

NEW POLYNOMIALS FOR THE SOLUTION OF CERTAIN CLASSES OF SINGULAR INTEGRAL EQUATIONS

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NEW POLYNOMIALS FOR THE SOLUTION OF CERTAIN CLASSES OF SINGULAR INTEGRAL EQUATIONS

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DEDICATION

*I would like to dedicate this thesis to my loving family.
To my parents, Father and Mother for their support and guidance.*

*To my LOVELY Wife, ELHAM
For her Patience, Love and Support.*

*And my children, Mohammad, Yamen and Eleen
Also Masa,
I Hope this INSPIRES you to work hard in order to achieve your dreams
and a better future.*

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ABSTRAK

Dalam tesis ini, kita mencadangkan satu kelas polinomial berortogon baru $Z_{(i,n)}^k(x), i = \{1, 2, 3, 4\} n = 0, 1, 2, \dots$, diberi nama masing-masing polinomial Chebyshev diperluas jenis pertama, kedua, ketiga dan keempat, peringkat k , dengan k integer positif ganjil dan polinomial tersebut adalah perluasan polinomial Chebyshev. Didapati bahawa polinomial Chebyshev diperluas jenis pertama - keempat ini adalah berotogon terhadap fungsi pemberat tertentu dalam selang $[-1, 1]$. Cirian utama bagi jujukan polinomial Chebyshev diperluas $\{Z_{(i,n)}^{2m+1}(x)\}_{m=0}^k; i = \{1, 2, 3, 4\}; n = 0, 1, 2, 3, \dots$ akan diterangkan apabila k adalah integer ganjil yang cukup besar. Telah ditunjukkan bahawa jujukan Chebyshev diperluas jenis pertama dan kedua menumpu kepada salah satu nilai $\{-1, 0, 1\}$ masing-masing bergantung kepada $n = \{2l + 1, 2(2l + 1), 4l\}, l = 0, 1, 2, \dots$. Tambahan pula, jujukan Chebyshev diperluas jenis ketiga dan keempat menghampiri dua nilai $\{-1, 1\}$ bergantung masing-masing kepada $n = \{1, 2, 5, 6, \dots\}$ dan $\{0, 3, 4, 7, 8, \dots\}$. Hubungan jadi semula dan kaitan antara satu sama lain telah dibentang dan dibuktikan. Sifat spektral polinomial berotogon baru yang dicadangkan telah diperolehi dan digunakan untuk menyelesaikan persamaan berikut:

- Kelas khas bagi Persamaan Kamiran Singular Berlogaritma (jenis pertama dan kedua) peringkat k (LogSIEk).
- Kelas khas persamaan Kamiran Singular jenis pertama dan kedua peringkat k (SSIEk).

Nilai eigen dan fungsi eigen berkaitan telah didapati untuk (LogSIEk) jenis pertama. Untuk jenis kedua tidak homogen (LogSIEk) penyelesaian hampiran yang sangat jitu telah diperolehi. Bagi (SSIEk) jenis pertama, kita mempertimbangkan kes berikut:

- Kes (I): Penyelesaian tak terbatas di kedua titik hujung $x = \pm 1$,
- Kes (II): Penyelesaian terbatas di kedua titik hujung $x = \pm 1$.
- Kes (III): Penyelesaian terbatas di titik hujung $x = -1$ dan tidak terbatas di $x = 1$.
- Kes (IV): Penyelesaian terbatas di titik hujung $x = 1$ dan tidak terbatas di $x = -1$.

Telah didapati bahawa penyelesaian hampiran adalah sangat jitu untuk keempat-empat kes tersebut.

Contoh berangka dan perbandingan dengan kaedah lain juga diberikan untuk menunjukkan ketepatan dan keberkesanan kaedah yang dicadangkan. MATLAB digunakan bagi tujuan pengiraan untuk mendapatkan penyelesaian berangka.

