiency.

Hwang & Weiss (2002) discussed the measurement of QoS network service performance based on bandwidth, delay and delay jitter, throughput and loss rates. Paschalidis and Liu (2002) pointed out the importance of multiservice networks such as assisting ISPs in spending their allocations, increasing the effectiveness of network usage by giving incentives to customers, to aid well established market view since new services can gain more sustainability. Gottinger (2011) explained that utility function of a user can be in the form of probability of packet loss, average packet delay, probability of packet tail, delay of maximum packet and also throughput.

Crowcroft et al. (1996) contended that commercial internet providers prefer to charge by using the flat fee mechanism. They seek to explain the new internet pricing scheme model based on the aspects of metering which is calibrated by a router and shows the usage statistics as per packet level, the class that shows the requirement of performance of the flow such as data application that tolerates delay but needs more reliability. The pricing aspect is determined through service class and flow usage. Tariff is the service cost for using the parameter, subscription means to pay in advance and a service token is applied when the user is willing to have a reliable service level for a stipulated period.

DaSilva (2000) discusses the connection between pricing schemes and management function of traffic in QoS networks. The arrangement of prices is to obtain optimal pricing that renders an impact on user behavior during fair allocation of resources. He mentioned that the utility function is the preference of users resulting from the user's sensitivity in QoS changes. In reality, utility is considered a function of how much of the network's resources can be utilized by the users. The flat rate scheme is applied to light users to avoid penalizing the heavy users. However, a new problem of over capacity consumption is created due to uncontrollable congestion and that can result in overuse of resources. The usage-sensitive pricing scheme can handle the congestion problem but it creates other problems such as the difficulty to plan expenditure due to uncertainty of the network and billing costs that can result in lower number of internet users.

Gu et al. (2011) discussed pricing schemes based on QoS levels in different allocations that control congestion and load balance. Multiple class QoS networks require differentiated pricing schemes for allocations of different levels of service traffic. Kumaran et al. (1999) talked about creating a new model based on users and service providers. The market preference to competitive market equilibrium has been established and satisfies the multiple class QoS classes in which the profit decreases with increasing number of service providers and eventually the prices will be on par with the marginal cost.

Bouch & Sasse (1999) examined the connection between QoS characteristics in the network with quality requirement of users applying for the network. The models showed dynamic user behavior and pricing schemes that control the real-time quality allocation application. The model is used for predicting user behavior towards the QoS level they had received. The results of their findings are that predictability and consistency of QoS is crucial, the pricing scheme is crucial to have a QoS, having reliable service protocols and new integrated service mechanisms is to present alternatives in solving the problem. Through multiple class QoS, the profile of the support task is supported. There exist a direct connection between QoS profile application identification in packets and users requesting QoS.

Shakkottai & Srikant (2007) viewed the relationship between congestion control, routing and scheduling of wired networks as fair allocation of resources. By assuming the utility function to be continuously differentiable, non-decreasing and strictly concave, then the utility can be seen as diminishing on the assumption of concavity, maximum unique value in closed and bounded sets. Shakkottai et al. (2008) also discussed the flat fee pricing scheme and the simplicity price as maximum revenue. That scheme can approximate nonlinear prices of revenue maximizing in networks. The drawback of the rule is due to nonlinearity and does not reflect the price observed in reality. Eltarjaman et al. (2007) stated that in network models, it is assumed that n users can be split into k categories. Each category can apply the same service offered by application server in a shared link with total bandwidth C_{tot} but has different demand frameworks and different price sensitivities. Alderson et al. (2006) discussed issues related to ISP problems dealing with network topology such as link cost, router technology that impacts the availability of topology to network creators and dealing with equipment for routing that is adopted to tackle network traffic flow. Other areas of issues deal with customer constraint in providing network service and the SLA, as a business contract with the customers.

