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Half Circle Position for Arc Cracks in Half Plane

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Abstract. In this paper, the problem of arc cracks that lie in the boundary of half circle in an elastic half plane is investigated. The complex potential variables with free traction boundary condition is used to formulate the problem into a singular integral equation. The singular integral equation is solved numerically for the unknown distribution dislocation function with the help of curve length coordinate method. The numerical results have shown that our results are in good agreement with the previous works. Stress intensity factors for different cracks position are presented.

1. Introduction

The complex variable method formulates the cracks problems into integral equations, which can be solved by different approaches. The curve coordinate method in conjunction with the appropriate quadrature formulas is one of them. The curve coordinate method provides an effective way to mapped the configurations of the cracks into a straight line on a real axis that require less collocation points, therefore give faster convergence. This approach was suggested to solve a singular integral equation (SIE) for curved crack problems in plane elasticity by Chen [1, 2], and hypersingular integral equation (HSIE) for multiple curved cracks problem in plane elasticity by Nik Long and Eshkuvatov [3], also investigated for the interaction between curved and inclined cracks [4, 5]. Recently, Rafar et al. used it with HSIE to study the behavior of the stress intensity factor for multiple inclined or curved cracks in circular positions in plane elasticity [6].

In addition, for elastic half plane this method has also proven its effectiveness with HSIE for the curved crack problem [7], and with SIE for multiple cracks [8].

In this paper, the problem of the arc cracks in half circular positions in an isotropic half plane elasticity is proposed. The problem is formulated into singular integral equation based on the modified complex potential with free boundary condition. The final solution is obtained by using the curve coordinate method in conjunction with Gauss quadrature rule. The numerical examples demonstrate the effectiveness of this method with more complicated problem. Our results are identical with previous works.



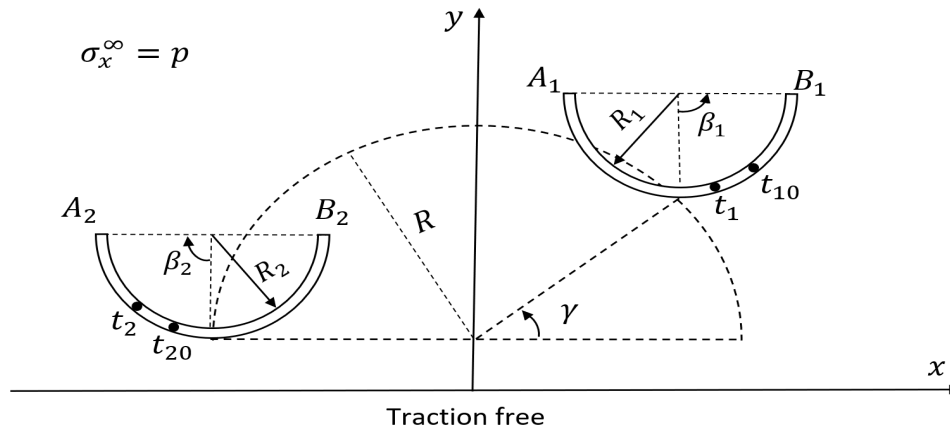


Figure 1: Two semi-circular arc cracks placed in a half circular position in an elastic half plane.

2. Formulation of the problem

Assume that, two arc cracks lie on the boundary of a virtual half circular with radius R subjected to uniaxial tension $\sigma_x^\infty = p$ as shown in Figure 1. The problem of a single arc crack in an elastic half plane is formulated into singular integral equation by using modified complex potential as follow [8]

$$\frac{1}{\pi} \int_L \frac{g'(t)dt}{(t-t_0)} + \frac{1}{2\pi} \int_L M_1(t, t_0)g'(t)dt + \frac{1}{2\pi} \int_L M_2(t, t_0)\overline{g'(\bar{t})}d\bar{t} = [N(t_0) + iT(t_0)], \quad (t_0 \in L), \quad (1)$$

where the kernels are defined as

$$M_1(t, t_0) = \left(-\frac{1}{(t-t_0)} + \frac{1}{(\bar{t}-\bar{t}_0)} \frac{d\bar{t}_0}{dt_0} - \frac{1}{(\bar{t}-t_0)} + \frac{(\bar{t}-t)}{(t-\bar{t}_0)^2} + \frac{d\bar{t}_0}{dt_0} \left(\frac{2t_0(\bar{t}-t)}{(t-\bar{t}_0)^3} + \frac{(3\bar{t}_0-\bar{t})}{(t-\bar{t}_0)^2} + \frac{2\bar{t}_0(\bar{t}_0-\bar{t})}{(t-\bar{t}_0)^3} \right) \right), \quad (2)$$

