

Forecasting the mortality rates of Malaysian population using Lee-Carter method

Cite as: AIP Conference Proceedings **1750**, 020009 (2016); <https://doi.org/10.1063/1.4954522>
Published Online: 21 June 2016

Nuraini Ngataman, Rose Irnawaty Ibrahim, and Mazlynda Md. Yusuf



View Online



Export Citation

ARTICLES YOU MAY BE INTERESTED IN

[Forecasting the mortality rates of Malaysian population using Heligman-Pollard model](#)
AIP Conference Proceedings **1870**, 040019 (2017); <https://doi.org/10.1063/1.4995851>

[Evaluating the performance of the Lee-Carter method and its variants in modelling and forecasting Malaysian mortality](#)

AIP Conference Proceedings **1635**, 762 (2014); <https://doi.org/10.1063/1.4903668>

[Analysis of mortality trends by specific ethnic groups and age groups in Malaysia](#)

AIP Conference Proceedings **1605**, 1002 (2014); <https://doi.org/10.1063/1.4887727>



Your Qubits. Measured.

Meet the next generation of quantum analyzers

- Readout for up to 64 qubits
- Operation at up to 8.5 GHz, mixer-calibration-free
- Signal optimization with minimal latency

Find out more



Forecasting the Mortality Rates of Malaysian Population Using Lee-Carter Method

Nuraini Ngataman^{1, a)}, Rose Irnawaty Ibrahim^{1, b)} and Mazlynda Md Yusuf^{1, c)}

¹*Faculty of Science and Technology, Universiti Sains Islam Malaysia, Bandar Baru Nilai, 71800 Nilai, Negeri Sembilan, Malaysia*

^{a)} Corresponding author: nuraini.ngataman@gmail.com

^{b)} rose.irnawaty@usim.edu.my

^{c)} mazlynda@usim.edu.my

Abstract. The population of many countries might undergo dramatic changes in the coming decades due to continuous increases in life expectancy. The sustained reduction in mortality rates and its systematic underestimation has been attracting the significant interest of researchers in recent years because of its potential impact on population size and structure, social security systems, and (from an actuarial perspective) the life insurance and pensions industry worldwide. Among all projection methods, the Lee-Carter method has been widely accepted by the actuarial community. This paper explores the use of the Lee-Carter method to forecast the mortality rates for Malaysian population. The index of the level of mortality for each gender, and the shape and sensitivity coefficients for 18 age groups were obtained through the Lee-Carter method. The Singular Values Decomposition (SVD) is used to forecast the general index for the time period that goes from 2011 to 2030. Since the model involves nonlinear equations that are explicitly difficult to solve, the Matrix Laboratory Version 7.0 (MATLAB 7.0) software will be used in the study. The empirical data sets of Malaysia population for the period of 1981-2010 and for both genders will be considered.

INTRODUCTION

Malaysia is categorised as one of the developing countries that has experienced improvement in health. Mortality rate is one of the indicator for health status as well as life expectancy at birth. Mortality rate is also known as death rate, while life expectancy is defined as the average number of years a person expect to live. It is stated that the life expectancy for Malaysian population has increased for the past 10 years. In 2002, the estimated life expectancy at birth increased from 70.8 years for male and 75.3 for female to 72.3 years for male and 77.2 for female [1]. As life expectancy increases, the mortality rate tend to decrease as both indexes have an inverse relationship to each other. These have brought the importance of studying the mortality forecasting especially for both actuaries and demographers.

This paper explores the use of Lee-Carter method in modeling and forecasting the mortality rate for Malaysian population. The method is applied to 18 different age groups of males and females data, from year 1981 to 2010. The forecasting period is 20 years, which is year 2011 to 2030. Lee-Carter method was proposed by R. D. Lee and L. R. Carter in 1992. Results from the model shows that it almost fits the pattern of age-specific mortality in the United States starting from year 1933 [2]. The method has been widely used as an extrapolative method in fitting and forecasting mortality for United State population [2], Argentina [3], Sweden [4] and other countries. Although Lee-Carter method has been applied to seven different time series for Sweden data, the performance for males and females is identical for the same time period in almost all cases [4]. In addition, it is declared that Lee-Carter model had successfully predict the declining trend of the death rates in the future as the study compared the forecast values with actual values for year 2001 to 2009 for Malaysian population [5].

