COMPARISON BETWEEN PARAMETRIC MORTALITY MODELS FOR OLD AGE MORTALITY IN MALAYSIA

Nur Idayu Ah Khaliludin[i] Norrlaili Shapiee [ii] & NurLiyana Juhan [iii]

[i] (*Corresponding author*). Universiti Sains Islam Malaysia. nuridayu@usim.edu.my
[ii] Universiti Sains Islam Malaysia. norrlaili@usim.edu.my
[iii] Universiti Malaysia Sabah. Liyana87@ums.edu.my

Abstract

An Unexpected increase in life expectancy and low mortality rates for old people has lead Malaysia to be an ageing nation. As the ageing issue has gained prominence as mortality rates, study on the mortality rates and modelling these rates is significant so that the government and private sectors can prepare immediately and adequately for the specific needs of the aged such as the health care and retirement plan. However, modelling the mortality rates for old age is challenging as the mortality data often exhibit irregular patterns due to randomness and uncertainties. Several mortality models have been formulated to model the death rates. This research applies seven mortality models to the Malaysian mortality data from the years 2010 to 2016. The results show that the model established by Khaliludin et. al (2021) significantly improves Malaysian mortality estimation for old age in terms of accuracy and prediction performance, as well as its ability to capture important mortality features such as accident hump, mortality crossover, and deceleration of mortality at old ages.

Keywords: *Mortality model, Ageing, Parametric, Survival model.*

INTRODUCTION

The population for every country is inevitably ageing. Population ageing is defined as an increasing proportion of older people out of the total population (United Nations, 2020). As reported by the Department of Statistics Malaysia (2020) in the "Key Findings Population and Housing Census of Malaysia 2020", Malaysia is expected to experience an ageing population in 2030. As a matter of fact, four states have become the ageing states in 2020 which include Perak (8.9%), Kedah (7.9%), Perlis (7.9%) dan Sarawak (7.5%) (Department of Statistics of Malaysia, 2022).

This increment has changed the Malaysian population age structure from a progressive population pyramid to a regressive population pyramid. The population pyramids in 1957 and 2010 are shown in Figure 1.

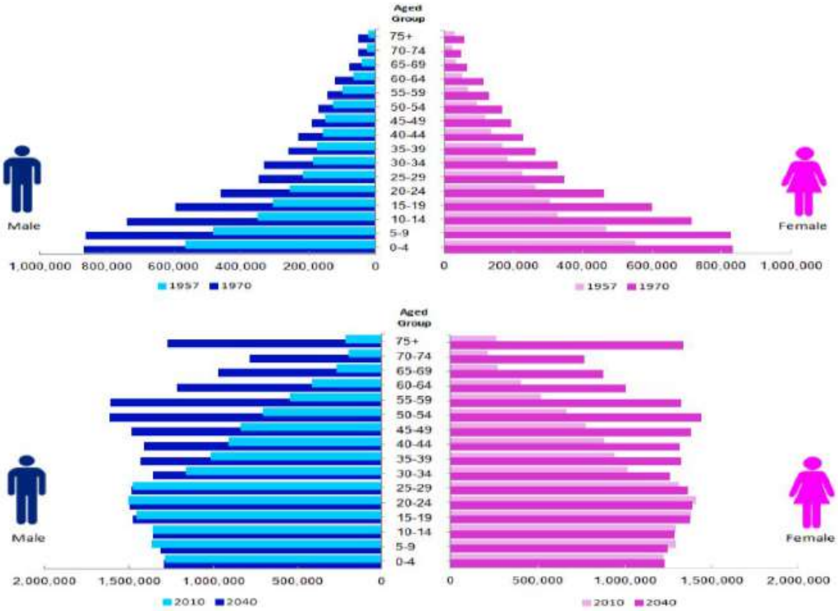


Figure 1 Population age structure in 1957, 1970, 2010 and 2040 for males and females in Malaysia.

Figure 1 shows a concave population age structure in 1957 with a broad base and pointed peak. However, the shape of this pyramid changed to a convex slope with a much smaller base and wider top. This age structure is expected to remain as such until the year 2040. The first question raised is what would happen to this population age structure after the year 2040. This can be solved by modelling historical mortality rates to forecast future mortality.