ABSTRACT

In this thesis, we propose a new classes of orthogonal polynomials $Z_{(i,n)}^k(x)$, $i = \{1, 2, 3, 4\}$ $n = 0, 1, 2, \dots$, namely Extended Chebyshev polynomials (ECPs) of the first, second, third and fourth kinds respectively, of order k , where k is positive odd integer and the polynomials are the extension of Chebyshev polynomials. It is found that ECPs of first - fourth kinds are orthogonal with respect to a certain weight functions on the interval $[-1, 1]$. The main characteristics of ECPs sequences $\{Z_{(i,n)}^{2m+1}(x)\}_{m=0}^k$; $i = \{1, 2, 3, 4\}$; $n = 0, 1, 2, 3, \dots$ are described and explained when k is sufficiently large odd integer. It is shown that ECPs sequences of the first and second kinds converge to one of the three values $\{-1, 0, 1\}$ depending on $n = \{2l + 1, 2(2l + 1), 4l\}$, $l = 0, 1, 2, \dots$ respectively. Moreover ECPs sequences of the third kind approaches into two values $\{-1, 1\}$ depending on $n = \{1, 2, 5, 6, \dots; 0, 3, 4, 7, 8, \dots\}$ respectively, and ECPs sequences of the fourth kind approaches into two values $\{-1, 1\}$ depending on $n = \{2, 3, 6, 7, \dots; 0, 1, 4, 5, 8, 9, \dots\}$ respectively. Recurrence relations and the relationships between each other kinds are proved. Spectral properties of the proposed new orthogonal polynomials are obtained and used to solve the following equations:

- Special class of the Logarithmic Singular Integral Equations (first and second kind) of order k (LogSIEk).
- Special class of Singular Integral Equations of the first kind of order k (SSIEk).

Eigenvalues and corresponding eigenfunctions are found for the LogSIEk of the first kind. For the second kind inhomogeneous LogSIEk highly accurate approximate solution are obtained. For SSIEk of the first kind, we have considered the following cases:

- Case (I): The solution is unbounded at both end points $x = \pm 1$,
- Case (II): The solution is bounded at both end points $x = \pm 1$.
- Case (III): The solution is bounded at the end point $x = -1$ and unbounded at $x = 1$.
- Case (IV): The solution is bounded at the end point $x = 1$ and unbounded at $x = -1$.

It is found that the approximate solution is highly accurate for the four cases. Numerical examples and comparisons with other methods are also provided to illustrate the effectiveness and accuracy of the proposed methods.

في هذه الاطروحة عرف الباحث نوع جديد من كثيرات الحدود المتعامدة على الفترة $[-1, 1]$ في فضاء هلبرت ، تم تقسيم هذه الكثيرات إلى اربعة انواع رئيسية مسماه بكثيرات حدود شيبشيف الموسعة من النوع الاول ، النوع الثاني، الثالث والنوع الرابع.

وجد الباحث أن متتالية شيبشيف الموسعة من النوع الاول والثاني تقترب الى ثلاث حالات وذلك عندما تكون قيم K كبيرة بشكل كافي، وفي مايتعلق بالنوع الثالث والرابع تقرب هذه الدوال إلى قيمتين هي $\{-1, 1\}$ حيث وجد الباحث أن كثيرات حدود شيبشيف الموسعة متعامدة للدوال الوزنية المعينة وعلى الفترة $[-1, 1]$.

على نفس الصعيد استخدمت هذه الكثيرات في حل المعادلات التكاملية المنفردة، اللوغارتمية منها والكوشية، إضافة إلى دراسة بعض خصائص هذه الكثيرات ووضع النظريات المتعلقة بها، تم وضع بعض النظريات المتعلقة بالتعامد .

أوجد الباحث أيضا الدوال الذاتية والقيم المميزة المتناظرة للمعادلات التكاملية اللوغارتمية المتجانسة منها والغير متجانسة منها واستخدام مفكوك متسلسلة شيبشيف الموسعة لإيجاد الحل لها. نوع اخر من المعادلات المنفردة للرتبة

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الحالة الاولى: الحل غير محدود عند الاطراف.
الحالة الثانية : الحل محدود عند الاطراف.
الحالة الثالثة: الحل محدود عند الواحد.
الحالة الرابعة : الحل محدود عند السالب واحد.
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ABBREVIATIONS

ADM	Adomian Decomposition Method.
CPV	Cauchy Principal Value.
CSIEs	Cauchy Singular Integral Equations.
ECP	Extended Chebyshev polynomial.
ECPs	Extended Chebyshev polynomials.
HSIEs	Hyper-Singular Integral Equations.
IE	Integral Equation.
IEs	Integral Equations.
LogSIE_k	Special Logarithmic Singular Integral Equations for order k .
LSIEs	Logarithmic Singular Integral Equations.
PMs	Projection Methods.
QMs	Quadrature Methods.
SIEs	Singular Integral Equations.
SSIE_k	Special Singular Integral Equations for order k .