



Leak detection of non-isothermal transient flow of hydrogen-natural gas mixture

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ABSTRACT

Transient pressure wave detection analysis to detect the location of leakage of non-isothermal flow in an inclined pipeline containing hydrogen-natural gas mixture is investigated. The governing equations are solved using the reduced order modelling technique. The effects of inclination angles, mass ratio of gas mixture and temperature change on the pressure and celerity waves in an inclined pipeline are discussed. The solutions for isothermal flow in a horizontal pipeline show good agreement with published results. For non-isothermal flow an increase in the mass ratio lead to an increase in the pressure and celerity waves, while the leak location and amount of leak discharge decrease. However, it is noted that the amount of leak discharge is still higher than that of isothermal flow. It is also observed that an increase in the inclination angle increases the pressure drop and leak discharge but the celerity wave and the leak location do not seem to be affected. Thus, to reduce the leak discharge, the inclination angle of the pipeline should be reduced and further, to ensure that leakage does not occur before the calculated leak position, the mass ratio of hydrogen to natural gas should not be more than 0.5.

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1. Introduction

Gas distribution pipeline networks are systems with hundreds or thousands of kilometers of pipes which contain compression stations and other devices like valves. These types of systems work at high pressure and use compression stations to supply the gas over long distances with enough energy. In many pipeline simulations, rapid changes in the solution are present due to the disturbances generated by the fluctuations in operation of system controlling devices such as valves, compressors and pressure regulators (Chaczykowski, 2010). As a result, non-isothermal gas flow models need to be considered to account for sharp changes in the gas pressure, temperature and flow rate.

For the non-isothermal flow in a pipeline, the gas properties can be assumed to be varied or not constant over any cross section in a pipeline. When a gas mixture is subjected to a temperature change, some properties, such as the density and viscosity, will change accordingly. In some situations, these changes are large enough to have a substantial influence on the flow characteristics. Transient non-isothermal modelling of different pipeline operating

conditions is widely used, especially when testing various control strategies proposed for the gas transmission system. Solution of the non-isothermal flow model requires certain types of pipeline thermal models as a component of the flow model responsible for heat flow in the gas and through the pipeline walls. Depending on the form of heat transfer term, there are two models for obtaining the amount of heat exchanged by the gas with its surrounding environment, which are steady state and unsteady heat transfer (Abbaspour and Chapman, 2008; Chaczykowski, 2010; Ebrahimzadeh et al., 2012; Osiaadacz and Chaczykowski, 2001; Tentis et al., 2003). However, all these works have been considered mostly in fluids involving natural gas only but have not yet been applied in hydrogen natural-gas mixture.

Subani and Amin (2015) and Elaoud and Hadj-Taïeb (2008) studied the transient flow in hydrogen-natural gas mixtures, but they assumed that the flow is isothermal and the pipeline is laid horizontally. Later, Uilhoorn (2009) solved the gas flow of mixed hydrogen with natural gas and transport via high pressure in an inclined pipeline. Hydrogen is always combined with natural gas to enhance storage capacity, quicken burning capability and reduce air pollution (Agaie, 2014). According to Elaoud and Hadj-Taïeb (2008), during the transition period towards a full development of hydrogen market, the transmission costs by the construction of

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new network pipelines become relatively expensive. Many petroleum companies utilize the existing natural gas pipeline networks to transported hydrogen and natural gas in the same pipeline to reduce the transportation cost.

The existing pipeline is designed and constructed specifically for natural gas only. The chemical and physical properties of hydrogen differ significantly from those of natural gas. Addition of even a small quantity of hydrogen to natural gas (10%–20% hydrogen) (Veziroglu and Barbir, 1992) requires special attention especially on the leakage problem because hydrogen is a reactive gas and has high pressure (Subani et al., 2015). The problem of hydrogen or hydrogen-natural gas mixture release appears to be a major potential risk that should be predicted (Elaoud and Hadj-Taieb, 2009; Elaoud et al., 2010).

From the 3751 pipeline incidents, that were recorded between 1994 and 1999, two third of them were as a results of pipe leaks (Lydell, 2000). Leakage in pipelines can cause serious problems related not only to the environment or safety but also to the economy (Elaoud et al., 2010). Leaks also waste natural resources and create public health risks. Risk of leakage through pipelines is well studied for natural gas (Turner and Mudford, 1988; Wilkening and Baraldi, 2007), but not for hydrogen or hydrogen-natural gas mixtures. Subani et al. (2015), Elaoud and Hadj-Taieb (2008) and Elaoud et al. (2010) determined the leakage of hydrogen-natural gas mixtures in pipelines, but they assumed the transient isothermal flow. They leave out the gravity and inclination terms in the momentum equation, and they do not consider the energy equation in their models.

The main objective of this paper is to determine and locate leakage on non-isothermal transient flow in the inclined pipeline system of hydrogen-natural gas mixtures based on transient pressure wave analysis. The effect of hydrogen mass ratio on the transient flow of hydrogen-natural gas mixture and leak discharge is analyzed. The effect of body force due to the inclined pipeline is also presented. The effect of temperature change on the non-isothermal flow model is determined by considering the unsteady heat transfer term in the energy equation. The transient non-isothermal flow is created by the sudden closure of a downstream shut-off valve. The governing equations are solved by the reduced order modelling (ROM) technique (Agaie and Amin, 2014; Behbahani-Nejad and Shekari, 2008, 2010; Subani and Amin, 2015; Subani et al., 2015).

2. Mathematical formulation

The governing equations consist of three coupled non-linear hyperbolic partial differential equations. The flow is assumed to be one dimensional, non-isothermal, and compressible and to include transient condition. The fluid is assumed to be a homogeneous mixture of hydrogen and natural gas.

2.1. Governing equation

From the principle of conservation of mass, momentum and energy laws, the governing equations for the transportation of hydrogen-natural gas mixtures in an inclined pipeline are given by the following:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + P)}{\partial x} + \frac{f \rho u |u|}{2D} + \rho g \sin \theta = 0 \quad (2)$$

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho u e + P u)}{\partial x} = \rho q - \rho u g \sin \theta \quad (3)$$

where ρ is defined as density, u is the gas velocity where the modulus sign is to ensure that the frictional force shall always act opposite to the direction of motion, P is the pressure, f is the coefficient of friction, D is the diameter of the pipeline, g is the gravitational force, θ is an angle between the friction force and the x direction, and q is heat transfer.

These governing equations (1) (2) and (3) will be used in transient analysis of non-isothermal hydrogen-natural gas mixture in pipeline where kinetic energy $u^2/2$ is neglected.

2.2. Equation of state

An equation of state for a gas relates the variables of pressure, density and temperature. The equation of state for perfect gas, which is commonly used in the gas industry, is given by:

$$P = \rho R T \quad (4)$$

where R is the specific gas constant and T is the temperature.