Nonetheless, the main challenge in modelling the mortality rates of Malaysian people is how to deal with the random variations which exist due to the low number of deaths and people surviving in old age. Furthermore, based on Figure 2, the death counts typically fluctuate due to the natural variability in the mortality process within the population at risk. Therefore, the mortality model ensures a harmonious link from one age to another as well as estimating the missing probabilities.

Many researchers suggest modelling the old age mortality rates using the exponential-based models. In Malaysia, the Department of Statistics of Malaysia which is the governing body to publish the Malaysian mortality rates applied the Gompertz (1825) exponential model in modelling these rates (Department of Statistics Malaysia, 2021). The Gompertz model is the oldest model yet, it is the easiest model to be fitted as it only consists of two parameters.

As more and better mortality data becomes available, studies have discovered mortality deceleration in the adult life spans. Hence, logistic-based models have been proposed to models these mortalities such as Beard (1963). Furthermore, starting from 2010, instead of increasing with increasing rate, both older age male and female mortality curves appear to grow at a decreasing rate. Hence, their shapes can no longer be modelled using the existing exponential models (Khaliludin *et al.*, 2021).

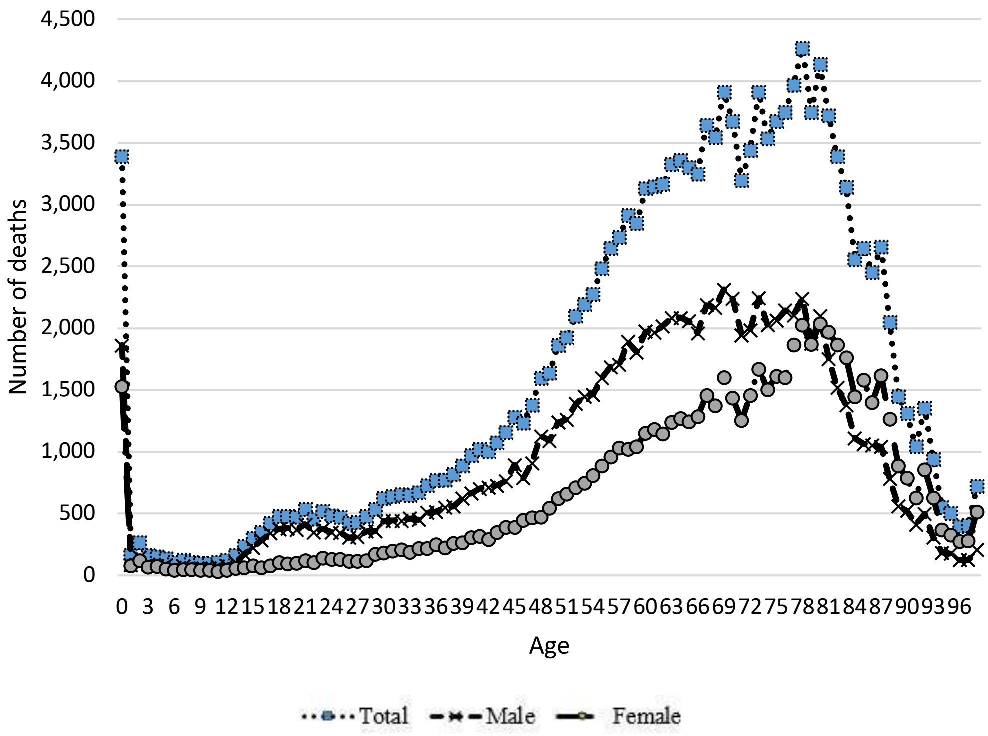


Figure 2 Death count from age 0 to 98 in 2016

This paper selects seven mortality models namely Gompertz, Makeham, Heligman and Pollard, Beard, Kannisto, Wilmoth and Khaliludin models and applies the models to the Malaysian mortality data to determine which model is the best model to estimate the mortality in Malaysia

METHODOLOGY

This study aims to compare seven mortality models that can calculate the accurate Malaysian mortality rates for old ages which is the age between 60 to 98. As each model has a distinct curve, any changes in the mortality pattern will cause the models to fit poorly on the data. The formula for each of the models are listed in Table 1. $\hat{m}_x$ is the fitted mortality rate estimated from the crude or empirical mortality rate, m_x and age is presented by x. All the comparative models estimated the observed mortality rates directly but the Heligman Pollard (HP) model estimated the conditional probability of death which is denoted by $\hat{q}_x$. This research employs the Nelder-Mead method as the optimisation strategy.