Garcell et al. (2008) described ways to solve internet optimization, which includes system definition as an interest function and viewing it differently via points and system mathematical definition. Wu et al. (2009) focused on problems of web services arrangement and explained the model of multi-dimension QoS. On that framework, the web service QoS model can be created and described in detail. Wang et al. (1996) analyzed the framework of pricing problems in integrated service networks that have guaranteed QoS. They attempted to improve the three phases to obtain the capacity schedule and optimal price.

Marzolla & Mirandola (2010) proposed terms for performance prediction of service-based systems that consist of performance indicating how fast it takes to complete a service request; time interval indicating the time period needed to complete a service request; dependability indicating the capability of the web service to conduct conditional required functions; price setting by ISPs and reputation indicating the users perception to the service. They considered QoS based on the point of view of ISPs and users and level of development, which is design and run time. In addition, QoS is based on metrics that consist of performance; dependability, which is reliability and availability; price that is fixed or proportional; reputation of web service and other customers. Lastly, QoS is based on the method of evaluation gives information about models, ontology and monitoring.

Karsten et al. (1999) discussed about charging schemes in service networks. They focused on obtaining mechanisms to enable extension of network capacity at required locations on the user's expense called virtual resource mapping, using principles of economics to optimize the framework of pricing. Moreover, they contended that communication services from an economic point of view can be classified into non-storable resource availability and high fixed costs and low variable costs. Tektas & Kasap (2008) explained about monopoly in pricing model strategy that is based on pay-per-volume and pay-per-time set by the network. They concluded that ISPs will benefit by providing pricing schemes based on volume since this scheme is an alternative to numerous users and the pay-per volume scheme will benefit the network provider and can prevent from bursting the networks. Atov & Harris (2003) discussed network design problems in engineering frameworks, especially on problems of link capacity expansion in communication networks. The problem is then transformed into a sub-problem to solve multiservice IP networks. Muthuswamy et al. (2012) discuss the path vector method in serving the customers by formulating the optimization problem of objective function profit subject to risk limitations. Ramsey pricing model was deployed and the results show the optimal solution was obtained.

Keon (2003) considered multiple service pricing schemes in a single telecommunication network. Service has QoS requirement that is guaranteed on behalf of the customer. Services are characterized by service types such as video or data, the origin and the user's destination of connection. Formulation of the problem is conducted as a nonlinear integer problem related to revenue optimization. The model consists of classes that yield optimal prices and resources as a QoS requirement for each service class. Lee & O'Mahony (2000) discussed the requirement of policy based on the execution environment to handle support and flexibility when charging the network. They also proposed a network approach to provide charging support to the service provider's network. Policy of charging is specified as a set of rules applied in a variety of network nodes that include routers and other traffic elements.

Roberts (1998) explained that QoS in a multiservice network depends on the service model to identify service classes and indicate the networks resources that are shared between numerous flows and also depends on the rule of sizing indicated from prediction. QoS requirements should be transparent regarding time and integrity of transferred data, the stream that is tolerable to loss and the delay that is not crucial. Accessibility is an aspect required for QoS, which is the probability of refusal and delay admission for arranging in blocking case and throughput, which is a measurement of QoS for elastic traffic as document size per time for transferring the document.

Some Recent studies for continuous pricing scheme were proposed by Safari et al. (2014), by considering the optimal pricing strategy for specific service as function of time. Their proposed model was created then comparing with the existing approaches available. The models focus on continuous models solved heuristically. Other continuous pricing model scheme was proposed by Castillo, et al. (2013) where in their research, the dynamic pricing scheme proposed by setting up the model as a partial differential equation (PDE) and solving it numerically. The pricing scheme proposed mainly for pricing companies. Their work utilizes the PDE background by utilizing necessary and sufficient condition of Lagrange. So by solving the boundary conditions the pricing scheme involving company debt can be calculated. Fruchter & Sigué (2013) explained the pricing scheme of services that involve subscription fee. The optimal pricing scheme was set up by formulating into optimization problem which involves the fee to subscribe the service. The Lagrangian optimality conditions were utilized. The results show the connection among the subscription fee of the service. Some users will pay attention to one fee and ignore the other fees.