For compressible flow, the relation of equation of state with the celerity pressure wave c is denoted as:

$$P = \rho c^2 \quad (5)$$

From relation of perfect gas, the specific heat at constant volume C_v and internal energy e are defined as:

$$C_v = \frac{R}{r - 1} \quad (6a)$$

$$e = C_v T \quad (6b)$$

Substituting (6a) and (6b) into (4), then the perfect gas equation is written as:

$$P = \rho C_v (r - 1) \frac{e}{C_v} = (r - 1) \rho e \quad (7)$$

2.3. Hydrogen-natural gas mixture equation

Density of hydrogen and natural gas is defined as:

$$\rho_h = \frac{m_h}{V_h} \text{ and } \rho_g = \frac{m_g}{V_g} \quad (8)$$

where m_g , m_h , V_g and V_h are defined as the mass of natural gas and hydrogen and volume of natural gas and hydrogen, respectively.

For hydrogen-natural gas mixtures, the hydrogen mass ratio will be used in determining the mixture density, where the mass ratio of the mixture is given as:

$$\phi = \frac{m_h}{m_h + m_g} \quad (9)$$

From the definition of density of gas, the density of hydrogen-natural gas mixture is defined as:

$$\frac{1}{\rho} = \frac{V_m}{M_m} \quad (10)$$

where $V_m = V_h + V_g$ and $M_m = m_h + m_g$.

Substituting (8) and (9) into (10), the density of hydrogen-

natural gas mixture is defined according to the mass ratio ϕ , given by:

$$\frac{1}{\rho} = \frac{m_h}{\rho_h(m_h + m_g)} + \frac{m_g}{\rho_g(m_h + m_g)} \quad (11)$$

$$\rho = \left[\frac{\phi}{\rho_h} + \frac{(1-\phi)}{\rho_g} \right]^{-1}$$

The density of hydrogen and natural gas according to isentropic laws is written as:

$$\rho_h = \rho_{h0} \left(\frac{P}{P_0} \right)^{\frac{1}{n'}} \quad (12a)$$

$$\rho_g = \rho_{g0} \left(\frac{P}{P_0} \right)^{\frac{1}{n''}} \quad (12b)$$

where each ρ_{h0} and ρ_{g0} represents the initial density of hydrogen and natural gas, respectively. P is transient pressure and P_0 is permanent pressure, n' and n'' represent the flow process index of hydrogen and natural gas, respectively.

To solve the governing equations (1) (2) and (3) numerically, the density of the hydrogen-natural gas mixture ρ must be expressed according to the gas pressure P . Substituting (12a) and (12b) into (11), the expression of the average density of the mixture is given by:

$$\rho = \left[\frac{\phi}{\rho_{h0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n'}} + \frac{(1-\phi)}{\rho_{g0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n''}} \right]^{-1} \quad (13)$$

For compressible flow, the celerity of pressure wave is defined as:

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s \text{ thus } c = \left(\frac{\partial \rho}{\partial P} \right)_s^{-\frac{1}{2}} \quad (14)$$

where the subscript s denotes the condition of constant entropy.

By taking the derivative of (13) with respect to P , and substituting into (14), then the celerity pressure wave yields:

$$c = \left[\frac{\phi}{\rho_{h0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n'}} + \frac{(1-\phi)}{\rho_{g0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n''}} \right] \times \left[\frac{1}{P} \left\{ \frac{\phi}{n' \rho_{h0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n'}} + \frac{(1-\phi)}{n'' \rho_{g0}} \left(\frac{P_0}{P} \right)^{\frac{1}{n''}} \right\} \right]^{-\frac{1}{2}} \quad (15)$$

2.4. Boundary conditions

The boundary conditions depend on the valve operational time and the types of closure. The boundary conditions at the initial point, $x = 0$, are given by:

$$\rho(0, t) = \rho_0(t) \quad (16a)$$

$$\rho u(0, t) = m_0(t) \quad (16b)$$

$$T(0, t) = T_0(t) \quad (16c)$$

where ρ_0 , m_0 and T_0 are defined as density, mass flux and temperature at the inlet pipeline, respectively.

The boundary conditions at the end point $x = L$ are:

$$\rho(L, t) = \rho_L(t) \quad (17a)$$

$$\rho u(L, t) = m_L(t) \quad (17b)$$

$$\rho u e(L, t) = h_L(t) \quad (17c)$$

where ρ_L , m_L and h_L are defined as density, mass flux and heat flux at the outlet pipeline, respectively.

2.5. Initial conditions

The initial conditions that are assumed to be in a steady state condition at $t = 0$ are:

$$\frac{\partial \rho u}{\partial x}(x, 0) = 0 \quad (18a)$$

$$\frac{\partial \rho u^2}{\partial x}(x, 0) = -c^2 \frac{\partial \rho}{\partial x} - \frac{f \rho u |u|}{2D} - \rho g \sin \theta \quad (18b)$$

$$\frac{\partial(\rho u e)}{\partial x}(x, 0) = -\frac{\partial P u}{\partial x} + \rho q - \rho u g \sin \theta \quad (18c)$$

3. Numerical solution

To solve the governing equations (1) (2) and (3) numerically, ROM is employed (Agaie, 2014; Agaie and Amin, 2014; Subani and Amin, 2015; Subani et al., 2015; Behbahani-Nejad and Shekari, 2008, 2010; Behbahani-Nejad et al., 2004). These governing equations are transformed in the flux vector form as:

$$\frac{\partial Q}{\partial t} + \frac{\partial E(Q)}{\partial x} - H(Q) = 0 \quad (19)$$

where

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho e \end{bmatrix}, \quad E(Q) = \begin{bmatrix} \rho u^2 + P \\ \rho u e + P u \end{bmatrix} \text{ and } H(Q) = \begin{bmatrix} 0 \\ -\frac{f \rho u |u|}{2D} - \rho g \sin \theta \\ \rho q - \rho u g \sin \theta \end{bmatrix}$$

3.1. Implicit steger-warming flux vector splitting scheme

The implicit Steger-Warming flux vector splitting method (FSM) is used as the numerical scheme (Hoffmann and Chiang, 2000; Subani et al., 2015). The governing equation (19) is discretized using FSM, and the resulting FSM scheme in the delta notation is:

$$\left[\frac{\Delta t}{\Delta x} A_{i+1}^{n(-)} \right] \Delta Q_{i+1} - \left[\frac{\Delta t}{\Delta x} A_{i-1}^{n(+)} \right] \Delta Q_{i-1} + \left[I + \frac{\Delta t}{\Delta x} (A_i^{n(+)} - A_i^{n(-)}) - \Delta t B_i^n \right] \Delta Q_i = -\frac{\Delta t}{\Delta x} [E_i^{n(+)} - E_{i-1}^{n(+)} + E_{i+1}^{n(-)} - E_i^{n(-)}] + \Delta t H_i^n \quad (20)$$

where

$$\Delta Q = Q^{n+1} - Q^n \quad (21)$$

and I is the identity matrix, and A and B are the Jacobian matrices defined as:

$$A = \frac{\partial E(Q)}{\partial Q} \text{ and } B = \frac{\partial H(Q)}{\partial Q} \quad (22)$$