$$M_2(t, t_0) = \left(\frac{1}{(\bar{t}-\bar{t}_0)} - \frac{(t-t_0)}{(\bar{t}-\bar{t}_0)^2} \frac{d\bar{t}_0}{dt_0} - \frac{1}{(t-\bar{t}_0)} + \frac{(t-\bar{t})}{(\bar{t}-t_0)^2} + \frac{d\bar{t}_0}{dt_0} \left(\frac{(t-t_0)}{(t-\bar{t}_0)^2} \right) \right), \quad (3)$$

and $g'(t)$ is defined as

$$g'(t) = -\frac{2Gi}{\kappa+1} \frac{d\{(u(t)+iv(t))^+ - (u(t)+iv(t))^- \}}{dt}, \quad (t \in L), \quad (4)$$

and $((u(t)+iv(t))^+ - (u(t)+iv(t))^-)$ represents the displacements jump at a point t of the upper and lower faces of the crack L . In Eq. (4) the expression $d\{\}/dt$ should be defined as in [9, 10]. The single valuedness condition is

$$\int_L g'(t) dt = 0. \quad (5)$$

Whereas for two arc cracks we consider the distribution dislocation functions $g'_1(t_1)$ and $g'_2(t_2)$, respectively, for crack-1 (L_1), and crack-2 (L_2). Let $N_1(t_{10}) + iT_1(t_{10})$ be the tractions

applied on the crack-1 at the point t_{10} , and the tractions at t_{20} of crack-2 is $N_2(t_{20}) + iT_2(t_{20})$, then the equations for the two cracks subjected to traction can be written as [8]

$$\sum_{j=1}^2 \left\{ \frac{1}{\pi} \int_{L_j} \frac{g'_j(t_j) dt_j}{(t_j - t_{k0})} + \frac{1}{2\pi} \int_{L_j} M_1(t_j, t_{k0}) g'_j(t_j) dt_j + \frac{1}{2\pi} \int_{L_j} M_2(t_j, t_{k0}) \overline{g'_j(t_j)} dt_j \right\} = [N_k(t_{k0}) + iT_k(t_{k0})], \quad t_{k0} \in L_k \text{ for } k = 1, 2, \quad (6)$$

where the kernels are expressed as

$$M_1(t_j, t_{k0}) = \left(-\frac{1}{(t_j - t_{k0})} + \frac{1}{(\bar{t}_j - t_{k0})} \frac{dt_{k0}}{dt_{k0}} - \frac{1}{(\bar{t}_j - t_{k0})} + \frac{(\bar{t}_j - t_j)}{(t_j - t_{k0})^2} + \frac{dt_{k0}}{dt_{k0}} \left(\frac{2t_{k0}(\bar{t}_j - t_j)}{(t_j - t_{k0})^3} + \frac{(3t_{k0} - \bar{t}_j)}{(t_j - t_{k0})^2} + \frac{2t_{k0}(t_{k0} - \bar{t}_j)}{(t_j - t_{k0})^3} \right) \right),$$

$$M_2(t_j, t_{k0}) = \left(\frac{1}{(\bar{t}_j - t_{k0})} - \frac{(t_j - t_{k0})}{(\bar{t}_j - t_{k0})^2} \frac{dt_{k0}}{dt_{k0}} - \frac{1}{(t_j - t_{k0})} + \frac{(t_j - \bar{t}_j)}{(t_j - t_{k0})^2} + \frac{dt_{k0}}{dt_{k0}} \left(\frac{(t_j - t_{k0})}{(t_j - t_{k0})^2} \right) \right).$$

Note that if $j = k$ in Eq. (6) then the three integrals denoted to the traction influence on the crack- k caused by dislocation distribution function, $g'_k(t_k)$, on crack- k , else the three integrals are the traction influence on the crack- k caused by dislocation distribution function, $g'_j(t_j)$, on crack- j .