METHODOLOGY

The data used in this study includes m_x , the central mortality rate that can be attained directly from Abridged Life Table that was provided by the Department of Statistics Malaysia. The 18 different age groups of each gender, starting from age 0, 1-4, 5-9, 10-14, 15-19 and so on for year 1981 to 2010 were considered in this study. The age-specific death rates in a single calendar year or also known as central mortality rate can be calculated by dividing the number of deaths for age group x with the average number of people living for age group x in year t [5].

Lee-Carter model is affected by two factors, which are age x and year t . The equation of Lee-Carter method is as follows:

$$\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} \quad (1)$$

where $m_{x,t}$ is the central mortality rate at age x and year t ; a_x is average (over time) log-mortality at age x ; b_x measures the response at age x to change in the overall level of mortality over time; k_t represents the overall level of mortality in year t ; and $\varepsilon_{x,t}$ is the residual [6].

The first step is to find the values of a_x . It is calculated by taking the average of log-mortality for each age group. Then, by applying the singular value decomposition (SVD) on the matrix of $\{\ln m_{x,t} - a_x\}$, the parameters of b_x and k_t were obtained. In order to obtain a unique solution, the SVD is applied with constraints that the estimated of b_x sum to one and the estimated k_t sum to zero [7]. On the other hand, two other methods can also be used to estimate the parameters; Weighted Least Square (WLS) and Maximum Likelihood Estimation (MLE). However, it was shown that SVD is the best method for obtaining the value of mortality index, k_t [4]. Since the model involves nonlinear equations that are explicitly difficult to solve, the Matrix Laboratory Version 7.0 (MATLAB 7.0) software is used to implement SVD. The result may show that the fitted and observed number of deaths is not the same. Therefore, to reconcile both fitted and observed number of deaths, the parameter of k_t is re-estimated for all t which satisfy the following equation:

$$\sum_x D_{x,t} = \sum_x E_{x,t} \exp(\hat{a}_x + \hat{b}_x \hat{k}_t) \quad (2)$$

where \hat{a}_x , \hat{b}_x and \hat{k}_t are the estimate values of a_x , b_x and k_t respectively [4], while $D_{x,t}$ is the total number of deaths of age x in year t and $E_{x,t}$ is the population (exposure to risk) of age x in year t [7]. Lastly, autoregressive integrated moving average (ARIMA) model is applied to forecast the dynamics of k_t . Besides ARIMA, space-state model (SSM) can also be used to forecast the mortality index. Even though the estimation values for both forecast models were similar, the prediction intervals for the index k_t are slightly more accurate when using ARIMA model [3]. In most cases, k_t can be effectively captured by ARIMA (0,1,0) model. It is also known as random walk with drift, which can be expressed as follows:

$$k_t = k_{t-1} + d + e_t \quad (3)$$

where d is the drift parameter and e_t is an error term. Forecast mortality rates are derived by using the values of \hat{a}_x and \hat{b}_x [6]. Statistical Package for the Social Sciences (SPSS) version 15.0 is used to forecast the values of k_t in 20 years period of time. Then, all values of the three parameters are used to forecast the mortality in the next 20 years.