Research on optimal pricing, basically focus more on practical pricing scheme, but some of the researches do not give any information regarding the interaction between the resources in pricing scheme such as bandwidth allocation and user' price sensitivity. Lee et al. (2013) seek the optimal pricing scheme by utilizing the service by considering the pricing problem as an economic model. The optimization problem was formulated and solved numerically. Then the tiered and non tiered models were compared. The results show that the tiered service model in network can gain better profit. Other research (Borgia, 2014) also describes the technological part of internet or is called Internet of thing (IoT). The review concludes that to support IoT, a number of researches has been done but mostly do not include all aspect of areas such as communication, billing and pricing. So the needs to cover those aspects are critical to solve. Other economical point of view on pricing scheme was recently discussed by (Pal & Hui (2013) in cloud service market. In this service, the company is able to produce new application program to the customers. The model was based on game theory involving Nash equilibrium. The optimization problem of cloud service was formulated and the optimality condition is tested by KKT conditions of Lagrangian multiplier. The results show that the proposed pricing scheme works optimally in the one tier and multi tier services.

Some researches attempt to compare the pricing scheme models proposed. Sen (2013) compare the fixed and dynamic pricing scheme in getting the revenue. Two heuristic dynamic pricing schemes were proposed. The research concludes that the proposed dynamic pricing schemes show better performance with smaller gap in getting the optimality. Strategic pricing involving in supply chain was proposed by Zhang et al. (2014) and Wei et al. (2013). Competition in market consists of manufacturer and retail in a supply chain condition. The equilibrium of game theory in form of Stackelberg game theory was investigated and the policy should be set up. The function investigated was in linear form. The investigation shows that the same pricing strategy does not always occur between the manufacturer and the retailer. The consumer will affect the pricing decisions. Both researches utilize the game theory problem and dealing with allocation, pricing, and ordering of the services. The difference between these two researches is in the profit gain for manufacturer and retailer. In Wei et al. (2013) model, the manufacturer will gain higher profit but the retailer not always the same conditions. When dealing the internet policies of a manufacturer, both manufacturer and retails are able to decide how to maximize the profit. So, using Nash or Stackelberg game theory the solution of equilibrium can be achieved (Chen et al., 2013).

The recent discussion about bandwidth allocation in data networks is due to Divakaran et al. (2014). A model to reserve the bandwidth is proposed. Heuristic bandwidth allocation problem is formulated as an optimization problem. The model can enhance the users by the ability of the users to request and to choose the bandwidth.

Simulation results show the model effectiveness by showing the allocation of the bandwidth required.

2.3 Multiservice Network Architecture

Ninan (2004) described the connection system between the networks and the users who utilize the system. He described the circuit of multiple users as in Figure 4.

FIGURE 4: Network Pipes in a Multiservice Network (Ninan, 2004)

2.4 Quality of Service (QoS)

According to Karsten et al. (2000), QoS is used by network providers. QoS models for the internet are grouped into:

i. RSVP/IntServ: the scope is to provide end-to-end services. Implementation tools for the RSVP/IntServ model are signaling and admission control.
ii. DiffServ: the scope is inter-domain on peering between dominants. PHBs are forwarding behavior inside the element of networks. In DiffServ, the SLA is allowed to gain a bilateral contract.

iii. Best-effort: the scope is end-to-end due to all network resources being in end-systems. It is either over provided or price-controlled best effort in addition to time scale, due to the announcement of price and capability of set prices.

