Entanglement Classification Operator Model Development of SU(2)

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Abstract

The study presents the development of the $SU(2)$ operator model for entanglement classification within the Local Unitary (LU) protocol. The operator model was systematically designed, specifically tailored for the pure three-qubit quantum system to accurately identify the distinct classes of entanglement. By leveraging the special unitary group, $SU(2)$, this operator model development benefits researchers to gain a better understanding of the essence of entangled states, assisting researchers in effectively harnessing them to numerous potential applications in quantum computing, quantum cryptography, quantum teleportation and other respective fields. Hence, the study contributes significantly to advancing quantum information science. Furthermore, the foundation of this study establishes a broad pathway for advancements and future innovation in quantum information science and technologies.

Keywords: *Special unitary group; Local Unitary; LU; Entanglement classification.*

1.0. Introduction

Quantum entanglement is regarded amongst the most important phenomenon in quantum information science. It describes a phenomenon where multiple particles carrying vital information are somehow correlated with each other regardless the distance between them. The principal concept of quantum entanglement has attracted researchers, innovators and others from various science, technology and engineering fields. Over the past several decades, quantum entanglement has enabled numerous scientific breakthroughs to have been made namely in quantum computing, quantum cryptography and quantum teleportation [1-9].

The utilization of quantum entanglement in various quantum fields, particularly quantum computing is extremely crucial in order to successfully implement its complex programs. To fully utilize the quantum entanglement, a vital process to differentiate types of entanglement is essential. Entanglement classification as the name suggests, is a process of classifying different types of entanglement [4,10]. The classification of entangled states is needed to fully understand the nature of quantum entanglement and its application. By classifying entangled states, the most suitable quantum states for specific quantum tasks can be allocated effectively and efficiently. Some existing key protocols for entanglement classification are namely, Local Unitary (LU), Local Operations and Classical Communication (LOCC), and Stochastic Local Operations and Classical Communication (SLOCC) [11]. These protocols basically suggest that quantum states can be manipulated using local operations and classical communication between different part of its subsystems.

This study considered entanglement of multiqubit quantum systems under the LU protocol, specifically three-qubit pure quantum system, which is integral to many practical quantum applications.

LU operations are a transformation applied to quantum states, and the protocol preserves the overall structure and entanglement properties of quantum states. Additionally, LU protocol does not require any communication or interaction between quantum subsystems, which is a key feature that preserves the independence of local operations. For a three-qubit quantum systems, there exist six entanglement classes under Local Unitary (LU) which include separable ($A-B-C$), bi-separable ($A-BC$, $B-AC$ and $C-AB$) and genuinely entangled states (W and GHZ) [10-13].

The main goal of this study is to design and develop an operator model, SU(2), under the LU protocol for entanglement classification of a three-qubit pure quantum system. This work aims to bridge theoretical concepts with practical applications, thereby enhancing the understanding and utilization of entangled states. The remainder of this study is organized as follows: Section 2 describes the methodology followed by section 3 which presents the result and finally, section 4 concludes the study.

2.0 Research Methods

The SU(2) operator model was developed through a systematic process, using three sets of 2×2 matrices to serve as generators, after appropriate sets of generators were determined. As part of the development process, suitable sets of parameters were selected to ensure an accurate and adaptable operator model was developed. It was then proceeded to the full development phase from the generated matrix produced. The selection of appropriate parameters was crucial for achieving accuracy during the development.

It is important to develop a precise operator model, so that the operator model is able to be further extended as the SU(2) represents a single-qubit quantum system. It is greatly convenient if it is capable of extending to a multiqubit quantum system. Validation of the operator model was conducted to ensure its reliability and accuracy. The developed model was benchmarked against previous studies, with experimental comparisons conducted to evaluate its performance. Fig. 1 provides an overview of the modelling process of SU(2), highlighting key steps from the initial parameter selection to the final validation phase.