From (20), the Jacobian matrix A is split into two parts (positive and negative) as denoted by A^+ and A^- :

$$A^+ = \frac{u}{2c} \begin{bmatrix} -u+c & \frac{u+c}{u} & \frac{(r-1)(c-u)}{cu} \\ -(u^2+c^2) & \frac{(u+c)^2}{u} & \frac{(r-1)(u+c)}{u} \\ \frac{rc^2(u+c)}{r-1} & \frac{rc^2(u+c)}{u(r-1)} & \frac{rc(u+c)}{u} \end{bmatrix} \quad (23a)$$

$$A^- = \frac{u}{2c} \begin{bmatrix} u-c & \frac{c-u}{u} & \frac{(r-1)(u-c)}{cu} \\ (u-c)^2 & -\frac{(u-c)^2}{u} & \frac{(r-1)(c-u)}{u} \\ \frac{rc^2(u-c)}{r-1} & -\frac{rc^2(u-c)}{u(r-1)} & \frac{rc(u-c)}{u} \end{bmatrix} \quad (23b)$$

while E^+ and E^- are the positive and negative parts of the flux vector E given by:

$$E^+ = \begin{bmatrix} \frac{\rho(u+c)}{2} \\ \frac{\rho(u+c)^2}{2} \\ \frac{r\rho c^2(u+c)}{2(r-1)} \end{bmatrix} \text{ and } E^- = \begin{bmatrix} \frac{\rho(u-c)}{2} \\ \frac{\rho(u-c)^2}{2} \\ \frac{r\rho c^2(u-c)}{2(r-1)} \end{bmatrix} \quad (24)$$

When (20) is applied to each grid point, a block tridiagonal of systems of algebraic equations is obtained. The system of equations is solved at each time step and results in ΔQ . The values of Q at the next time step are calculated using (21).

To perform the Eigen analysis and to construct ROM technique based on the flow eigenmodes, the finite difference equation in (20) is linearized. The linearization can be achieved by assuming steady state results in which stability perturbation is used to obtain the transient solution at each nodal point. For linearization, the flow field variables at each time step are considered as:

$$Q^{n+1} = Q^0 + \hat{Q}^{n+1} \quad (25)$$

where $Q^{n+1} = Q^0 + \hat{Q}^{n+1}$ represents the corresponding steady state values, and $\hat{Q}$ represents the perturbation values.

Substituting (25) into (20) yields:

$$\begin{aligned} & -\left[\frac{\Delta t}{\Delta x} A_{i-1}^{0(+)}\right] \hat{Q}_{i-1}^{n+1} + \left[\frac{\Delta t}{\Delta x} A_{i+1}^{0(-)}\right] \hat{Q}_{i+1}^{n+1} \\ & + \left[I + \frac{\Delta t}{\Delta x} (A_i^{0(+)} - A_i^{0(-)})\right] \hat{Q}_i^{n+1} - \Delta t B_i^0 \hat{Q}_i^{n+1} = \hat{Q}_i^n \end{aligned} \quad (26)$$

The above linearization (26) is represented in the form of the following eigenvalue problem:

$$W^0 \hat{Q}^{n+1} = I \hat{Q}^n + V^{n+1} \quad (27)$$

where $V^{n+1} = -\left[\frac{\Delta t}{\Delta x} A_{i-1}^{0(+)}\right] \hat{Q}_{i-1}^{n+1} + \left[\frac{\Delta t}{\Delta x} A_{i+1}^{0(-)}\right] \hat{Q}_{i+1}^{n+1}$ and $W^0 = \left[I + \frac{\Delta t}{\Delta x} A_i^{0(+)} - A_i^{0(-)}\right] - \Delta t B_i^0$ and V is defined as a vector consisting of the imposed values by the boundary conditions and W^0 is the coefficient of the matrix represented by the left hand side terms of (27). The finite difference equation (26) is the linearized form of FSM.

3.2. Reduced order modelling

To construct the ROM technique, the zero eigenvalue in the Eigen system of matrix is required. For zero forcing function V , we need consider the homogeneous part of (27) by setting:

$$\hat{Q}_i = x_i \exp(\lambda_i t) \alpha_i \exp(iz_i x) \quad (28)$$

where λ_i represents eigenvalues, x_i is eigenvector and $z_i = \exp(\lambda_i \Delta t)$.

Equation (28) is used to obtain the following generalized eigenvalue problem. Then, the general form of the diagonal matrix which contains the eigenvalues and eigenvectors are written as:

$$ZW^n X = IX \quad (29)$$

where Z represents the diagonal matrix of eigenvalues at each time step, and X is the matrix with columns that are the right eigenvector.

On the other hand, the column matrix with left eigenvectors satisfies the following relation:

$$(W^0)^T YZ = IY \quad (30)$$

where Y is the matrix with rows that are the left eigenvectors.

The eigenvectors are then normalized to satisfy the following orthogonality conditions:

$$Y^T W^0 X = I \text{ and } Y^T I X = Z \quad (31)$$

Agai (2014), Agai and Amin (2014) and Subani et al. (2015) analyzed the eigenmodes based on the time basis, then (28) reduces the gas flow behaviour to the sum of individual nodes:

$$\hat{Q} = X \hat{c} \quad (32)$$

where $\hat{Q}$ and $\hat{c}$ are denoted by:

$$\hat{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} \text{ and } \hat{c} = \begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \vdots \\ \hat{c}_n \end{bmatrix} \quad (33)$$

Multiplying (32) by $Y^T W^0$ we have:

$$\hat{c} = Y^T W^0 Q \quad (34)$$

Substituting (31) and (27) and then multiplying by Y^T gives a set of N uncoupled equations for the nodal coordinates $\hat{c}$:

$$\hat{c}^{n+1} = Z \hat{c}^n + Y^T V^{n+1} \quad (35)$$

Since the orthogonality conditions are satisfied, the eigenmodes can be retained to construct ROM by using (35).

3.3. Treatment of boundary conditions

The boundary conditions have been developed for the governing equations (1) (2) and (3). The finite difference equations developed for the treatment of boundary conditions are given as:

$$\rho_0^{n+1} = \rho_0^n + \frac{\Delta t}{\Delta x} (m_0^n - m_1^n) \quad (36a)$$

$$m_0^{n+1} = m_0^n - \frac{\Delta t}{\Delta x} \left[\left(\frac{m_1^{n2}}{\rho_1^n} + c_1^{n2} \rho_1^n \right) - \left(\frac{m_0^{n2}}{\rho_0^n} + c_0^{n2} \rho_0^n \right) \right] - \frac{f}{2D} \frac{m_0^n |m_0^n|}{\rho_0^n} \Delta t - \rho_0^n g \sin \theta \quad (36b)$$

$$h_0^{n+1} = h_0^n - \frac{\Delta t}{\Delta x} \left[\left(\frac{h_1^{n2} m_1^n}{\rho_1^n} + c_1^{n2} m_1^n \right) - \left(\frac{h_0^{n2} m_0^n}{\rho_0^n} + c_0^{n2} m_0^n \right) \right] + \rho_0^n q + m_0^n g \sin \theta \quad (36c)$$

where m_0 and m_1 are the mass flux before and after leak point, ρ_0 and ρ_1 are density of gas mixture before and after leak point, c_0 and c_1 are celerity wave of gas mixture before and after leak point, and h_0 and h_1 are the heat flux of gas mixture before and after leak point, respectively.