2.5 Network Service Charging Terminology

Network service charging terminology is adopted from Stiller et al. (2001) to enable us to resolve the differences in definitions.

i. Accounting refers to summarized information in connection to utilization of customer's service.
ii. Billing refers to record of charge collected, summarizing content of charge and providing information and details to the customer regarding the charge list.
iii. Charge calculation refers to the amount calculated for a given accounting record.
iv. Charging refers to all tasks required to calculate the final content of a billing record.
v. Pricing includes the specifications and setting of prices for networking resources and services in an open market situation. The process can be combination of technical considerations such as consumption of resources or application of the tariff theory.

2.6 Utility and Optimization Objectives

Utility is the amount of money a user is willing to pay for certain QoS guarantees. Utility is also often expressed as a function of the amount of resources made available to users by the network. Cocchi et al. (1991) proposed the utility for four application types.

i. Email – assuming that the utility is a decreasing function of the average delay and percentage of messages not delivered within a delay bound of five minutes.
ii. File transfer.
iii. Remote login – assuming that user satisfaction decreases with average packet round-trip time.
iv. Real time voice - parameters are the average one-way delay and the percentage of voice packets not conforming to a tight delay bound of 100 milliseconds.

Characterizing utility can be used as a function of the resources allocated to the flow or the call, as an indication of expected performance by considering it as an elastic application, for example, real time voice and video, which employs constant bit rate coding, requires a fixed amount of bandwidth for adequate QoS. It is called the elasticity of their demand for bandwidth so the function is a step function as shown in Figure 5(a).

FIGURE 5: (a) Elastic Application, (b) Inelastic Application and (c) Partially Elastic Application

Another kind of performance is called inelastic application, for example, traditional data such as email are elastic since it tends to tolerate variations in delay and takes advantage of minimal amounts of bandwidth as seen in Figure 5(b). The last kind of performance is called partially elastic, where the utility function is in Figure 5(c) (some minimum bandwidth are nevertheless required).

Wang & Schulzrinne (1999) explained that a bandwidth function shows the utility function and has formulation: $U(x) = U_0 + Wlog\frac{X}{X_m}$ with X being the minimum bandwidth requirement, W=utility sensitivity to bandwidth, and U_0 as the base cost that the user obtains at the lowest QoS level and X_m will be the minimum bandwidth in class m. Lee et al. (2005) stated that the properties of utility function $U_i(x_i)$ such as U_i , should be an increasing function and x_i is the allocated rate for customer I, thus U_i should be twice as differentiable and strictly a concave function.

2.7 Pricing Scheme in Multiservice Networks

Sain & Herpers (2003) investigated the pricing for multiservice networks by considering prices, capacity allocation and QoS in maximizing the ISP profit. They formulated the model called OPCQP and solved the optimization problem by using the OPL Studio. The parameters and decision variables used are as follows.

Parameters:

C : Total capacity

d_s : Capacity needed to provide full QoS unit of service s

m_s : minimum level of QoS needed for service s

n_s : maximum number of users for service s

p_s : user price sensitivity of service s

S : number of service

Decision variables:

a_s : Reserved share of the total capacity for service s

q_s : QoS level for service s

x_s : number of users of service s

The description of the optimization model proposed by Sain & Herpers (2003) is as follows.

$$\text{Max} \sum_{s=1}^{S} q_s p_s x_s \tag{1}$$

Subject to

$$q_s d_s x_s \le a_s C, s = 1, 2, \ldots, S \tag{2}$$

$$\sum_{s=1}^{S} q_s d_s x_s \le C \tag{3}$$

$$\sum_{s=1}^{S} q_s d_s x_s = 1 \tag{4}$$

$$0 \le a_s \le 1, s = 1, 2, \ldots, S \tag{5}$$

$$m_s \le q_s \le 1, s = 1, 2, \ldots, S \tag{6}$$

$$0 \le x_s \le n_s, s = 1, 2, \ldots, S \tag{7}$$

2.8 Pricing Scheme in Multiple Class QoS Networks

The research leading to these pricing schemes were carried out by Byun & Chatterjee (2004), Yang et al. (2003) , Yang (2004), Yang et al. (2004) and Yang et al. (2005).

