

# Probabilistic Thinking of Undergraduate Male Mathematics Education Students in Solving Probability Problems Involving Binomial Distribution Based on Field-Independent Cognitive Styles

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## Abstract

This study explores the probabilistic thinking of a male undergraduate mathematics education student characterized by a masculine gender trait and a field-independent (FI) cognitive style. The subject, referred to as MLI (Male, Field-Independent), was selected through purposive sampling based on gender traits, results from the Group Embedded Figures Test (GEFT), and mathematical ability assessed through UTBK SBMPTN questions. Most students tested were classified as having moderate mathematical ability, and the MLI subject was selected accordingly. Using a descriptive qualitative approach with a case study design, the study aimed to analyze the subject's approach to solving a probability problem involving binomial distribution. Data were gathered through a problem-solving task administered at the Faculty of Teacher Training and Education, Universitas Sembilanbelas November Kolaka. The data analysis process included data reduction, presentation, and conclusion drawing. Findings revealed that the MLI subject successfully applied Polya's four-step problem-solving model understanding the problem, devising a plan, carrying out the plan, and looking back. Although formulas were not explicitly written, the subject demonstrated clear logical structure and accurate reasoning. Reflective thinking was confirmed through interviews. These results highlight the need to develop probabilistic thinking in mathematics education to support effective teaching and real-world decision-making.

**Keywords:** Probabilistic Thinking; Field-Independent Cognitive Style; Binomial Distribution; Probability Problem Solving; Male Undergraduate Students

## 1.0 Introduction

In the Indonesian higher education curriculum, students enrolled in mathematics education program prospective mathematics teachers are systematically prepared to become professional educators in the field of mathematics. According to Law No. 14 of 2005 on Teachers and Lecturers, a mathematics teacher's professionalism comprises four core competencies: pedagogical, personal, social, and professional. These competencies are cultivated through academic supervision and structured coursework throughout the undergraduate program. Moreover, Law No. 12 of 2012 on Higher Education emphasizes the importance of facilitating the development of students' cognitive, affective, and psychomotor domains. To support this goal, Presidential Regulation No. 8 of 2012 on the Indonesian National Qualifications Framework (KKNI) was introduced to guide the formulation of learning outcomes at every level of education. For undergraduate students (S-1), learning outcomes are classified under KKNI Level 6, which includes the ability to apply specialized knowledge, utilize science and technology in solving problems, adapt to various situations, master both general and specific theoretical concepts, formulate procedural solutions, make informed decisions based on data analysis, and demonstrate responsibility in both individual and organizational contexts. These outcomes are aligned with the national standards for higher education in Indonesia [1]. At KKNI Level 6, the expected learning outcomes for mathematics education students include mastery of theoretical concepts in areas such as number theory, algebra, geometry and measurement, statistics and probability, trigonometry, and calculus, and the ability to formulate procedural solutions to problems. Teaching mathematics in general and instructing mathematics at junior schools in particular not only create favorable conditions for students to develop essential and core competencies but also help students enhance mathematical competencies as a foundation for a good study of the subject and promote essential skills for society, in which mathematical communication skill is an important one [2].

The topic of probability is addressed in the Mathematical Statistics I course in higher education. At this level, students are introduced to concepts such as combinatorics (including factorial, permutation, and combination), conditional probability, independent events, Bayes' theorem, discrete and continuous random variables, and probability distributions. These topics pose challenges for many students, particularly in solving probability-related problems. Solving such problems requires logical reasoning and the ability to provide sound arguments, explanations, and justifications. Success in problem-based learning requires both strong information literacy to search for, evaluate, and use information effectively, and argumentation skills to generate coherent arguments [3]. These skills are essential for students not only during their academic careers but also in their professional and everyday lives, enabling them to become critical thinkers and informed decision-makers. Consequently, it is crucial to revise the curriculum and instructional strategies used in teaching probability to undergraduate mathematics education students. This aligns with findings from Hokor and Hokor [4,5], which suggest that educators should incorporate strategies that enhance probabilistic thinking, particularly because this type of thinking is essential in understanding probability and avoiding misconceptions. To foster probabilistic thinking among mathematics education students, problem-solving tasks rooted in real-life contexts should be emphasized in coursework. According to Pehkonen [6] states that "Problem solving has generally been accepted as a means for advancing thinking skills". In addition, [7-12] which concluded that in probabilistic thinking, students can develop their good probabilistic thinking intuitively, classically, and continuously, besides that students can also develop ideas.