RESULTS AND DISCUSSION

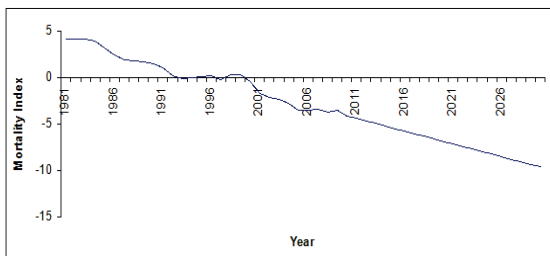
The estimated values of a_x and b_x for each gender are shown in Table 1. The exponential value of a_x , e^{a_x} is the general shape across age of the mortality schedule [2]. The pattern is the same for male and female, which the age pattern tend to decrease from age <1 until 10-14 age group and getting higher as the age increases. This is supported

as most countries where the value of a_x is high at the beginning of life then declines until it reaches a minimum, at 5-14 age group and increases until at the highest age group [5]. On the other hand, b_x shows the sensitivity by showing which rates decline rapidly and which rates decline slowly to the changes of k_t [2]. For male, the most sensitive rate is at age <1, while for female is in age group of 1-4. This result is supported as the mortality is highly sensitive to changes in the general mortality index, k_t in the <1 and 1-4 age group compared to older age group, which the mortality is not too sensitive to the changes in k_t [5].

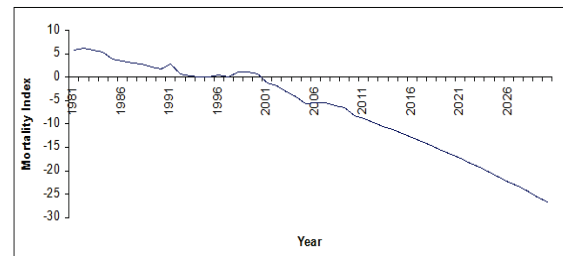
TABLE 1. The estimated values of a_x and b_x .

Age Group	Male		Female	
	a_x	b_x	a_x	b_x
<1	-4.58874	0.21819	-4.79789	0.14016
1-4	-7.17500	0.20246	-7.29932	0.14707
5-9	-7.69355	0.14254	-7.98275	0.10208
10-14	-7.57592	0.08755	-7.99052	0.06867
15-19	-6.72473	0.02523	-7.66146	0.05215
20-24	-6.42173	0.03629	-7.44536	0.05745
25-29	-6.36976	0.02299	-7.28233	0.05820
30-34	-6.20203	-0.00623	-7.00483	0.05824
35-39	-5.97799	-0.00459	-6.64846	0.05313
40-44	-5.64457	0.01149	-6.23076	0.04402
45-49	-5.23070	0.02618	-5.75841	0.03326
50-54	-4.73502	0.04741	-5.24443	0.03799
55-59	-4.26789	0.04639	-4.75278	0.03511
60-64	-3.77576	0.04449	-4.20830	0.03894
65-69	-3.33119	0.03548	-3.69834	0.03099
70-74	-2.85870	0.03174	-3.14801	0.02680
75-79	-2.46999	0.01941	-2.67623	0.00698
80+	-1.71158	0.01298	-1.83689	0.00876

Figure 1 shows the fitted and forecasted value of k_t for each gender. The similar pattern of mortality index can be seen as the index tend to decrease year by year. This trend is similar for other countries as well; United States [2], Argentina [3] and Sweden [4]. As the mortality index decreases, the mortality rate also tend to decrease because of the improvement in standard of living and advanced technology especially in health and medical services.