The oldest model which is the Gompertz model is proposed by Gompertz (1825). It is perhaps one of the most notable models in the history of mortality and survival modelling. He proposed an exponential increase in death rates with age (from about 35 to 80 years of age). His mortality function increases from α at time 0 to ∞ at time ∞ at an increasing rate of β. The oldest model which is the Gompertz model is proposed by Gompertz (1825). It is perhaps one of the most notable models in the history of mortality and survival modelling. He proposed an exponential increase in death rates with age (from about 35 to 80 years of age). His mortality function increases from α at time 0 to ∞ at time ∞ at an increasing rate of β. The parameter α represents the slope of the function or the baseline hazard and the parameter β represents the rate at which the mortality rate increases. Makeham (1867) extended the Gompertz model by adding a constant term representing the mortality of young people to the force of mortality which is symbolised by c in the Table 1. This constant also explained the risk of death from all causes which independent of age. The other terms shared similar meaning as Gompertz model.

Table 1: List of functions of the comparative models.

Name	Function
Gompertz	$\widehat{m}_x = \alpha exp(\beta x)$
Makeham	$\widehat{m}_x = \alpha exp(\beta x) + c$
Beard	$\widehat{m}_x = \frac{\alpha exp(\beta x)}{1+c exp(\beta x)}$
Kannisto	$\widehat{m}_x = \frac{\alpha\ exp(\beta x)}{1+\alpha exp(\beta x)}$
Heligman Pollard	$\frac{\hat{q}_x}{1-\hat{q}_x} = A^{(x+B)^C} + D\ exp[-E\ log(\frac{x^2}{F})] + GH^x$
Wilmoth	$\widehat{m}_x = \frac{exp(\alpha+\beta x)}{1+exp(\alpha+\beta x)}$
Khaliludin	$\frac{\alpha}{1+exp\left(-\frac{x-\beta}{\varsigma}\right)}$

Heligman and Pollard (HP) model is also another extension of Gompertz model. The first term represents a child mortality pattern in which A reflects the infant mortality rate, B reflects the mortality rate of one-year old children and C reflects the rate of mortality declines up to early adult life. The second part represents accident mortality in early adult life. This pattern is also known as accident hump in the demographic literature. The last term which is the Gompertz exponential, reflects the pattern of adult mortality. However, it is often difficult to estimate the parameter as there are eight parameters in total.

Likewise, Beard (1963) also derived his formula from the Gompertz function. Let us suppose that the whole population follows the Gompertz function but an older individual aged x slightly deviates with a scalar Z. Beard referred to Z as the longevity factor or better known as the frailty component. They further assumed that Z follows a Gamma distribution. Thus, the final formula for the Beard model is given by equation in the Table 1 where α is the scale parameter, β is the rate of increment and c is the frailty. It should also be noted that the numerator of Beard equation is the Gompertz formula.

On the other hand, Kannisto observed an empirical result and developed one of the simplest forms of the logistic model for mortality data at high ages. Yet, it should be noted that the model is established for the oldest old age, particularly those beyond 80 years of age. In his formula, α denotes the baseline hazard or the slope of the mortality curve and β denotes the rate of increment of mortality rates.

Wilmoth *et al.* (2007) derived the Beard model by modifying the limiting parameter α to be one to ensure that the probability of death, $\hat{q}_x$ will not exceed 1. The parameter α is the point of departure at age 0 and the parameter β corresponds to the rate of increase of mortality from age x to the age $x+1$. Both parameters are assumed to be positive, and this condition is enforced as the model is estimated.