4. Leak detection

The problem is focused on transients occurring in a single inclined pipeline with a compressor at the upstream end and a sudden closure valve at the downstream end as illustrated in Fig. 1. A single leak is supposed to exist at an intermediate section of the pipeline and located at a distance X_L from the compressor. The pipeline with a length L is divided into two segments, which are Pipe 1, from the compressor to the leak, and Pipe 2 from the leak to the valve.

4.1. Leak modelling

The leak orifice can be simulated based on the flow rate through a small orifice between the high-pressure pipeline and the environment. The discharged flow Q_L from the orifice can be calculated by the given equation:

$$Q_L = \left(\rho_L C_d A_L \sqrt{2P_L / \rho_L} \right) / X_L \quad (37)$$

where C_d is a discharge coefficient, A_L is the orifice area of the leak

with radius r_L , P_L and ρ_L are the pressure and the density of gas mixture at the leak position, respectively and X_L is the distance of leak from the reservoir.

The analysis of transient pressure wave of hydrogen-natural gas mixtures is based on the properties of transmission and reflection of pressure wave caused by a sudden closure of a downstream valve. When the initial pressure wave reaches the leak, it will produce a reflection when it arrives back at the downstream end section. The time difference between the initial transient wave and the reflected wave is then measured, and the position of leakage X_L in the pipeline is calculated by:

$$X_L = L - \frac{\Delta t_L c}{2} \quad (38)$$

where X_L is the distance to the leak from the upstream end section, Δt_L is the time difference between the initial transient wave and the reflected wave and c is the transient celerity wave.

By considering the partial reflection of pressure waves which take place at the leak, the discharged behaviour of the leak may be obtained by:

$$Q_L = \frac{A}{c} (\Delta P_0 - \Delta P_L) \quad (39)$$

where Q_L is the leak discharge for transient flow, ΔP_0 and ΔP_L are pressure rises produced by the sudden closure of the downstream end valve, assuming the pipe to be intact or containing a leak, respectively.

4.2. Boundary conditions when leak occurs

The transient flow is created by the sudden closure of the valve at the downstream end ($x = L$). At this point the discharge flow of leakage is $Q_2(L, t) = 0$. At the upstream end ($x = 0$ and $t > 0$), the condition of the reservoir is given by the constant pressure $P_1(0, t) = P_0$.

At the leak point, (37) is implemented as the internal boundary condition. According to Elaoud et al. (2010), the two relationships that relate the upstream pressure and discharge flow to the downstream pressure and discharge flow are:

$$P_1 = P_2 = P_p \quad (40a)$$

$$Q_1 = Q_2 + \left(\rho_p C_d A_L \sqrt{2P_p / \rho_p} \right) / X_L \quad (40b)$$

where P_1 , P_2 and P_p are the pressure before, after and at leak point, respectively (refer to Fig. 1). Q_1 and Q_2 are the discharge flow at upstream and downstream of the leak, respectively, and $(\rho_p C_d A_L \sqrt{2P_p / \rho_p}) / X_L$ is the leakage mass flow rate.

5. Results and discussion

Fig. 2 illustrates the transient pressure distribution with a sudden closure valve for an intact pipeline with different mass ratio ϕ . The above method of ROM is validated by comparison of the pressure in the case of isothermal and non-isothermal flow. It is observed that the present results obtained for the pressure on isothermal flow give approximately the same results as Subani et al. (2015). These graphs are plotted for natural gas ($\phi = 0$), hydrogen-natural gas mixture ($\phi = 0.5$) and hydrogen ($\phi = 1$). As indicated in Fig. 2, the pressure on non-isothermal flow is higher compared to the pressure on isothermal flow. The pressure is also higher for hydrogen compared to the natural gas and hydrogen-natural gas mixture.

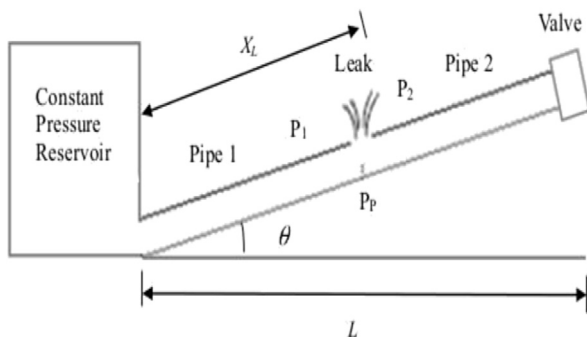


Fig. 1. Hydrogen-natural gas mixture installation with leakage in an inclined pipeline.

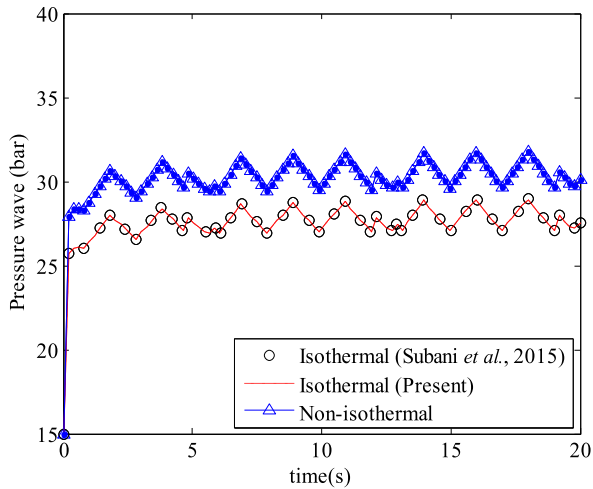
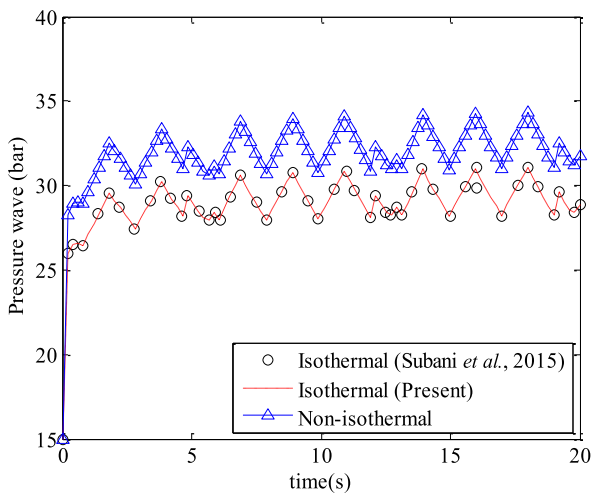
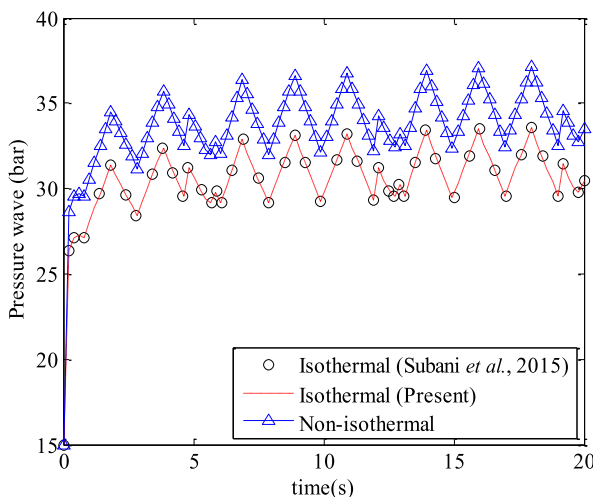
a) Natural gas ($\phi = 0$)b) Mixture ($\phi = 0.5$)a) Hydrogen ($\phi = 1$)