(a) Fitted and forecasted mortality index for male

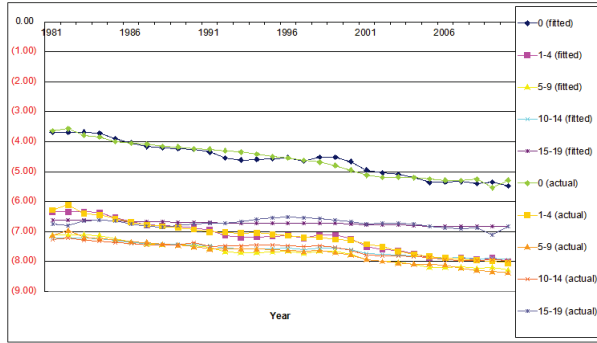


(b) Fitted and forecasted mortality index for female

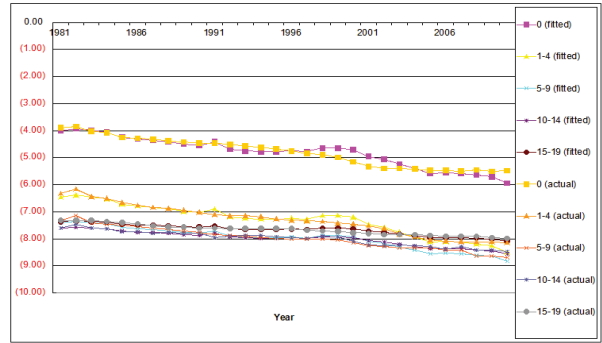
FIGURE 1. The fitted and forecasted values of mortality index, k_t for male and female.

The actual and fitted log age-specific death rate for each gender is shown in Figure 2. The actual values were obtained from Abridged Life Table, while the fitted values were computed from the values of parameters that have been estimated earlier; \hat{a}_x , \hat{b}_x and \hat{k}_t . As can be seen from Figure 2, the fitted log age-specific death rates move along with the actual values for the whole time period of study, starting from year 1981 to 2010. This was supported

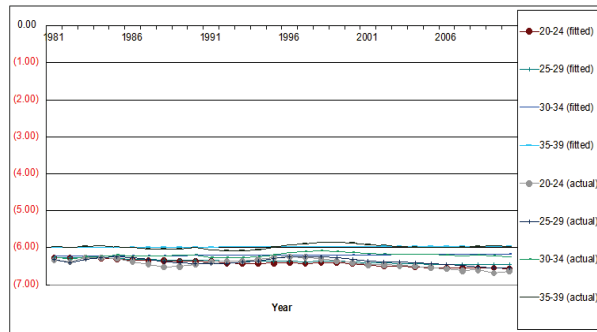
by previous study, which the fitted values match up quite well with the actual rates for each gender in year 1980 to 2000 and also from overall population in year 1971 to 2000 for Malaysian population [5].



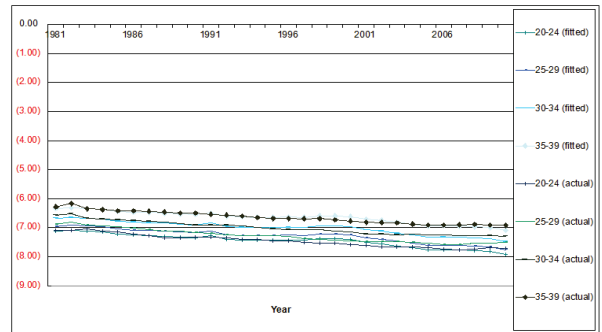
(a) Actual and fitted $\ln m_x$ for male adolescent



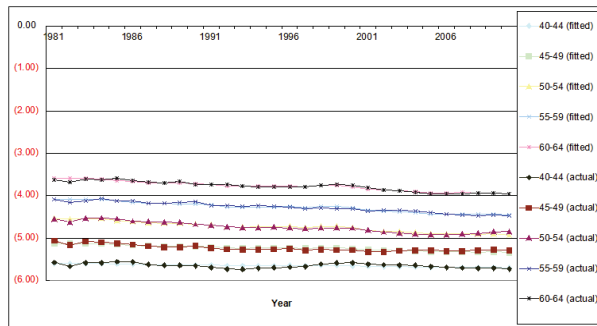
(b) Actual and fitted $\ln m_x$ for female adolescent