2.8.1 Pricing Scheme Proposed by Byun & Chatterjee (2004)

Byun & Chatterjee (2004) proposed the pricing scheme for QoS networks to maximize the revenue function of ISPs.

Parameters:

α : Base price if the ISP intends to recover the cost

β : Quality premium if the ISP intends to let the user select the class

n : network service class

Decision variables:

α_i : Base price if the ISP intends to compete in a market competition

β_i : Quality premium if the ISP intends to promote certain services

I_q^i : Quality index for ith class

u_i^* : Total bandwidth consumption of ith service class

The model formulation proposed by Byun & Chatterjee (2004) is as follows.

$$\text{Max} \sum_{i=1}^{n} (\alpha + \beta_i I_q^i) u_i^* \tag{8}$$

Subject to

$$\sum_{i=1}^{n} u_i^* \le C \tag{9}$$

$$0 \le I_q^i \le 1, \text{for } \forall I_q \tag{10}$$

$$(\alpha + \beta_i I_q^i) \ge (\alpha + \beta_{i-1} I_q^{i-1}) \text{ for } i > 1 \tag{11}$$

$$I_q^i \ge I_q^{i-1}, \text{ for } i > 1 \tag{12}$$

2.8.2 *Pricing Scheme in Single Bottleneck Link Multiple Class QoS networks proposed by Yang (2004)*

Yang (2004) proposed internet pricing in multiple class QoS networks. She proposed the model and solved the model formulation by using the auction method. The model begins from single bottleneck links and is later generalized into multiple bottleneck links.

2.8.2.1 Pricing Scheme in Single Bottleneck Link Multiple Class QoS networks proposed by Yang (2004) By Setting up the Base Price as a Fixed Value

Yang (2004) set up the parameters and the variables as follows.

Parameters:

α_j : Base price for class j

C : Total bandwidth

V_i : Minimum bandwidth required by customer i

c : upper bound value of user's i price sensitivity at class j

Decision variables:

$Z_{ij} = \begin{cases} 1, & \text{if user } i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

The model formulation of internet pricing in multiple class QoS networks proposed by Yang (2004) is as follows.

$$\text{Max} \sum_{j=1}^{2}\sum_{i}(\alpha_j Z_{ij} + W_j \log\frac{\tilde{X}_{ij}}{L_{m_j}}) \tag{13}$$

Subject to

$$\sum_{j=1}^{2}\sum_{i} X_{ij} \le Q, i = 1,\ldots,n \tag{14}$$

$$\tilde{X}_{ij} \geq L_{m_j} - (1 - Z_{ij}), i = 1, \ldots, n; j = 1, 2, \ldots \tag{15}$$

$$W_j \leq W_{ij} + (1 - Z_{ij}), i = 1, \ldots, n; j = 1, 2, \ldots \tag{16}$$

$$\tilde{X}_{ij} \geq V_i - (1 - Z_{ij}), i = 1, \ldots, n; j = 1, 2, \ldots \tag{17}$$

$$\tilde{X}_{ij} \geq X_j - (1 - Z_{ij}), i = 1, \ldots, n; j = 1, 2, \ldots \tag{18}$$

$$\tilde{X}_{ij} \geq 0 + (1 - Z_{ij}), i = 1, \ldots, n; j = 1, 2, \ldots \tag{19}$$

$$\tilde{X}_{ij} \geq 0, i = 1, \ldots, n; j = 1, 2, \ldots \tag{20}$$

$$L_{m_j} \geq 0, j = 1, 2, \ldots \tag{21}$$

$$W_j \geq 0, j = 1, 2, \ldots \tag{22}$$

$$\tilde{X}_{ij} \leq X_j, i = 1, \ldots, n; j = 1, 2, \ldots \tag{23}$$

$$Z_{ij} = \begin{cases} 1, i \text{ is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases} \tag{24}$$

$$0 \leq W_j \leq c, i = 1, \ldots, n; j = 1, 2, \ldots; c \in [0, 1] \tag{25}$$

2.8.2.2 Pricing Scheme in Single Bottleneck Link Multiple Class QoS networks proposed by Yang (2004) By setting up the Base Price as a Variable

Yang (2004) also set up the parameters and decision variables if the base price is a decision variable.