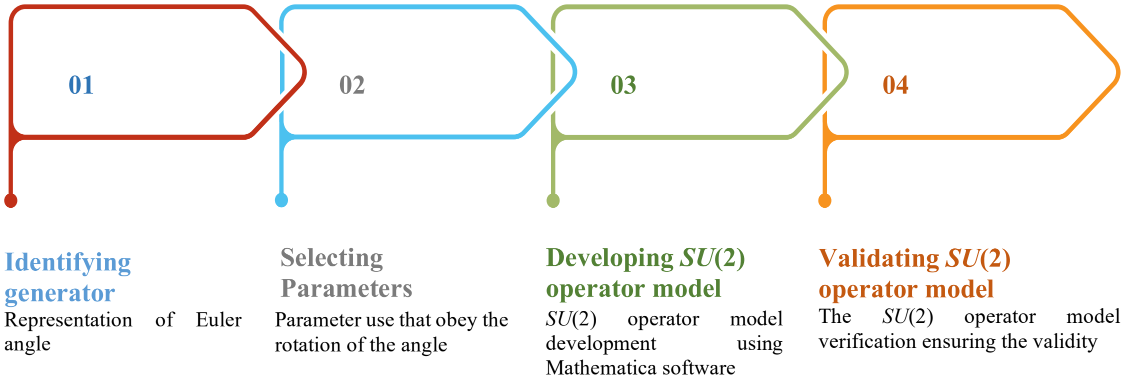
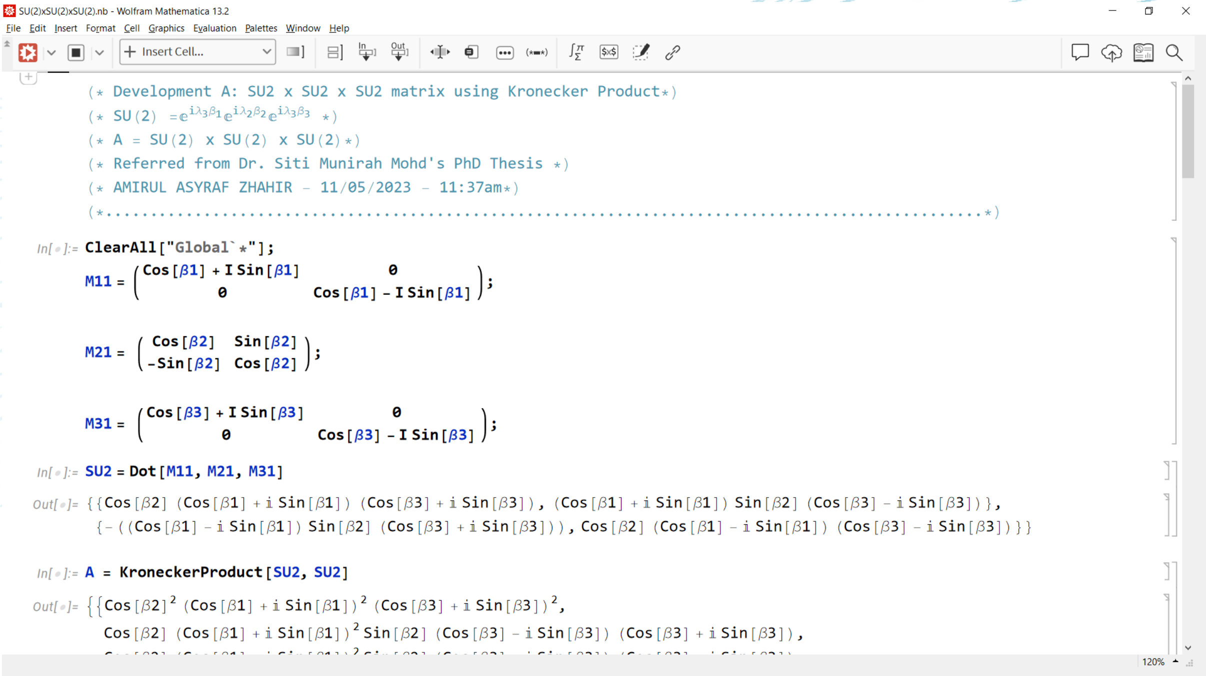


Fig. 1. Modelling process of $SU(2)$

3.0 Results

The $SU(2)$ operator model was successfully developed and validated for entanglement classification. Fig. 2 illustrates the $SU(2)$ operator model successfully developed derived on equation (1). The operator model demonstrated robust classification capabilities, aligning with the theoretical framework and outperforming some existing approaches in terms of precision and scalability.

$$SU(2)=e^{i\sigma_3\beta_1}e^{i\sigma_2\beta_2}e^{i\sigma_3\beta_3} \tag{1}$$



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(* Development A: SU2 x SU2 x SU2 matrix using Kronecker Product*)
(* SU(2)  =e^(iλ3β1)e^(iλ2β2)e^(iλ3β3)  *)
(* A = SU(2) x SU(2) x SU(2)*)
(* Referred from Dr. Siti Munirah Mohd's PhD Thesis *)
(* AMIRUL ASYRAF ZHAHIR - 11/05/2023 - 11:37am*)
(*...........................................................................................*)

In[•]:= ClearAll["Global`*"];
M11 = (Cos[β1] + I Sin[β1]           0
                0            Cos[β1] - I Sin[β1]);

M21 = ( Cos[β2]  Sin[β2]
       -Sin[β2]  Cos[β2]);

M31 = (Cos[β3] + I Sin[β3]           0
                0            Cos[β3] - I Sin[β3]);

In[•]:= SU2 = Dot[M11, M21, M31]
Out[•]= {{Cos[β2] (Cos[β1] + i Sin[β1]) (Cos[β3] + i Sin[β3]), (Cos[β1] + i Sin[β1]) Sin[β2] (Cos[β3] - i Sin[β3])},
  {-((Cos[β1] - i Sin[β1]) Sin[β2] (Cos[β3] + i Sin[β3])), Cos[β2] (Cos[β1] - i Sin[β1]) (Cos[β3] - i Sin[β3])}}

In[•]:= A = KroneckerProduct[SU2, SU2]
Out[•]= {{Cos[β2]^2 (Cos[β1] + i Sin[β1])^2 (Cos[β3] + i Sin[β3])^2,
   Cos[β2] (Cos[β1] + i Sin[β1])^2 Sin[β2] (Cos[β3] - i Sin[β3]) (Cos[β3] + i Sin[β3]),
```

Fig. 2. Development of $SU(2)$

4.0 Conclusion

This study presents the design and development of a novel *SU*(2) operator model, tailored to entanglement classification of a pure three-qubit quantum system utilizing the special unitary group *SU*(2), within the LU protocol. The developed model not only advances the theoretical understanding of entangled states but also serves as a practical tool for precise entanglement classification in applications where accuracy is critical.

By leveraging the properties of the special unitary group *SU*(2), this work contributes to the growing body of knowledge in quantum information science. The developed operator model offers significant potential for future work, extending its developments and applications to multiqubit or even a higher-dimensional quantum systems, paving the way for future advancements in quantum computing, quantum cryptography, quantum teleportation and beyond.

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