Fig. 2. Comparison of transient pressure wave distribution with sudden closure valve for an intact pipeline (horizontal) with different mass ratio ϕ (Subani et al., 2015).

5.1. Description of problem

According to Subani et al. (2015), increasing values of pressure are better to increase the performance of energy when the gas flows in pipelines. Pressure distribution is an important parameter to calculate the discharged amount of gas released due to pipeline leakage in (37). This parameter is also needed in the detection and locates the position of leaks in the pipeline.

To locate the leak by the transient flow analysis of hydrogen-natural gas mixtures, an iron pipe 0.4 m in diameter and 600 m long is considered. The pipe is connected to a compressor at the upstream end, while the sudden closure valve is at the downstream end. A mass flow $Q_0 = 55 \text{ kg/s}$ at a static temperature $T = 15^\circ\text{C}$ and an absolute pressure $P = 35 \text{ bar}$ are assumed. The properties of hydrogen and natural gas used in the calculations are those of Elaoud et al. (2010) and Subani et al. (2015).

5.2. Effect of temperature change

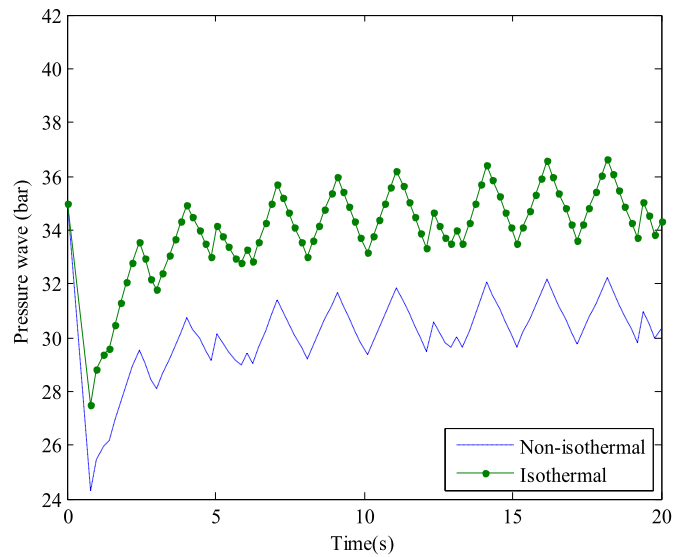
According to Chaczykowski (2010), transient non-isothermal gas flow models need to be considered to account for sharp changes in the gas pressure, temperature and flow rate. When transient flow are considered, it is assumed that the pressure changes occurred instantaneously, which means that no time is allowed for heat transfer to take place between the gas in the pipe and the pipe's surroundings (Osiaiecz and Chaczykowski, 2001). The effects on flow characteristics due to pipeline surrounding temperature change are determined. The inclination terms are included in the models in order to represent real situation of pipeline systems. According to Herran-Gonzalez et al. (2009), the pressure along the pipe has a strong dependence on the inclination term.

Fig. 3 shows the plots of the transient pressure as a function of time with a sudden closure valve at leakage $X_L = L/3$ for hydrogen-natural gas mixtures for isothermal and non-isothermal flow at horizontal pipeline. As indicated in Fig. 3, the transient pressure waves are strictly dependent on the mass ratio ϕ and temperature change. Based on this Fig. 3, increase in the mass ratio in an increase in the transient pressure distribution. For non-isothermal flow, the pressure drop at leak point is higher compared to the isothermal flow. Fig. 4 shows the plots of the transient pressure of hydrogen-natural gas mixture with $\phi = 0.5$ for isothermal and non-isothermal flow at different angle θ . An increase the inclination angle in an increase the pressure for non-isothermal flow after leak point, while the pressure drop decreases at leak point. As indicated in Figs. 3 and 4, the leak point is estimated at $t = 0.81 \text{ s}$.

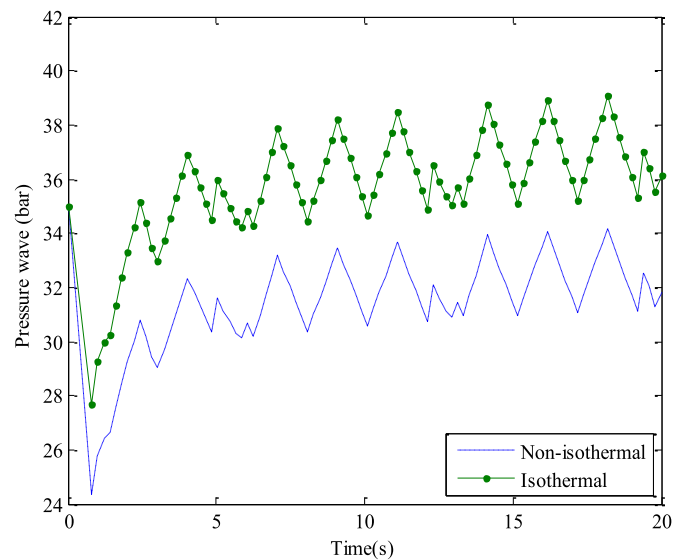
Fig. 5 presents the celerity wave distribution as a function of time with sudden closure valve at leakage $X_L = L/3$ for hydrogen-natural gas mixtures at different mass ratio ϕ in a horizontal pipeline. For non-isothermal flow, the celerity wave at leak point of the pipeline are 828.5 ms^{-1} , 970.4 ms^{-1} and 1125 ms^{-1} for $\phi = 0.25$, 0.5 and 0.75 , respectively. For isothermal flow, the celerity wave at leak point of the pipeline are 819.0 ms^{-1} , 960.3 ms^{-1} and 1085.0 ms^{-1} for $\phi = 0.25$, 0.5 and 0.75 , respectively.

Fig. 6 presents the celerity wave of hydrogen natural gas mixture in horizontal pipeline $\theta = 0^\circ$ and inclined pipeline $\theta = 15^\circ$. Based on Fig. 6, the celerity wave is not strong dependence on the inclination angle. There is no effect of inclination angle on the celerity wave.