Parameters:

C : Total bandwidth

V_i : Minimum bandwidth required by customer i

c : upper bound value of user's i price sensitivity at class j

Decision variables:

$Z_{ij} = \begin{cases} 1, \text{if user } i \text{ is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

X_{ij} : Final bandwidth obtained by user i for class j

L_{mj} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

α_j : Base price for class j

The model formulation of internet pricing in multiple class QoS networks proposed by Yang (2004) is as follows.

$$\text{Max } \sum_{j=1}^{2}\sum_{i}(\alpha_j + W_j \log \frac{\tilde{X}_{ij}}{L_{m_j}})Z_{ij} \tag{26}$$

Subject to constraints (14)-(25) and additional constraints as follows.

$$a \le \alpha_j \le b, j = 1,2,\ldots;[a,b] \in [0,1] \tag{27}$$

$$\alpha_j \ge \alpha_{j-1}, j > 1 \tag{28}$$

2.8.3 Pricing Scheme in Multiple Bottleneck Links Multiple Class QoS networks Proposed by Yang (2004)

When the pricing scheme occurs in multiple bottleneck links, then Yang (2004) proposed a model formulation with parameters and decision variables as listed below. There are two types of model formulations, which consider the base price as fixed or variable.

2.8.3.1 Pricing Scheme in Multiple Bottleneck Links Multiple Class QoS networks Proposed by Yang (2004) By setting up the Base Price as Fixed Price

The parameters and decision variables proposed by Yang (2004) are as follows.

Parameters:

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network.

α_j : Base price of a network service j, since the internet's level of service quality cannot guaranteed so α could be equivalent to the price of the best-effort service in the current internet architecture.

$\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l.

$\widetilde{W}_{ij}^{l}$: Price sensitivity for user i that shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

Decision variables are as follows.

$\tilde{X}_{ij}^{l}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user *i* in class *j*.

$Z_{ij} = \begin{cases} 1, & i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class *j*.

L_j : Bandwidth for class *j*.

X_j : Bandwidth assigned to each individual user in class *j*.

So, the mathematical model proposed by Yang (2004) is as follow.

$$\text{Max } \sum_{j=1}^{m}\sum_{i=1}^{n}(\alpha_j Z_{ij} + W_j \log\frac{\hat{X}_{ij}}{L_j}) \tag{29}$$

Subject to

$$\sum_{j=1}^{m}\sum_{i=1}^{n}\tilde{X}_{ij}^{l} \le C_l, l = 1,\ldots,k \tag{30}$$

$$\tilde{X}_{ij}^{l} = \hat{X}_{ij}, i = 1,\ldots,n; j = 1,\ldots m; l = 1,\ldots,k \tag{31}$$

$$\hat{X}_{ij} = \tilde{L}_{ij}^{l}, i = 1,\ldots,n; j = 1,\ldots m; l = 1,\ldots,k \tag{32}$$

$$\hat{X}_{ij} \ge Z_{ij}, i = 1,\ldots,n; j = 1,\ldots m \tag{33}$$

$$W_j \le \tilde{W}_{ij}^{l} + (1 - Z_{ij}), i = 1,\ldots,n; j = 1,\ldots m; l = 1,\ldots,k \tag{34}$$

$$L_j \le \tilde{L}_{ij}^{l} + (1 - Z_{ij}), i = 1,\ldots,n; j = 1,\ldots m; l = 1,\ldots,k \tag{35}$$