A modification of the Wilmoth model is done by Khaliludin et al (2021). In contrast to the Kannisto, Beard and Wilmoth model, this model lets the mortality data itself determine the maximum or limiting rate and thus, it lets the numerator be some number, α. Like any logistic model, the parameters β and ς in the model control the rate at which the mortality decreases or increases. Moreover, β is the age at which the change in the direction of the mortality curvature occurs and ς defines the 'wriggliness' or the sigmoidal shape of the curve.

MODEL ASSESSMENT

It is essential to assess the appropriateness of these mortality models. Graph analysis is always an excellent approach to observe the fitting of a model such as a scatterplot of fitted and observed values. This graph helps to determine the distribution of the fitted rate around the empirical rate.

The accuracy of a model is also gauged using loss functions. One of the most common choices for the loss function is the Root Mean Square Error (RMSE). RMSE computes the deviation of the estimated mortality rate from the crude mortality curve. A large RMSE indicates that the model produces more errors. In this function, N denotes the number of observations, T is the number of years, x is the age, t is the year, $m_{x,t}$ is the empirical mortality rate for age x in year t and $\hat{m}_{x,t}$ is the fitted mortality rate for age x in year t.

$$RMSE = \sqrt{\frac{1}{NT}\sum_{x}\sum_{t}\left(\frac{\hat{m}_{x,t} - m_{x,t}}{m_{x,t}}\right)^2}$$

The models are also compared in terms of Mean Absolute Percentage Error (MAPE) as shown by equation (3.37). The interpretation of MAPE is in the same way as RMSE. A lower value of MAPE suggests that the model performs well on the mortality data.

$$MAPE = \frac{1}{NT}\sum_{x}\sum_{t}\frac{|\hat{m}_{x,t} - m_{x,t}|}{m_{x,t}}$$

The comparison of the mortality models using these tests is conducted using the *k-fold* cross-validation. This type of cross-validation uses all data to minimise bias and variance. The mortality data are divided into *k* folds (*k* is chosen to be seven) of equal size and each of the folds is split into two parts which are the training and test sets. The test set is a measure of forecast performance. In contrast, the training sets focus on the performance of the model on unseen data and in reflecting the structure of the mortality data. A model is fit using all the data except the first subset. Then, the prediction error of the fitted model is calculated using the first held-out samples. The same operation is repeated for each fold and the model's performance is calculated by averaging the errors across the different test sets. The error gets smaller as long as the fitted values are close to the observed rates and will get larger if, for some of the observations, the fitted and observed mortality rates differ substantially. One of the aims of this method is to assess the model's ability to predict observations never seen during estimation. In general, the model that corresponds to the lowest test error will be selected as the best model. The optimal model should perform well in both training and test datasets.

RESULTS AND DISCUSSION

Seven mortality models are illustrated with data on Malaysian males and females from ages 60 to 98 over the period of 2010 to 2016 obtained from the Department of Statistics, Malaysia.

Table 2 and Table 3 summarise the result for RMSE and MAPE for all the mortality models under this study. Table 2 shows the quantitative amount of RMSE and MAPE for all the models fitted to the mortality dataset in Malaysia for both males and females and Table 3 ranks the results for each model. The model which has the lowest value of RMSE and MAPE is ranked as number one in Table 3. This means that the model produces less error and thus, becomes the best model for the Malaysian dataset.

As can be seen from Table 2 and Table 3, Khaliludin model outperforms all the existing mortality models for both males and females. Beard model becomes the best male model after the proposed model which differs by only 0.007 and 0.04 for RMSE and MAPE respectively. However, this model placed quite lower rank for female mortality dataset and Wilmoth model produced the most accurate result after the Khaliludin model. Kannisto model remains as the average model for both genders. As for the female mortality, Gompertz model, Makeham model and Heligman Pollard (HP) model that share Gompertzian trajectories performed very poorly on the Malaysian mortality dataset with Makeham model being the worst model out of the seven models. RMSE and MAPE may have assessed the quantitative measure for the accuracy of the mortality models but it is also important to analyse the mortality visually to have a better insight into how these models describe the mortality data.