In addition, the single-valuedness conditions for the dislocation distribution on crack-1 and crack-2 are described as, respectively,

$$\int_{L_k} g'_k(t_k) dt_k = 0, \quad \text{for } k = 1, 2. \quad (7)$$

The curve length coordinate method is utilized for mapping the two cracks on a real axis s with an interval of $2c_1$ for crack-1 and $2c_2$ for crack-2, the mapping functions $t_1(s_1)$ and $t_2(s_2)$ are defined as [1]

$$g'_k(t_k)|_{t_k=t_k(s_k)} = h_k(s_k) = \frac{H_k(s_k)}{\sqrt{c_k^2 - s_k^2}}, \quad \text{where } H_k(s_k) = H_{k1}(s_k) + iH_{k2}(s_k), \quad \text{for } k = 1, 2. \quad (8)$$

The integral equations (6) and (7) are solved for $g'_1(t_1)$ and $g'_2(t_2)$ by applying the following Gauss quadrature rules [11]

$$\frac{1}{\pi} \int_{-c}^c \frac{F(s)}{\sqrt{c^2 - s^2}(s - s_{0,m})} ds = \frac{1}{M} \sum_{i=1}^M \frac{F(s_i)}{(s_i - s_{0,m})}, \quad (9)$$

$$\frac{1}{\pi} \int_{-c}^c \frac{F(s)}{\sqrt{c^2 - s^2}} ds = \frac{1}{M} \sum_{i=1}^M F(s_i), \quad (10)$$

where M is some integer, and

$$s_i = c \cos \frac{(i - 0.5)\pi}{M}, \quad s_{0,m} = c \cos \frac{m\pi}{M}, \quad \text{for } i = 1, 2, \dots, M, \text{ and } m = 1, 2, \dots, M - 1. \quad (11)$$

3. Numerical Examples

The stress intensity factor (SIF) at the crack tips A_1 and B_1 of crack-1, and at the crack tips A_2 and B_2 of crack-2 (see Figure 1) can be calculated as

$$(K_1 - iK_2)_{A_k} = \sqrt{2\pi} \lim_{t \rightarrow t_{A_k}} \sqrt{|t - t_{A_k}|} g'_k(t) = \sqrt{\frac{\pi}{c_k}} H_k(-c_k), \quad k = 1, 2, \quad (12)$$

$$(K_1 - iK_2)_{B_k} = -\sqrt{2\pi} \lim_{t \rightarrow t_{B_k}} \sqrt{|t - t_{B_k}|} g'_k(t) = -\sqrt{\frac{\pi}{c_k}} H_k(c_k), \quad k = 1, 2. \quad (13)$$

3.1. Example 1

An arc crack with radius R has the central angle $2\beta = \pi/6$ is placed inclinedly in the upper half plane as shown by Figure 2 subjected to the remote tension $\sigma_x^\infty = p$ with free traction on the boundary of half plane. The tip A of the crack has a distance e from the boundary where $e = 0.2R \sin \beta$. The crack has an inclined angle α with respect to the horizontal axis. The SIFs at the crack tips A , and B are expressed as

$$\begin{aligned} K_{1A} &= F_{1A}(\alpha) p \sqrt{\pi b}, & K_{1B} &= F_{1B}(\alpha) p \sqrt{\pi b}, \\ K_{2A} &= F_{2A}(\alpha) p \sqrt{\pi b}, & K_{2B} &= F_{2B}(\alpha) p \sqrt{\pi b}, \end{aligned} \quad \text{where } b = R \sin \beta.$$

In calculation, we have chosen $\alpha = 0^\circ, 10^\circ, 20^\circ, \dots, 90^\circ$, and $M = 25$ in quadrature rule. The values of SIFs are listed in Table 1. The results agree very well with those in [7]. Observed that, the SIF value increases as the inclined angle α increases, and $F_{1A} > F_{1B}$ because the tip A closer to the boundary.

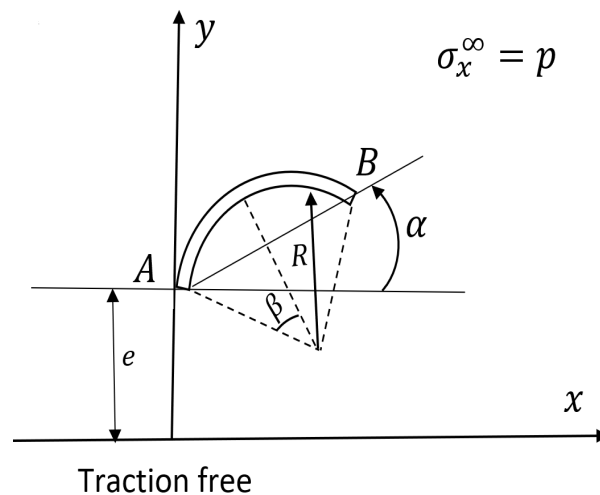


Figure 2: An inclined arc crack in an elastic half plane.