$$\hat{X}_{ij} \ge X_j - (1 - Z_{ij}), i = 1,\ldots,n; j = 1,\ldots m; l = 1,\ldots,k \tag{36}$$

$$\hat{X}_{ij} \le X_j, i = 1,\ldots,n; j = 1,\ldots m \tag{37}$$

$$\hat{X}_{ij} \ge 0, i = 1,\ldots,n; j = 1,\ldots m \tag{38}$$

$$L_j \ge 0, j = 1,\ldots m \tag{39}$$

$$W_j \ge 0, j = 1,\ldots m \tag{40}$$

$$Z_{ij} = \begin{cases} 1, & i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

2.8.3.2 Pricing Scheme in Multiple Bottleneck Links Multiple Class QoS networks Proposed by Yang (2004)By setting up the Base Price as a Variable

Parameters and decision variables for the model proposed by Yang (2004) that considers the base price as a variable are as follows.

Parameters:

C_l : Bandwidth capacity of link l that represents the set of links that flow i of class j crosses in the network.

$\tilde{L}_{ij}^{l}$: Minimum bandwidth for user i in class j on link l.

$\widetilde{W}_{ij}^{l}$: Price sensitivity for user i, which shows the satisfaction of the user i by receiving the bandwidth in class j on link l.

Decision variables:

$\tilde{X}_{ij}^{l}$: Bandwidth obtained by user i in class j on link l.

$\hat{X}_{ij}$: Final bandwidth obtained by user i in class j.

$Z_{ij} = \begin{cases} 1, & i \text{ is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$

W_j : Price sensitivity for class j.

L_j : Bandwidth for class j.

X_j : Bandwidth assigned to each individual user in class j.

Mathematical formulation for the model that considers the base price as a variable proposed by Yang (2004) is as follows.

$$\text{Max} \sum_{j=1}^{m} \sum_{i=1}^{n} (\alpha_j + W_j \log \frac{\hat{X}_{ij}}{L_j}) Z_{ij} \tag{42}$$

Subject to constraint (30)-(41) and additional constraints as follows:

$$\alpha_j \geq \alpha_{j-1}, j \geq 1 \tag{43}$$

$$a_j \leq \alpha_j \leq b_j, [a,b] \in [0,1] \tag{44}$$

2.9. LINGO 11.0 Software

To proceed with the large number of calculation in mathematical programming, it is best to perform the results by computer program named LINGO(2008) . LINGO enables user to input the model formulation, solve it, do analysis based on the solution and repeat the whole process (Schrage, 2009). The input of LINGO model is in model window to enable the user to input the mathematical model while the output is displayed in the report window. For example, the problem of solving the mathematical programming problem is as follows.

