CONFERENCE PROCEEDING

Application of Integration Method in Blood Flow System

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Abstract

There are a few theories about the types of fluid of the blood and one of them is that if the blood vessel does not contain a big amount of red blood cells, the blood is known as Power-law fluid. Power-law fluid has a shear rate depending on the obvious viscosity that categorizes non-Newtonian fluid in several types. In blood, the core region is modelled as a non-Newtonian fluid which is located at the centre of the blood vessel. By using the integration method, the velocity for the proposed mathematical model based on three different circumstances including the Power-law fluid for solute dispersion in blood flow with FHD effect, inclined artery and cholesterol stenosed artery are determined. The three circumstances which are considered for obtaining the velocity are done separately with the help of the given boundary conditions.

Keywords: *Power-law fluid, FHD effect, inclined artery, cholesterol stenosed artery*

INTRODUCTION

Nakayama and Boucher (1998) stated that fluid mechanics is the study of fluid in a two position that is the movement of fluid in motion or in stationary position. It is also a combined of hydraulics and hydrodynamics. Non-Newtonian fluid is a fluid which the viscosity is changed with the change of force or applied stress and it is not obey to the motion of the fluid. The power-law fluid is one of non-Newtonian fluid that been described by the two-parameter rheological model of a pseudoplastic fluid or a fluid whose viscosity decreases as shear rate increases.

Blood is mentioned in several passage of the Qur'an. The Qur'an discussed on the importance of the heart, blood and circulation both in humans and animals. As illustrated in the following verse: "We Cretaed man – We know what his soul whispers to him: We are closer to him that his jugular vein". (Surah Qaf verse 16).

There are few theories about the types of fluid of the blood. The types of fluid are different based on the size of the blood vessel. If the blood is located in the middle of the blood vessel, so the blood is known as Casson fluid. If the blood vessel is narrow, so it is known as Herschel-Bulkley fluid. If the blood vessel does not have a big amount of red blood cells, so the blood is known as Power-law fluid.

Integration is one of the operation in calculus in solving the problem including finding the area or volume of irregular objects. In physic, integration method is used to find time, velocity and displacement (Yek *et al*., 2017).

This research focuses on the blood in an artery that have been identified as the Power-law fluid. The momentum equations for each circumstance are solved by using integration to obtain the velocity of solute in blood flow system with FHD effect, inclined artery and cholesterol stenosed artery.

THE VELOCITY OF SOLUTE IN BLOOD FLOW SYSTEM WITH FHD EFFECT, INCLINED ARTERY AND CHOLESTEROL STENOSED ARTERY

In this section, the velocity, $\overline{u}$, of solute in blood flow system with FHD effect, inclined artery and cholesterol stenosed artery are presented. The momentum equations for each circumstances are stated as following:

The momentum equations for FHD flow is shown as follows (Bali & Awasthi, 2012):

$$\frac{d\bar{p}}{d\bar{z}}+\mu_0 M\frac{\partial\overline{H}}{\partial\bar{z}}=-\frac{1}{\bar{r}}\frac{d}{d\bar{r}}(\bar{r}\bar{\tau}). \tag{1.1}$$

Next, the momentum equations for inclined artery (pipe) is defined as follows (Sreenadh *et al*., 2011):

$$\frac{d\bar{p}}{d\bar{z}}-\bar{\rho}\bar{g}\sin\theta=-\frac{1}{\bar{r}}\frac{d}{d\bar{r}}(\bar{r}\bar{\tau}), \tag{1.2}$$

and Sharp in 1993 has determined the momentum equations for cholesterol stenosed artery (Sharp, 1993):

$$\frac{d\bar{p}}{d\bar{z}}=-\frac{1}{\bar{r}}\frac{d}{d\bar{r}}(\bar{r}\bar{\tau}). \tag{1.3}$$

In this study, a general equation in power-law fluid model is characterized by the flow curve (velocity gradient) as follows (Sharp, 1993):

$$-\frac{d\bar{u}}{d\bar{r}}=\frac{1}{\mu}(\bar{\tau})^n. \tag{1.4}$$

The boundary condition of Eq. (1.4) is given as follows:

$$\overline{u}=0 \text{ at } \bar{r}=\overline{R}(\bar{z}).$$

Firstly, the momentum equation for cholesterol stenosed artery are discussed. The boundary condition of Eq. (1.3) is given by

$$\bar{\tau} \text{ is finite at } \bar{r}=0\,. \tag{1.5}$$

From Eq. (1.3) and (1.5), the shear stress was found as

$$-\frac{d\overline{p}}{d\overline{z}}\left(\frac{0}{2}+x\right)=0(\bar{\tau})$$

$$\overline{\tau}=-\frac{d\overline{p}}{d\overline{z}}\left(\frac{\overline{r}}{2}\right). \tag{1.6}$$

Next, by substituting Eq. (1.6) into Eq. (1.3), it is found that

$$-\frac{d\overline{u}}{d\overline{r}}=\frac{1}{\overline{\mu}}\overline{\tau}^n$$

$$-\frac{d\overline{u}}{d\overline{r}}=\frac{1}{\overline{\mu}}\left[-\frac{d\overline{p}}{d\overline{z}}\left(\frac{\overline{r}}{2}\right)\right]^n. \tag{1.7}$$

Then, Eq. (1.7) is integrated to find the velocity of the equation.

$$\overline{u}=-\frac{1}{2^n\overline{\mu}}\left(-\frac{d\overline{p}}{d\overline{z}}\right)^n\left(\frac{\overline{r}^{n+1}}{n+1}+x\right)$$

$$0=-\frac{1}{2^n\overline{\mu}}\left(-\frac{d\overline{p}}{d\overline{z}}\right)^n\left[\frac{\left(\overline{R}(\overline{z})\right)^{n+1}}{n+1}+x\right] \tag{1.8}$$

$$0=-\frac{1}{2^n\overline{\mu}}\left(-\frac{d\overline{p}}{d\overline{z}}\right)^n\left[\frac{\left(\overline{R}(\overline{z})\right)^{n+1}}{n+1}\right]-\frac{1}{2^n\overline{\mu}}\left(-\frac{d\overline{p}}{d\overline{z}}\right)^n x$$

$$x=-\frac{\left(\overline{R}(\overline{z})\right)^{n+1}}{n+1} \tag{1.9}$$

Substitute (1.9) into Eq. (1.8)

$$\overline{u}=-\frac{1}{2^n\overline{\mu}}\left(-\frac{d\overline{p}}{d\overline{z}}\right)^n\left[\frac{\overline{r}^{n+1}}{n+1}+\left(-\frac{\left(\overline{R}(\overline{z})\right)^{n+1}}{n+1}\right)\right].$$

Lastly, Eq. (1.1) and Eq. (1.2) can be solved to boundary conditions (1.5) and with similar calculation as Eq. (1.3) it implies that the velocity for FHD effect is

$$\bar{u}=-\frac{1}{2^n\bar{\mu}}\left(-\frac{d\bar{p}}{d\bar{z}}-\mu_0 M\frac{\partial\bar{H}}{\partial\bar{z}}\right)^n\frac{\bar{r}^{n+1}}{n+1}-\frac{\bar{r}^{n+1}}{n+1}$$

and the velocity for inclined artery is

$$\bar{u}=-\frac{1}{\bar{\mu}}\frac{1}{2^n}\left(-\frac{d\bar{p}}{d\bar{z}}+\bar{\rho}\bar{g}\sin\theta\right)^n\left(\frac{\bar{r}^{n+1}}{n+1}\right)-\left(\frac{\bar{r}^{n+1}}{n+1}\right).$$

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