Table 2: MAPE and RMSE for male and female

Gender	Male		Female	
Model	RMSE	MAPE	RMSE	MAPE
Khaliludin	0.0242	0.0217	0.0174	0.0444
Gompertz	0.0406	0.1082	0.0246	0.1641
Makeham	0.0448	0.1155	0.0535	0.1264
Beard	0.0312	0.0673	0.0246	0.1640
Kannisto	0.0358	0.0936	0.0234	0.1633
Heligman Pollard	0.0432	0.1085	0.0276	0.1335
Wilmoth	0.0375	0.1098	0.0204	0.1640

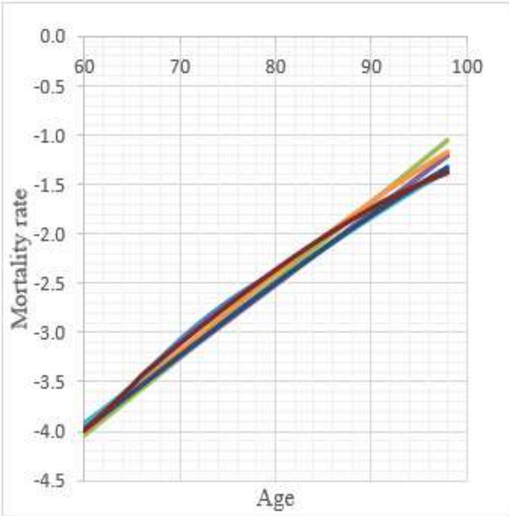
Table 3: Ranking of model based on RMSE and MAPE

Gender	Male		Female	
Model	RMSE	MAPE	RMSE	MAPE
Khaliludin	1	1	1	1
Gompertz	5	4	6	7
Makeham	7	7	7	2
Beard	2	2	5	6
Kannisto	3	3	3	4
Heligman Pollard	6	5	4	3
Wilmoth	4	6	2	5

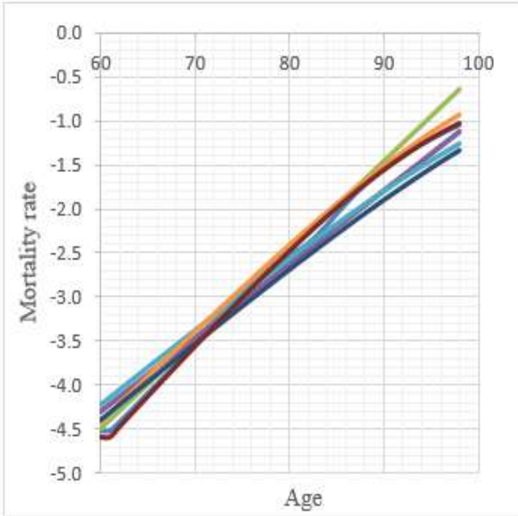
Figure 3 illustrates the mortality rates fitted for the older age males and females in Malaysia from 2010 to 2016. The crude mortality graphs are concave and the curves are especially pronounced for males than females in the latter years. As a rule of thumb, if the mortality model correctly explains the data, then the fitted mortality curve will roughly follow the empirical mortality curve. So, it passes the eyeball test. It can be seen in Figure 3 that the Khaliludin logistic model adheres acceptably well to this feature for all the years except in 2016, where the fitted mortality rates slightly deviate from the empirical mortality rates. This is mainly due to the parameters α and ς which constraint the rates to drop lower as age increases.

Heligman Pollard (HP) model which has the largest number of parameter performed poorly on the older age data. The main reason is that the estimates for older age fitted using HP model are linked by the younger ages and younger people have much lower mortality rates compared to what the elderly have. Furthermore, the last term of HP model which was particularly designed for the older ages was developed from the Gompertz model. The Gompertz model formulates an exponential increase of mortality rates which eventually overstates the mortality rates. It should be noted here that Gompertz model, Makeham model and HP model were designed to model the mortality rates for countries that have a higher life expectancy and lower mortality rates than Malaysia. Therefore, it is expected that these models overestimate the Malaysian mortality rates. Similarly, the Kannisto model is developed for the oldest age too. Thus, it also overstates the Malaysian mortality rates.