Table 1: The nondimensional SIFs for an arc crack in half plane (see Figure 2).

α	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
F_{1A}^*	0.0955	0.1428	0.3046	0.5398	0.8046	1.0562	1.2584	1.3848	1.4201	1.3607
F_{1A}^{**}	0.0957									1.3609
F_{2A}^*	-0.1611	-0.3634	-0.5193	-0.6007	-0.5913	-0.4913	-0.3146	-0.0849	0.1687	0.4154
F_{2A}^{**}	-0.1612									0.4154
F_{1B}^*	0.0955	0.0908	0.1482	0.2521	0.3918	0.5539	0.7221	0.8792	1.0083	1.0957
F_{2B}^*	0.1611	-0.0579	-0.2413	-0.3946	-0.5066	-0.5673	-0.5714	-0.5196	-0.4182	-0.2792

* Present study

** Chen [7]

3.2. Example 2

Let R_1 and R_2 are the radii for the first and second arc cracks, respectively, which are lie on the boundary of a virtual half circular with radius R subjected to uniaxial tension $\sigma_x^\infty = p$ as shown in Figure 3. The cracks have the central angle $2\beta_1 = 2\beta_2 = 2\beta = \pi/6$. The first crack is fixed on the x -axis whereas the second crack is placed on the boundary of a half circle with the varying angle, γ . The tip A_1 of the first crack has a distance e from the boundary where $e = 0.2R \sin \beta$. The SIFs at the crack tips A_1, B_1, A_2 , and B_2 are expressed as

$$\begin{aligned}
 K_{1A_k} &= F_{1A_k}(\gamma)p\sqrt{\pi b}, & K_{2D_k} &= F_{2D_k}(\gamma)p\sqrt{\pi b}, \\
 K_{1B_k} &= F_{1B_k}(\gamma)p\sqrt{\pi b}, & K_{2B_k} &= F_{2B_k}(\gamma)p\sqrt{\pi b},
 \end{aligned} \tag{14}$$

where $b = R \sin \beta$ for $k = 1, 2$.

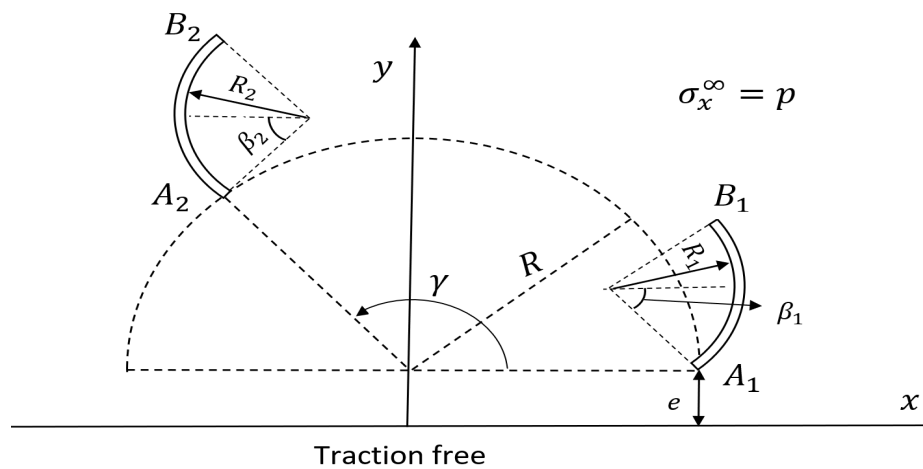


Figure 3: Two arc cracks placed in a half circular position in an elastic half plane.

In calculation, we have chosen $R_1 = R_2 = R$ and $\gamma = 40^\circ, 60^\circ, \dots, 180^\circ$. The values of SIFs are listed in Table 2. Observed that, the SIF increases as the angle position of the second crack, γ decreases. When $\gamma = 180^\circ$ the real parts of the SIF at both cracks tips are equal whereas the imaginary parts have the opposite sign.