According to Maxine & Constance [13] states that "probabilistic thinking is an inherently new way of processing information as the world view shifts from a deterministic view of reality". The quote explains that probabilistic thinking is a new way of processing information as the world view shifts from a deterministic view of reality. Lamprianou & Afantiti [14] stated that "probabilistic thinking is a mode of reasoning attempting to quantify uncertainty, as a tool for decision making". The quote explains that probabilistic thinking is a mode of reasoning attempting to quantify uncertainty, as a tool for decision making, while Biehler [15] states that there are several characteristics of probabilistic thinking as follows: (1) don't seek connection, (2) explaining and describing, (3) not a single data set, and (3) work with the model. The explanation of each characteristic is (1) in probabilistic thinking do not connect something in individual cases (special), but try to think on the regularity of repetition that may occur so that it is not just a guess but a certain calculation, (2) probabilistic thinking means explaining and describing variations of the probability model, (3) probabilistic thinking is not thinking of single data but the mechanism of generating an experiment that has several possibilities and (4) work with the "model" which is intended to work by making several possibilities, not with a fully known reality.), another case with Lau & Ranyard [16] probabilistic thinking as "the tendency to view the world in terms of uncertainty, the ascribing of different degrees of uncertainty either verbally or as a numerical probability". The quote explains that the tendency to describe the world in terms of uncertainty, ascribing different degrees of confidence either verbally or as a numerical probability. Based on the theoretical conjecture that the FI cognitive style of a masculine male in solving probability problems is interesting and important. Interesting because it can examine probabilistic thinking between subjects with FI cognitive style in solving probability problems. Important because it can provide scientific information in the field of cognitive psychology that the FI and FD cognitive styles of a student will also affect his probabilistic thinking in solving probability problems [17]. The importance of this research is done because probabilistic thinking as one of the thinking skills that must be owned by masculine male students who play a very important role and support student success in learning mathematics, then educators should know someone's probabilistic thinking in probability problems. Supratman, Budayasa & Rahaju [18] stated that quantitative analysis showed a significant role of probabilistic thinking and field-independent cognitive style in the probability problem-solving skills of feminine female undergraduate mathematics education students. Qualitative analysis, through interviews based on Polya's stages, provided deeper insights. During the interview, after completing the probability test, MPI struggled to apply the combination formula before using the binomial

distribution. However, in the "looking back" stage, MPI successfully wrote the final answer. These findings highlight the need to develop probabilistic thinking.

## 2.0 Methodology


### 2.1 Research Design

This study employed a descriptive qualitative approach, as outlined by [19] to investigate the differences in probabilistic thinking among undergraduate mathematics education students with a field-independent cognitive style, specifically focusing on gender-based variations. In this case, the gender of interest was feminine. The study adopted a case study design, concentrating on a single subject to explore, in depth, the phenomenon of probabilistic thinking in solving probability problems by a female undergraduate mathematics education student with a field-independent cognitive style.

### 2.2 Participants and Data Collection

One male undergraduate mathematics education student with a masculine gender trait and a field-independent cognitive style was purposefully selected as the subject. The selection process incorporated a mathematical ability test using UTBK SBMPTN questions, which are standardized and credible assessment tools used in Indonesia's national university entrance examination. The student's mathematical ability was classified as moderate, based on the grading criteria of Universitas Sembilanbelas November Kolaka: A (high), B (moderate), and C (low). Purposive sampling was employed, focusing on students who had completed the Mathematical Statistics I course. The Grade Point Average (GPA) of sixth-semester students was also considered, with specific attention given to those categorized as having moderate ability. This selection was further supported by recommendations from lecturers who taught the Mathematical Statistics I course, which includes the topic of probability. The probability problem-solving task was administered in Room B4 of the Faculty of Teacher Training and Education at Universitas Sembilanbelas November Kolaka. Data were collected from the test responses of the selected male subject with a field-independent cognitive style. The data analysis process was conducted in three stages: data reduction, data presentation, and conclusion drawing. The specific probability problem-solving task given to the subject is shown in Fig. 1.

2. A circular dart target board has a small circular center called the target center (Bullseye), the target center in area 1 consists of 2 parts, namely the red inner part (Double Bull) and the green outer part (Bull) while 20 circular areas numbered 1 to 20 are located around the board. Each circular area is further divided into 3 parts, namely in area 2 includes the target boundary line which is assumed to be inside the circle and gets a double value of 3, in area 3 includes the target boundary line which is assumed to be inside the circle and gets a double value of 2, while in area 4 is the outermost area of the target board, so that if someone throws the dart and hits a certain number, then he gets a value equal to the number, twice the number, or three times the number, depending on which part of the hit area is successively on the outside, the center, and the inside which is assumed that the boundary between 2 adjacent curves is given an iron plate, so that the arrow cannot stick in the boundary line. If the chance of someone hitting the center of the target is 0.04, the chance of hitting double value 3 is 0.45, the chance of hitting double value 2 is 0.20, and the chance of hitting the outermost region of the target board is 0.03 which is illustrated in the following figure



Then determine that the probability that 15 dart throws result in:

- Hit the center of the target exactly 5 times
- Getting a double score at most 2 times
- Scored double three more than 7 times
- Hits the outermost area of the target board 4 times

Fig. 1. Probability problem-solving task

### 2.3 Research Instruments

The primary research instrument was a set of probability problem-solving tasks designed to explore the subject's probabilistic thinking in depth. These tasks included indicators of probabilistic thinking. Before conducting the research, a validation process was undertaken to ensure that the instruments effectively measured students' probabilistic thinking abilities, particularly in solving mathematical problems related to probability. The instruments used in the selection of subjects included a mathematical ability test and the Group Embedded Figures Test (GEFT). The mathematical ability test comprised general mathematics questions covering statistics, geometry, and algebra sourced from the UTBK SBMPTN. The GEFT was used to classify subjects according to their cognitive styles, specifically to identify those with a field-independent cognitive orientation.

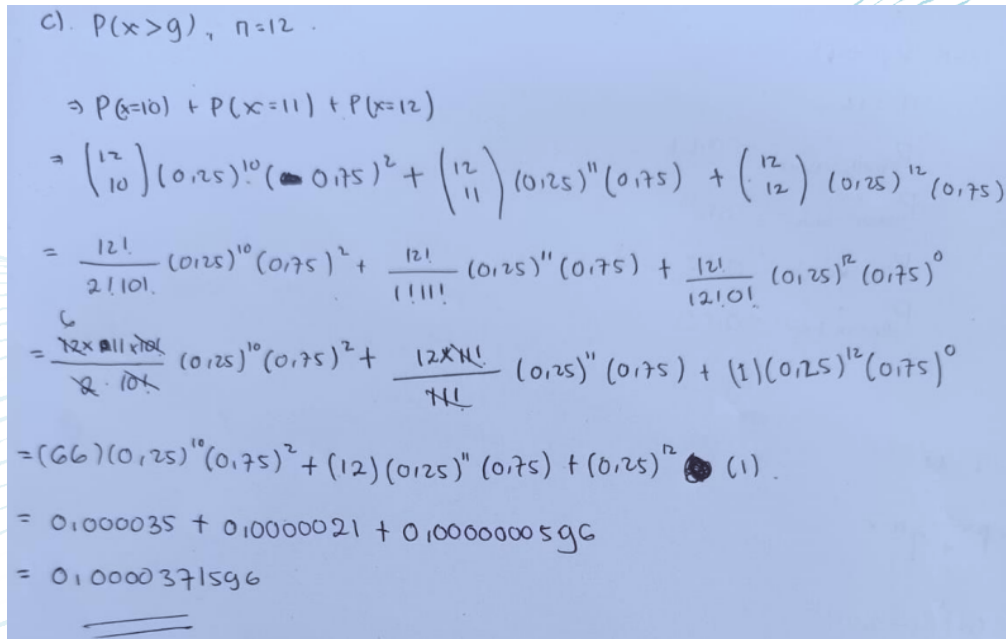
### 2.4 Data Analysis

Following the categorization of probabilistic thinking data, analysis proceeded through three stages: data reduction, data presentation, and conclusion drawing. The objective was to examine how the selected subject a feminine female undergraduate mathematics education student with a field-independent cognitive style approached and solved probability problems.

### 3.0 Results and Discussion

The initial stage involved administering the probability problem-solving test to 15 undergraduate mathematics education students from the 2020 cohort. The mathematical ability test results showed that none of the students were in the high category; 8 students (53.34%) were classified as having moderate ability, and 7 students (46.67%) as having low ability. In the 2021 cohort, similarly, no students scored in the high category; 7 students (58.34%) were categorized as moderate, and 5 students (41.67%) as low.

The GEFT was administered to 15 students from the 2020 cohort. Results revealed that 8 students exhibited a field-dependent (FD) cognitive style, consisting of 1 masculine male and 7 feminine females. The remaining 7 students demonstrated a field-independent (FI) cognitive style, comprising 1 masculine male and 6 feminine females. For the 2021 cohort, 5 students were identified as field-dependent (1 masculine male and 4 feminine females), while 7 were feminine females with a field-independent cognitive style. Based on purposive sampling, considering the mathematical ability and GEFT results, the study focused on one subject: a masculine male student from the 2020 cohort with a field-independent cognitive style, coded as MLI. This subject was recommended by the course instructor for Mathematical Statistics I and met the medium-level ability criterion. A detailed analysis of the MLI subject's responses to the probability problem-solving task (see Fig. 2) yielded the following insights:



$$\begin{aligned}
 & c). P(x > 9), n = 12 \\
 & \Rightarrow P(x=10) + P(x=11) + P(x=12) \\
 & = \binom{12}{10} (0.25)^{10} (0.75)^2 + \binom{12}{11} (0.25)^{11} (0.75) + \binom{12}{12} (0.25)^{12} (0.75)^0 \\
 & = \frac{12!}{2!10!} (0.25)^{10} (0.75)^2 + \frac{12!}{1!11!} (0.25)^{11} (0.75) + \frac{12!}{12!0!} (0.25)^{12} (0.75)^0 \\
 & = \frac{12 \times 11 \times 10!}{2 \cdot 10!} (0.25)^{10} (0.75)^2 + \frac{12 \times 11!}{11!} (0.25)^{11} (0.75) + (1)(0.25)^{12} (0.75)^0 \\
 & = (66)(0.25)^{10} (0.75)^2 + (12)(0.25)^{11} (0.75) + (0.25)^{12} (1) \\
 & = 0.000035 + 0.0000021 + 0.000000596 \\
 & = 0.0000371596
 \end{aligned}$$

Fig. 2. results of MLI subject's probability problem solving answers

The MLI subject demonstrated competence in solving the probability problem by successfully applying all four stages of Polya's problem-solving process. In the Understand the Problem stage, the subject was able to identify and extract relevant information from the given problem, particularly recognizing that the probability in question involved obtaining a double value of 3 in more than 9 attempts out of a total of 12 dart throws. He correctly interpreted this to mean the event occurring in exactly 10, 11, or 12 throws. Moving on to the Devising a Plan stage, the subject immediately implemented a solution strategy by applying the principles of the binomial distribution. Although he did not explicitly write the formulas for the binomial distribution or combinations, his solution was systematically structured and reflected a sound conceptual understanding of the underlying mathematical processes. In the Carrying Out the Plan stage, the subject executed the solution with a high degree of accuracy and fluency. He organized the steps logically and presented calculations in a clear and structured manner, ultimately arriving at an appropriate and correct final answer. Finally, during the Looking Back stage, although the subject did not document a written review of his work, follow-up interviews revealed that he had mentally retraced his entire problem-solving process. He revisited the known and unknown elements of the problem, reflected on the steps taken, and verified the correctness of the solution. This demonstrated his ability to engage in reflective thinking, an essential component of effective problem solving.

The findings indicate that the MLI subject successfully demonstrated all four stages of Polya's problem-solving model in addressing the probability task. In the Understand the Problem stage, the subject accurately interpreted the scenario and extracted the essential information needed to approach the solution, although the specific question posed in the problem was not explicitly restated in the written response. In the Devising a Plan stage, the subject did not provide the explicit formulas for the binomial distribution or combinations; however, he applied the binomial distribution strategy correctly, reflecting a solid understanding of the appropriate problem-solving method. During the Carrying Out the Plan stage, the subject executed the strategy competently, presenting the solution through a series of logical and systematically organized steps that led to a valid and appropriate conclusion. At the Looking Back stage, although the subject did not include a written review of the problem-solving process, data obtained through interviews confirmed that he had mentally reviewed and verified the accuracy of his solution. These results are consistent with findings from prior studies. For instance, Lin & Davidson-Shivers [20] found that individuals with a field-independent cognitive style are typically

more engaged in learning activities, enabling them to independently analyze, reconstruct, and resolve problems. Furthermore Yazici & Ertekin [21] reported notable gender-based differences in mathematics-related beliefs and confidence levels. Their study found that male pre-service teachers tend to exhibit higher levels of confidence in teaching mathematics, while their female counterparts are more likely to experience anxiety. Similarly, Li [22] identified significant gender differences in both students' and teachers' beliefs regarding the importance and difficulty of mathematical topics, with male teachers generally perceiving mathematics topics as more important than female teachers.

#### 4.0 Conclusion

Based on the findings and discussion, it can be concluded that the MLI subject demonstrated the ability to solve probability problems effectively by following Polya's four-step problem-solving process. At the Understand the Problem stage, the subject successfully identified and documented the essential information provided in the problem. During the Devising a Plan stage, the subject proceeded directly to apply the binomial distribution approach, although he did not explicitly write out the relevant formulas, including those for combinations. Nonetheless, the structure and coherence of his solution indicated that he understood the strategy required to solve the problem accurately. At the Carrying Out the Plan stage, the MLI subject displayed a high level of skill and accuracy, presenting a well-organized sequence of calculations that led to the correct final answer. In the Looking Back stage, although the subject did not include a written review of his solution, interviews revealed that he had mentally revisited the entire process from identifying the known and unknown elements to formulating and solving the problem demonstrating a thorough and reflective approach. These findings reinforce the importance of developing probabilistic thinking in higher education as a fundamental cognitive skill. Probabilistic thinking is a crucial component of the learning outcomes expected at the undergraduate level, as it supports students in applying mathematical knowledge and leveraging science and technology to solve real-world problems. Furthermore, it cultivates students' ability to adapt to diverse situations, make data-driven decisions, and master both general and specialized theoretical concepts in mathematics.

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