From Figs. 5 and 6, it is assumed that the leak occurs when Δt_l equals 0.81 s . By using (38), the leak location for hydrogen-natural gas mixture ($\phi = 0.5$) at horizontal pipeline ($\theta = 0^\circ$) on non-isothermal flow is estimated to be at:



a) Mixture ($\phi = 0.25$)



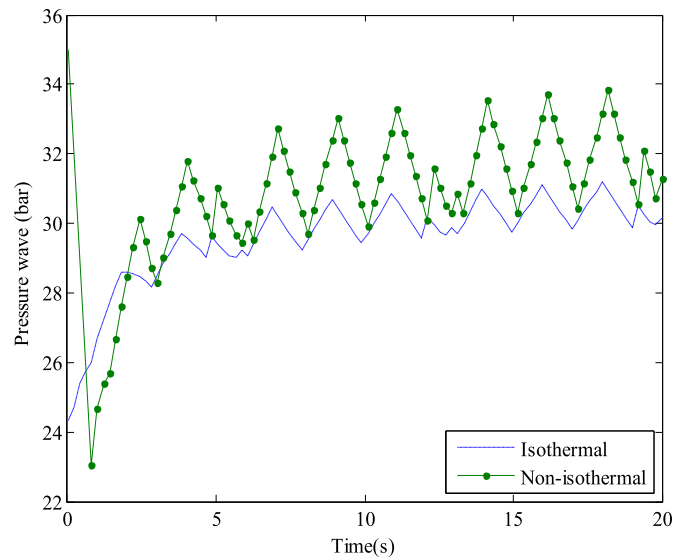
b) Mixture ($\phi = 0.5$)

Fig. 3. Transient pressure of hydrogen natural gas mixture for isothermal and non-isothermal flow when leakage occurs at $X_L = L/3$ in horizontal pipeline.

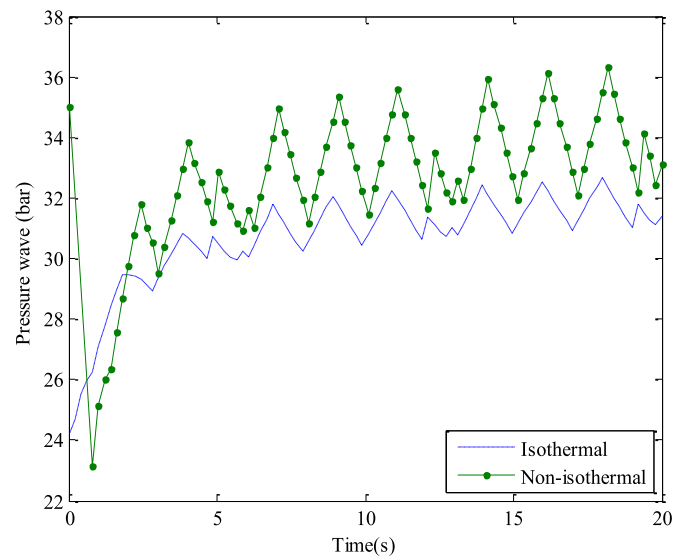
$$X_L = 600 - \frac{0.81 \times 970.3}{2} = 207m$$

Table 1 presents the calculated values of the leak location that occur at $X_L = L/3$ with different mass ratio ϕ and different angles of pipeline θ for isothermal and non-isothermal flow. From Table 1, we noted that the leak location is not strong dependence on the inclination angle θ , but dependence on the mass ratio of gas mixture. In this case, the mass ratio of hydrogen mixture should be considered.

The real location of the leak is supposed to be at 200 m when the leak is located at $X_L = L/3$. Based on Table 1, the closed calculated of leak location compared to the real location of the leak is 207 m,



a) $\theta = 0^\circ$



b) $\theta = 15^\circ$

Fig. 4. Transient pressure of hydrogen natural gas mixture with $\phi = 0.5$ for isothermal and non-isothermal flow when leakage occurs at $X_L = L/3$ with different angles θ .

which is hydrogen-natural gas mixture at mass ratio $\phi = 0.5$. For mass ratio $\phi = 0.75$, it is show that the leakage occurs before estimation point 200 m for both isothermal and non-isothermal flow. From these results we know that the leak position determined is greater than 200 m for the mass ratio of hydrogen less than 0.5, while the calculated leak location is determined before 200 m for the mass ratio of hydrogen greater than 0.5. These results show that the leak location's dependence on the mass ratio ϕ of gas mixtures. According to Elaoud et al. (2010), the downstream end point is more important when the leak is closer to the reservoir or compressor because the leak point can be detected earlier.

Table 2 represents the amount of leak discharge at leak position ($X_L = L/3$) with different mass ratio ϕ and different angles of pipeline θ for isothermal and non-isothermal flow. The transient

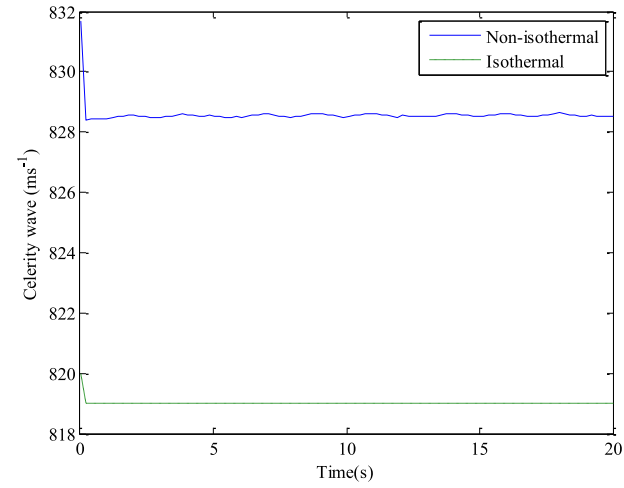
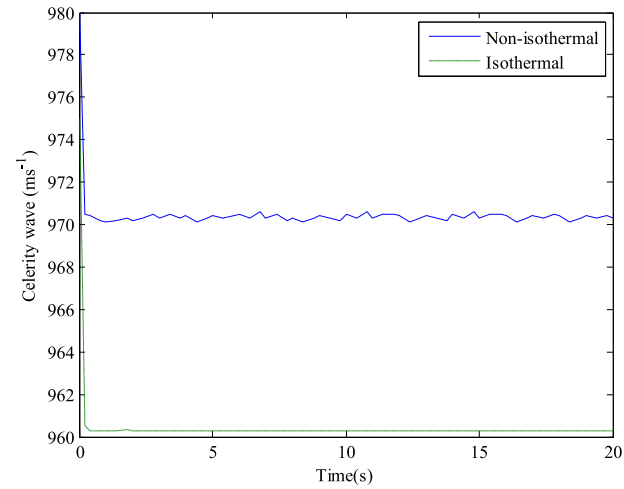
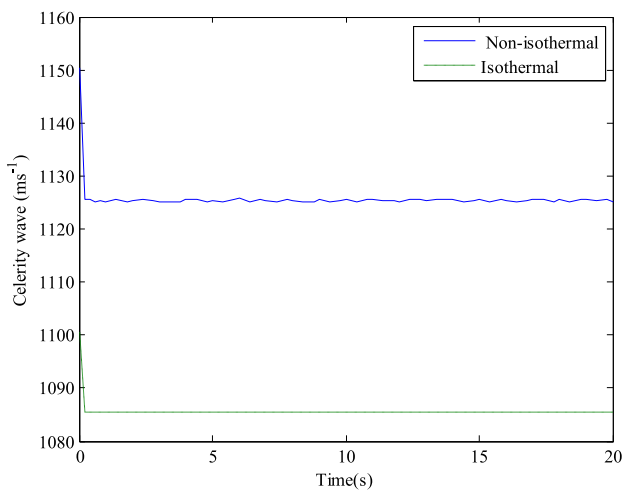
a) Mixture ($\phi = 0.25$)b) Mixture ($\phi = 0.5$)c) Mixture ($\phi = 0.75$)