Table 2: The nondimensional SIFs for two arc cracks in half plane (see Figure 3).

γ	F_{1A_1}	F_{2A_1}	F_{1B_1}	F_{2B_1}	F_{1A_2}	F_{2A_2}	F_{1B_2}	F_{2B_2}
40°	1.5315	-0.3538	1.3247	0.4223	1.2362	0.4067	1.1215	-0.2017
60°	1.4490	-0.3616	1.1724	0.3473	1.0669	0.3278	1.0555	-0.2215
80°	1.4297	-0.3613	1.1383	0.3270	1.0343	0.2982	1.0361	-0.2401
100°	1.4224	-0.3589	1.1264	0.3207	1.0206	0.2736	1.0218	-0.2588
120°	1.4151	-0.3568	1.1206	0.3192	1.0074	0.2503	1.0052	-0.2755
140°	1.4034	-0.3565	1.1147	0.3182	0.9980	0.2338	0.9900	-0.2896
160°	1.3853	-0.3586	1.1049	0.3157	1.0020	0.2292	0.9815	-0.2983
180°	1.3489	-0.3553	1.0794	0.3128	1.3489	0.3553	1.0794	-0.3128

3.3. Example 3

Assume that the two arc cracks center lie on the boundary of a virtual half circular with radius R subjected to uniaxial tension $\sigma_x^\infty = p$. Let R_1 and R_2 are radius for the first and second cracks, respectively, as illustrated in Figure 4. The SIFs at the crack tips A_1, B_1, A_2 , and B_2 are expressed as

$$\begin{aligned}
 K_{1A_k} &= F_{1A_k}(\gamma, R_2/R_1)p\sqrt{\pi b_k}, & K_{2D_k} &= F_{2D_k}(\gamma, R_2/R_1)p\sqrt{\pi b_k}, \\
 K_{1B_k} &= F_{1B_k}(\gamma, R_2/R_1)p\sqrt{\pi b_k}, & K_{2B_k} &= F_{2B_k}(\gamma, R_2/R_1)p\sqrt{\pi b_k},
 \end{aligned} \tag{15}$$

where $b_k = R_k \sin \beta_k$ for $k = 1, 2$.

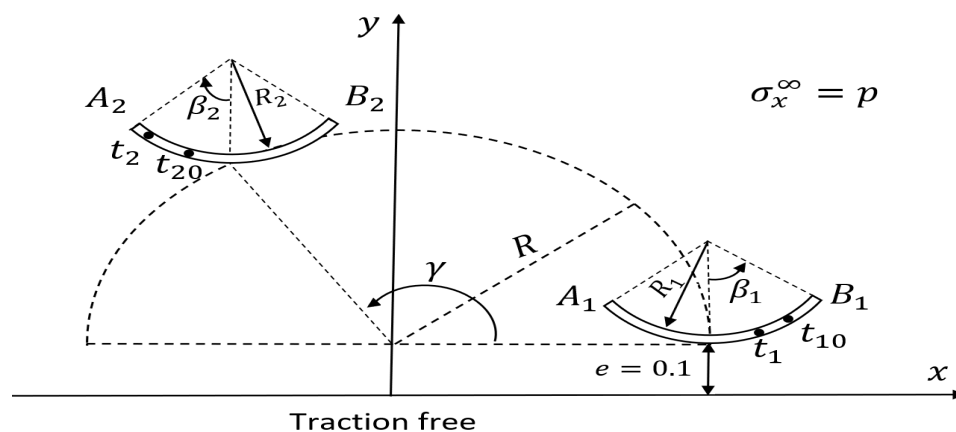


Figure 4: Two arc cracks placed in a half circular position in an elastic half plane.

Figure 5 shows the SIF when $\gamma = \pi, 3\pi/4, \pi/2$ and $\beta_1 = \beta_2 = \beta$ varies for $R_1/R = 0.9$, and $R_2/R_1 = 1$. Figures 5(a) and 5(b) shows that as γ increases the SIF at the tip A_1 , F_{1A_1} decreases, and SIF at the tip B_1 , F_{1B_1} increases, respectively. Figures 5(c) and 5(d) exhibit that at $\gamma = \pi/2$ and $\gamma = 3\pi/4$, the SIF at tips A_2 and B_2 (F_{1A_2} and F_{1B_2}) increase as β increases. Figure 5 also shows that as the two cracks are close to each other the SIF is higher at the tips A_1 and A_2 .