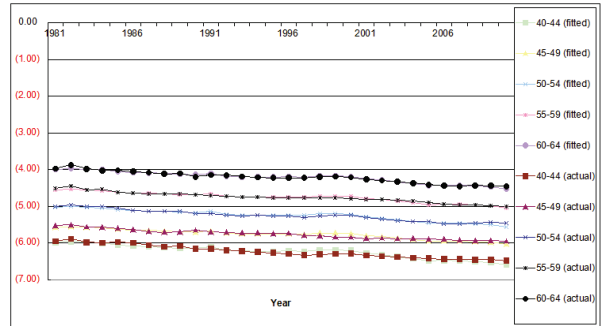
(c) Actual and fitted $\ln m_x$ for male adults



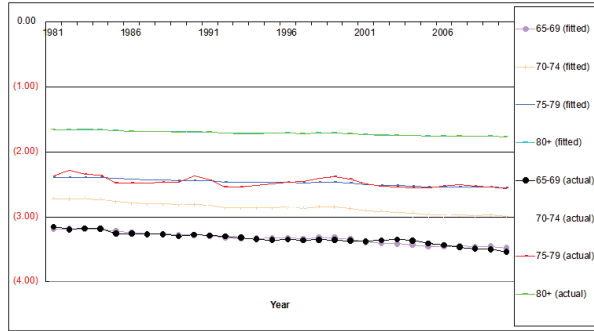
(d) Actual and fitted $\ln m_x$ for female adults



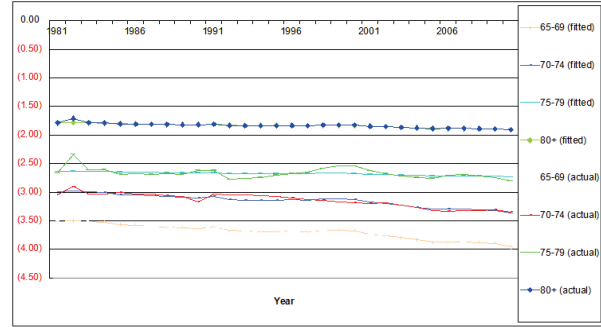
(e) Actual and fitted $\ln m_x$ for male middle age



(f) Actual and fitted $\ln m_x$ for female middle age

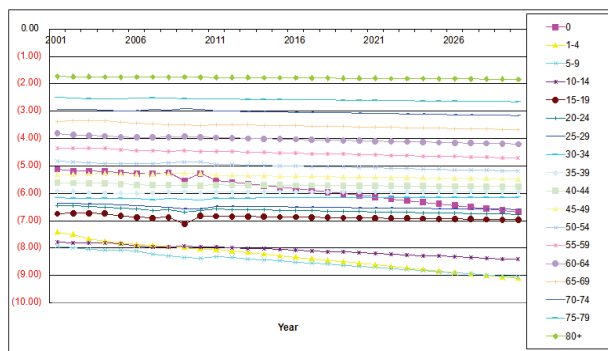


(g) Actual and fitted $\ln m_x$ for male elderly

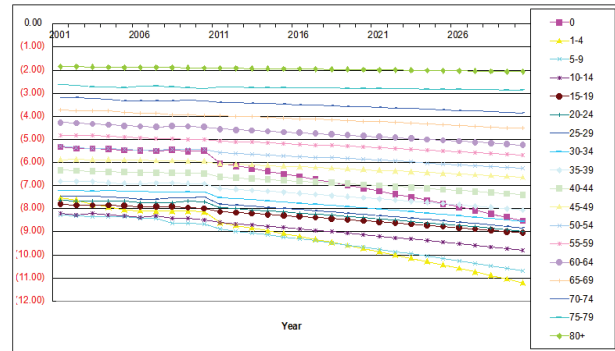


(h) Actual and fitted $\ln m_x$ for female elderly

FIGURE 2. The actual and fitted values of log age-specific death rate, $\ln m_x$ for male and female.



(a) Actual and forecasted values of $\ln m_x$ for male



(b) Actual and forecasted values of $\ln m_x$ for female

FIGURE 3. The actual and forecasted values of log age-specific death rate, $\ln m_x$ for male and female.