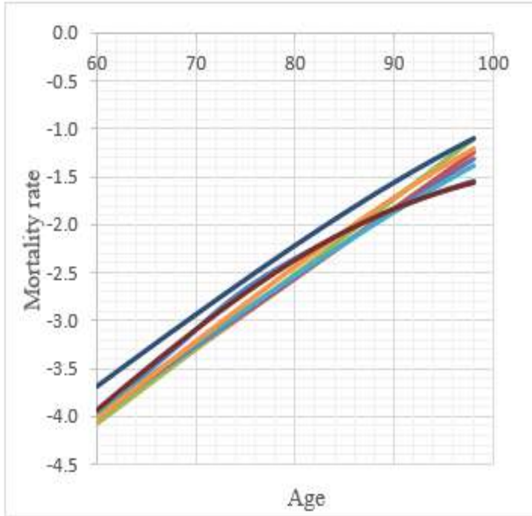
Besides that, along with the results from the RMSE and MAPE from Table 2 and Table 3, Figure 3 shows a straight increasing line when the mortality rates were fitted using the Gompertz model, Makeham model and HP model which again, strongly indicates that these models are poor models for the Malaysian mortality data.



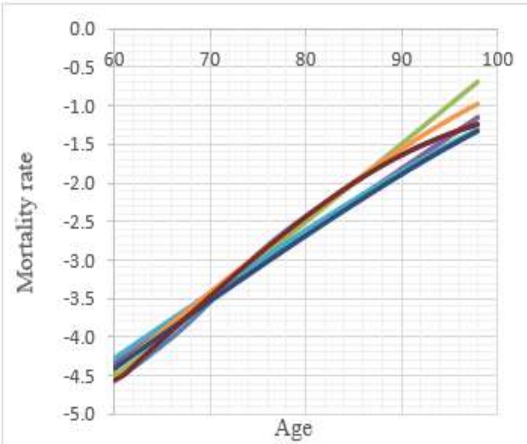
(a) Male 2010