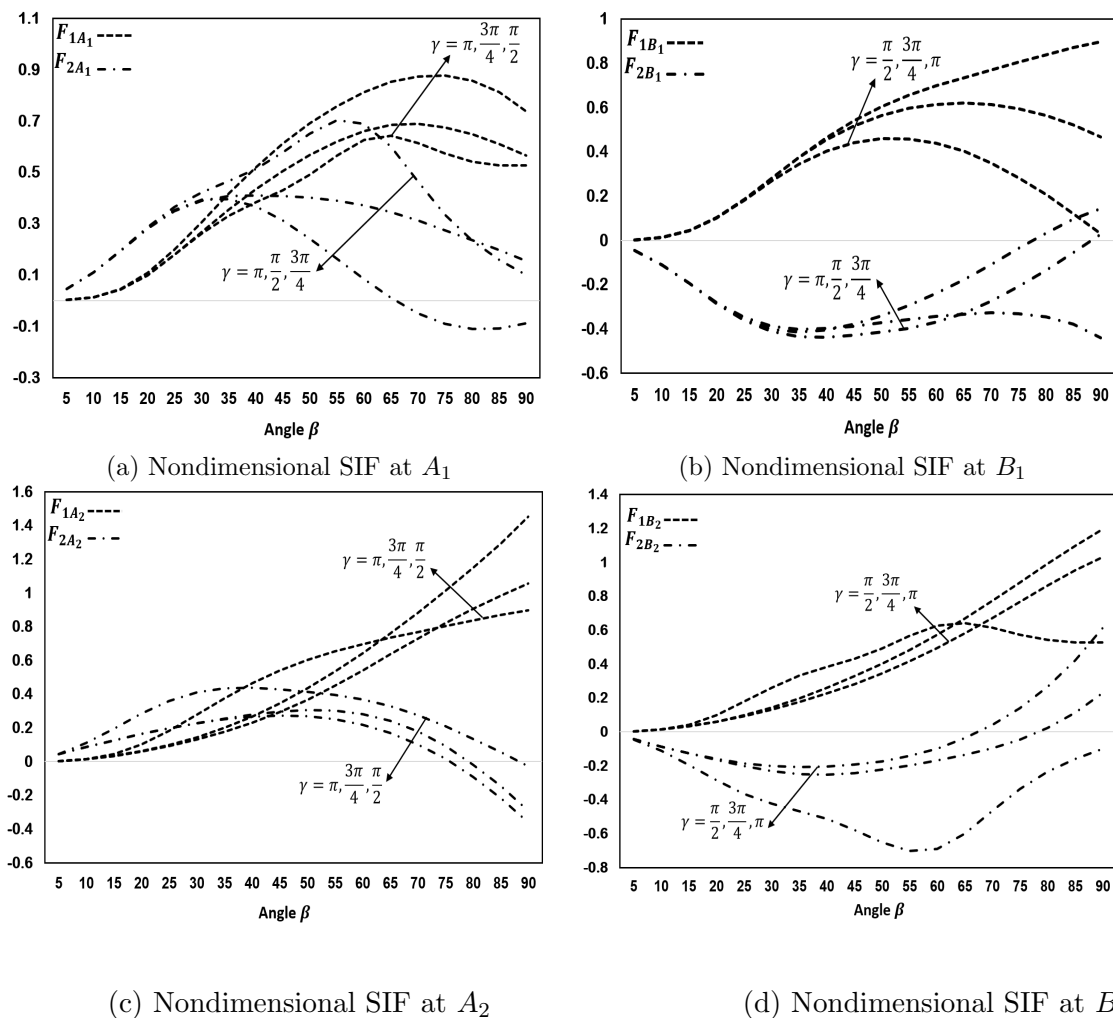


Figure 5: Non-dimensional SIFs for two arc cracks with changing β and $R_2/R_1 = 1$ for different angle of second crack position $\gamma = \pi, 3\pi/4, \pi/2$ (see Figure 4).

4. Conclusion

The arc cracks problems in a half circular position in half plane elasticity are investigated. The singular integral equation for two arc cracks is formulated. The behavior of the stress state at the crack tips is investigated by the numerical experiments. It is found that, the SIF increases as the angle γ decreases. The SIF is also affected by the position of the cracks.

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