Fig. 5. Celerity wave of hydrogen natural gas mixture for isothermal and non-isothermal flow when leakage occurs at $X_L = L/3$ in horizontal pipeline with different mass ratio ϕ .

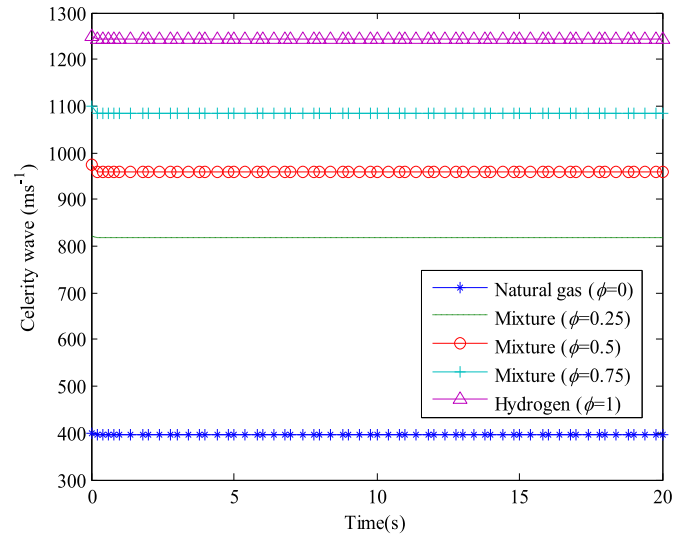
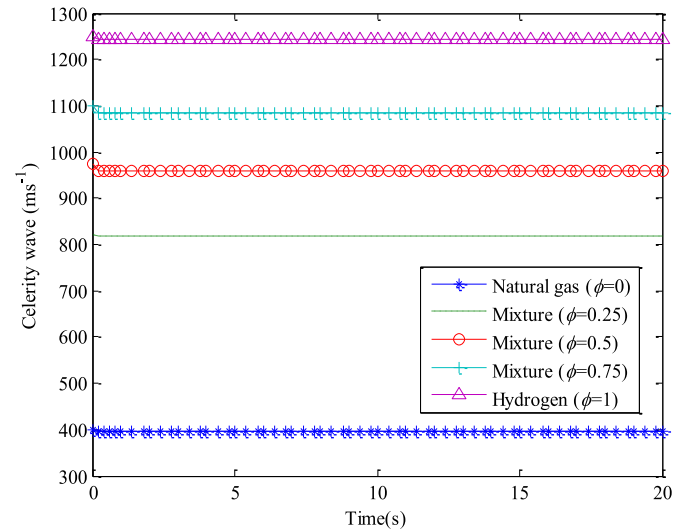
a) $\theta = 0^\circ$ b) $\theta = 15^\circ$

Fig. 6. Celerity wave of hydrogen natural gas mixture in horizontal $\theta = 0^\circ$ and inclined pipeline $\theta = 15^\circ$.

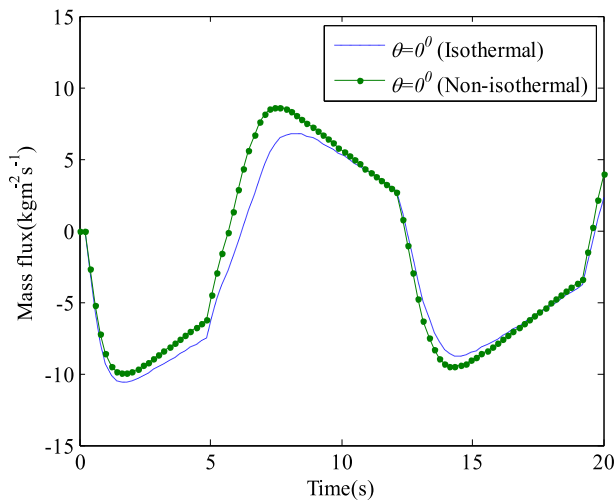
Table 1

Leak location for the hydrogen-natural gas mixture for isothermal and non-isothermal flow at leakage $X_L = L/3$.

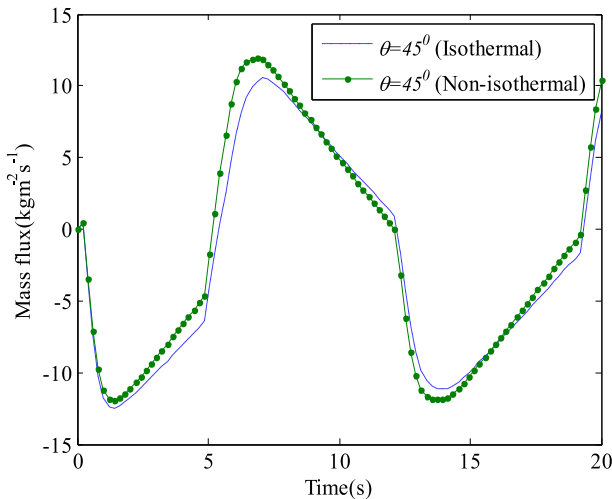
Gas mixture (ϕ)	Pipeline's angle (θ)	Leak location (m)	
		Isothermal	Non-isothermal
0	$\theta = 0^\circ$	439.4	432.3
	$\theta = 15^\circ$	439.4	432.3
0.25	$\theta = 0^\circ$	268.3	264.5
	$\theta = 15^\circ$	268.3	264.4
0.5	$\theta = 0^\circ$	211.1	207.0
	$\theta = 15^\circ$	211.1	207.0
0.75	$\theta = 0^\circ$	160.6	144.4
	$\theta = 15^\circ$	160.6	144.3
1	$\theta = 0^\circ$	95.8	90.9
	$\theta = 15^\circ$	95.8	90.9

Table 2
Leak discharge of hydrogen-natural gas mixture at $X_L = L/3$ for isothermal and non-isothermal flow.