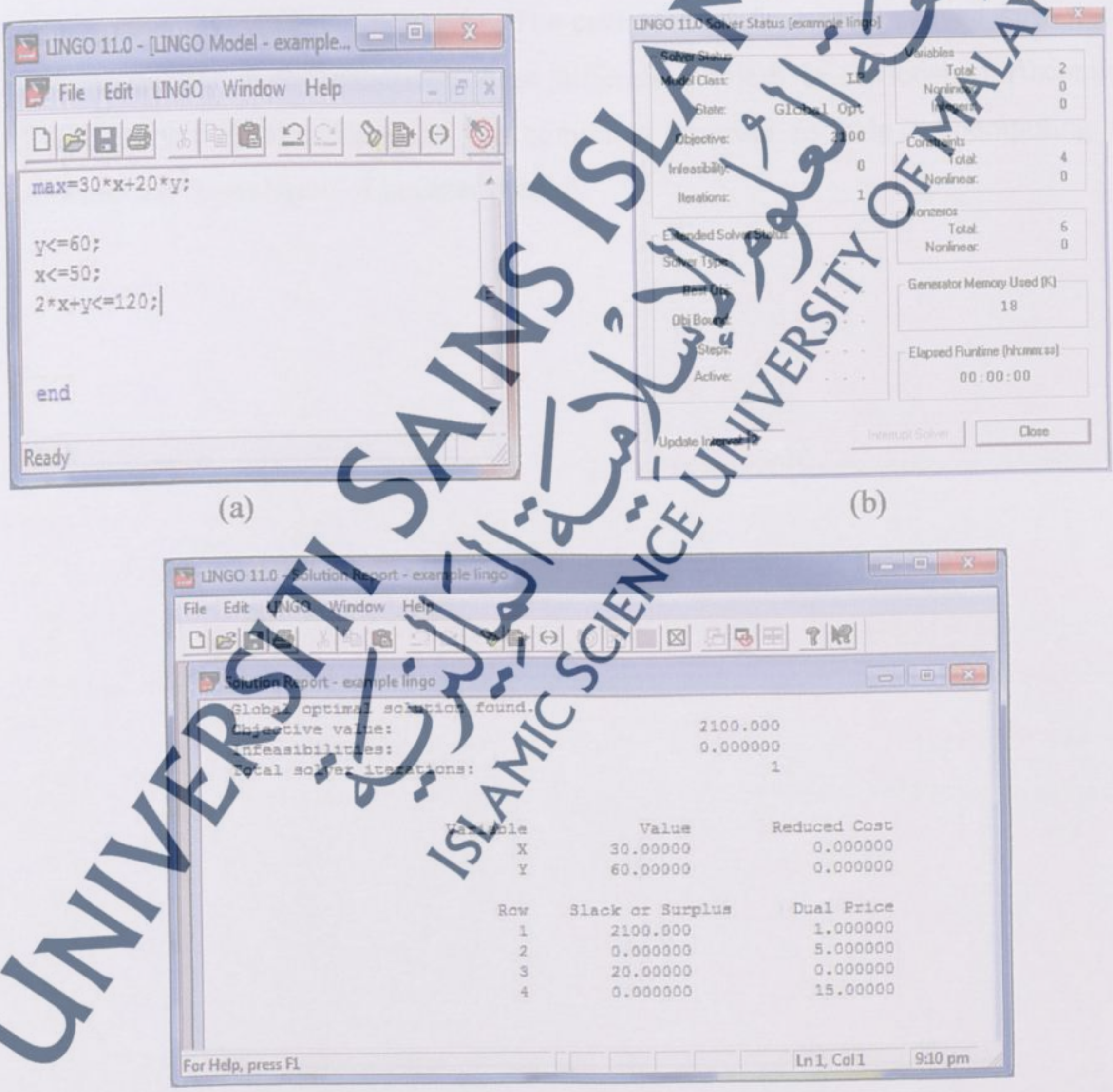
Max $30x+20y$

subject to

$$\begin{aligned} y &\le 60 \\ x \quad &\le 50 \\ 2x + y &\le 120 \end{aligned}$$

then by using LINGO 11.0, the solution will be described in Figure 6.

FIGURE 6: (a) The Model Window of LINGO 11.0, (b) Solver Status of LINGO 11.0 Report Window and (c) Solution Report of LINGO 11.0 Report Window



(a) (b)

(c)

Three sections of the output are shown in Figure 6(c). The first is variables and row section. The second will be the maximum solution to have x=30 and y=60 with optimal solution of 2100. This solution will have zero slack in row 2 and row 4, and also slack of 20 in row 3. The slack explain how far that the decision variables satisfy the constraints as equality. If satisfied, than the slack will be 0. If violated, than the slack will be negative.

2.10 Summary

Chapter 2 focuses on the benefits and drawbacks of the current internet pricing scheme. All the information discussed above highlights the development of internet charging and internet pricing in different networks. The critical need for new improved formulas to tackle problems of congestion by these different services is obviously forthcoming. LINGO 11.0 was introduced as the computer program to help in computing the mathematical formulation of internet pricing.