The projection of future age-specific death rates can be computed by using the value of forecasted mortality index k_t . It is stated that if k_t is linearly decline, then age-specific death rates will also be decreasing at its own constant exponential rates [7]. Figure 3 shows the actual values of log age-specific death rate for year 2001 to 2010 and projected rates for year 2011 to 2030. In 20 years time ahead, the rates are projected to continually decline for each gender in all age groups. It can be seen that for both gender, the age group of 80 years and above will continue to have the highest rate as the oldest age group has the highest mortality risk, while the lowest rates are conquered by the 1-9 age group. In 2030, the log death rate for male of age 80 years and above is forecasted to decline to -1.84 from -1.74 in 2001, and -2.07 from -1.84 for female. On the other hand, those in age group of 1-9 years is projected to have a decrement of almost 3.0 for female and almost 2.0 for male. The model will be considered having a good performance if the predicted rates correspond to the observed rates [4]. The different values, errors between predicted and observed rates for Malaysian population in year 2001 to 2009 were measured by [5], and it is concluded that the actual and forecasted death rates were almost the same by having the small error values for almost all age groups.

CONCLUSION

This paper applied Lee-Carter model to age-specific death rates for Malaysian population. The analysis was done for each gender in all 18 different age groups; <1, 1-4, 5-9, 10-14 and so on. Singular value decomposition is used to estimate the parameters for Lee-Carter model. The result shows that the fitted and observed log age-specific death rates are having the same pattern for both male and female, starting in year 1981 to 2010. Then, ARIMA (0,1,0) or random walk with drift method is used to forecast the mortality index in 20 years time. In computing the forecasting death rates, all the values of parameters estimation are used to project mortality. The mortality rates for all age

groups are expected to decline continually in the future. The eldest age group, which is 80 years and above is projected to continue to have the highest death rates due to the highest mortality risk compared to other age groups while those in age 1-9 year are projected to have the lowest death rate.

The Lee-Carter model fit Malaysian population data quite well as the fitted and observed data move along the same pattern for 30 years period of study, However, the model has under estimated the life expectancies for Malaysian population [5] . This may due to the limitation of age-specific death rates data in Malaysia. The result also shows that female mortality is projected to have lower rates than male for all 18 age groups.

ACKNOWLEDGMENTS

The authors would like to thank the Department of Statistics, Malaysia for providing the data and Universiti Sains Islam Malaysia for their financial support (PPT/GP/FST/30/14615).

REFERENCES

1. E. Tora, *Annual Report 2012*. (Ministry of Health, Malaysia, 2013), pp. 8-14.
2. R. D. Lee and L. R. Carter, *Journal of the American Statistical Association*, **87**(419), 659-671 (1992).
3. L. Andreozzi, M. T. Blacona and N. Arnesi, "The Lee-Carter Method for Estimating and Forecasting Mortality: An Application for Argentina" in *International Symposium on Forecasting-2011*, Prague Proceedings, pp. 1-17.
4. J. Z. Wang, "Fitting and Forecasting for Sweden: Applying the Lee-Carter Model", Master's thesis, Stockholm University, 2007.
5. W. H. W. Zakiatussariroh, Z. M. Said and M. R. Norazan, "Forecasting Malaysian Mortality by the Lee-Carter Method," in *International Congress-2013*, World Academy of Science, Engineering and Technology, pp.1-7.
6. H. Booth and L. Tickle, *Annals of Actuarial Science*, **1**, 3-43 (2008).
7. W-S. Chan, S-H. Li and S-H. Cheung, "Testing Deterministic versus Stochastic Trends in the Lee-Carter Mortality Indexes and Its Implications for Projecting Mortality Improvements at Advanced Ages" in *Living to 100 and Beyond Symposium-2008*, (Society of Actuaries, Orlando, Fla), pp. 1-50.