Gas mixture (ϕ)	Pipeline's angle (θ)	$\Delta P_0 - \Delta P_l$ (N/m ²)		Leak discharge (kg/s)	
		Isothermal	Non-isothermal	Isothermal	Non-isothermal
0	$\theta = 0^\circ$	0.63×10^5	0.76×10^5	20.65	23.86
	$\theta = 15^\circ$	0.71×10^5	0.80×10^5	23.28	25.12
0.25	$\theta = 0^\circ$	1.06×10^5	1.31×10^5	16.83	20.56
	$\theta = 15^\circ$	1.14×10^5	1.42×10^5	18.10	22.28
0.5	$\theta = 0^\circ$	1.00×10^5	1.19×10^5	13.54	15.94
	$\theta = 15^\circ$	1.10×10^5	1.38×10^5	14.89	18.49
0.75	$\theta = 0^\circ$	0.79×10^5	1.21×10^5	9.47	13.98
	$\theta = 15^\circ$	0.90×10^5	1.35×10^5	10.78	15.60
1	$\theta = 0^\circ$	0.79×10^5	1.02×10^5	7.94	10.55
	$\theta = 15^\circ$	0.90×10^5	1.14×10^5	8.67	11.79



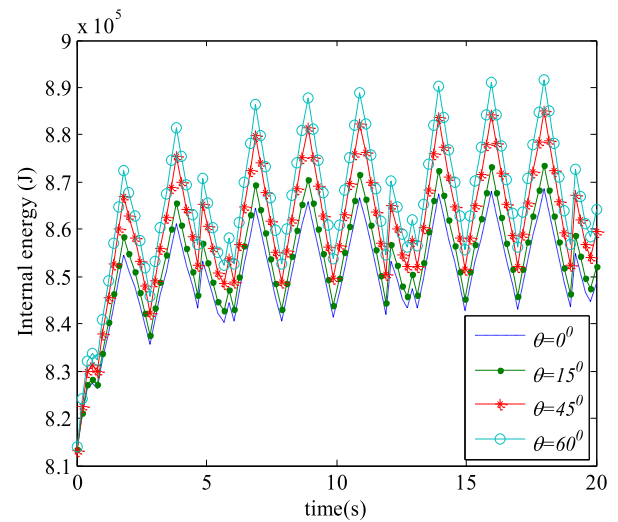
a) $\theta = 0^\circ$



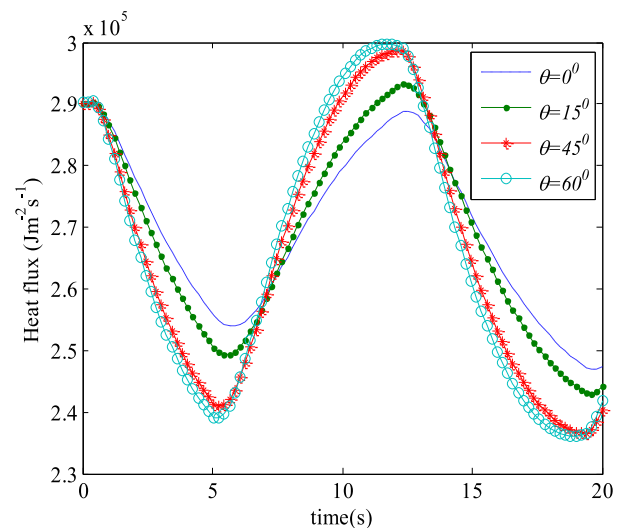
b) $\theta = 45^\circ$

Fig. 7. Mass flux of hydrogen natural gas mixture ($\phi = 0.5$) for isothermal and non-isothermal flow when leakage occurs at $X_L = L/3$ for different angles θ .

leak discharge amount (Q_L) can be calculated from (39). These results are obtained by analysing the plots of Figs. 3 and 5. The amount of leak discharge for non-isothermal flow located at $X_L = L/3$ for mass ratio $\phi = 0.5$ in a horizontal pipeline $\theta = 0^\circ$ is predicted to be:



a) Internal energy



b) Heat flux

Fig. 8. Effect on internal energy and heat flux of hydrogen natural gas mixture ($\phi = 0.5$) with different angles of θ when leakage occurs at $X_L = L/3$.

$$Q_L = \frac{0.13}{828.42} (1.19 \times 10^5) = 15.94 \text{ kg/s}$$

From Table 2, the results obtained clearly show that the amount of leak discharge increases when the inclination angle increases, but it decreases when the mass ratio increases.

The leak discharge is higher in inclined pipe is due to the increase in pressure difference and celerity wave [see (39)]. The increase in pressure in an inclined pipe is due to the increase in potential energy (and density) effected by an increase in height. For example, a pipe of length 600 m effects a change in height of 300 m at inclination angle of $\theta = 30^\circ$. Prediction of the leak location closer to reservoir and amount of leak discharge release are more important to estimate because pipelines contain hazardous and flammable gas and risks for public health (Subani et al., 2015; Elaoud and Hadj-Taïeb, 2009; Elaoud et al., 2010).

Fig. 7 presents the mass flux distribution as a function of time at different angles of θ . In this Fig. 7, it can be shown that for the same interval, increase in the angles of the pipeline in the mass flux results in an increase in the combustion capacity. This would affect the flow transportation, and the energy performance is better compared to a low angle θ .

Fig. 8 presents the effect of temperature change on the internal energy and heat flux for the hydrogen-natural gas mixture ($\phi = 0.5$) at different angles of θ . From the obtained results, an increase in the inclination angles results in an increase in the internal energy and heat flux, which also increases the burning capacity and gives better performance of hydrogen-natural gas mixture transportation in pipelines.

6. Conclusion

A technique based on the analysis of transient pressure waves to detect and locate leakage of non-isothermal flow in an inclined pipeline containing hydrogen-natural gas mixture has been presented. The transient pressure wave is created by the sudden closure of a downstream shut-off valve to ensure the attainment of minimum pressure at the downstream end within a short time. The boundary conditions are imposed by assuming that a reservoir exists at the upstream and a sudden closure valve at the downstream. The governing equations for such flows are three couple non-linear hyperbolic partial differential equations of unsteady, compressible and non-isothermal one-dimensional flow, which are solved using the reduced order modelling technique. The effect of body force, mass ratio of gas mixture and temperature change on the transient pressure and celerity waves due to the inclined pipeline are discussed. The leak location and amount of leak discharge due to leakage are also calculated. For an isothermal flow at a horizontal pipeline, the solutions obtained show good agreement with published results. It is also noted that the leak position is further from the reservoir and the amount of leak discharge is lower than that of non-isothermal flow. For non-isothermal flow, an increase in the mass ratio ϕ lead to an increase in the pressure and celerity wave, while the leak location and amount of leak discharge decrease. Leakage is estimated to occur at about 200 m and it is found that the mass ratio of hydrogen to natural gas should not be more than 0.5 to ensure that leakage does not occur before the estimated leak position. It is also observed that an increase in the inclination angle θ increases the pressure drop and leak discharge but the celerity wave and the

leak location do not seem to be affected.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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