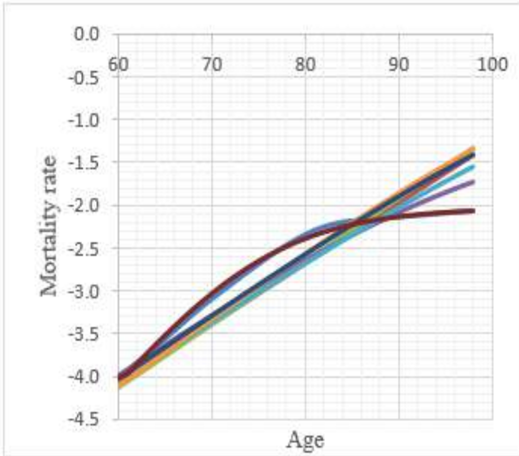
(b) Female 2010



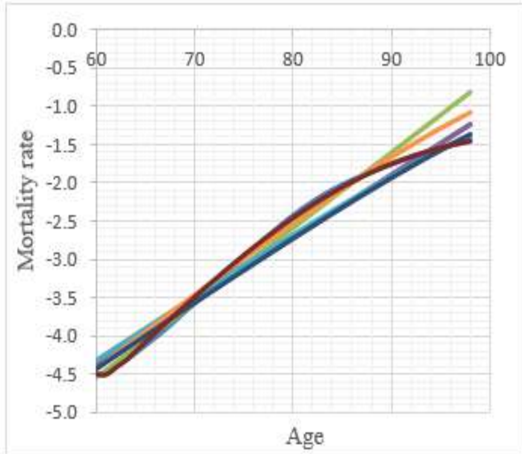
(c) Male 2011



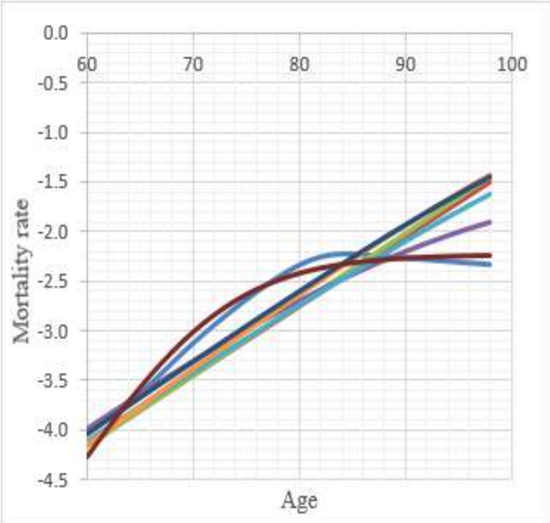
(d) Female 2011



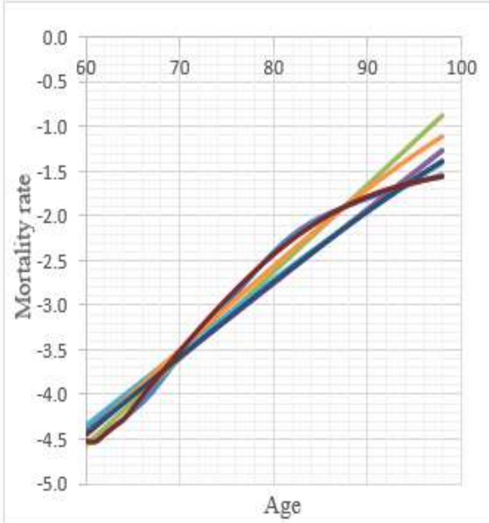
(e) Male 2012



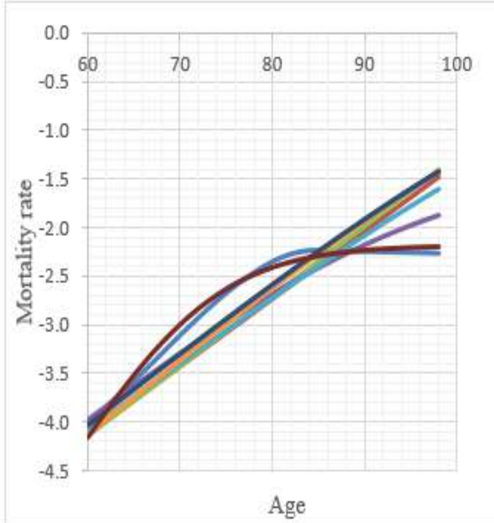
(f) Female 2012



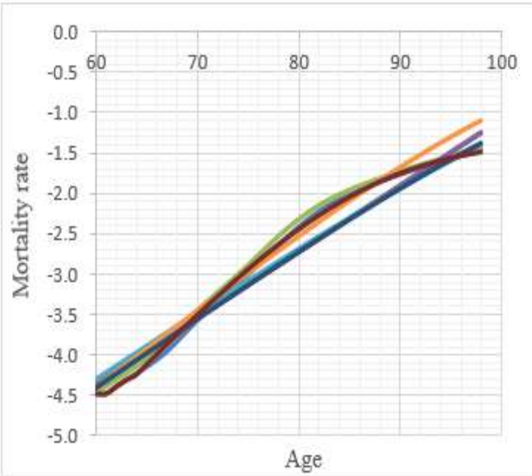
(g) Male 2013



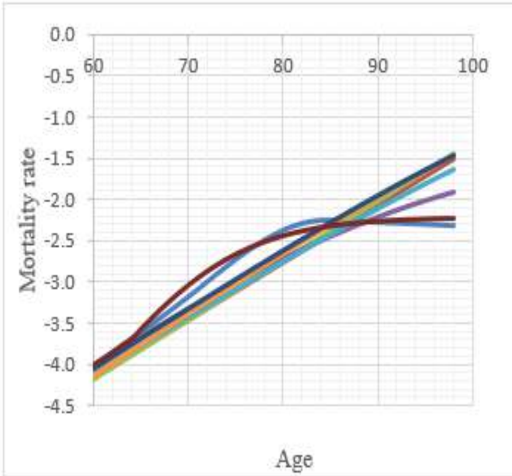
(h) Female 2013



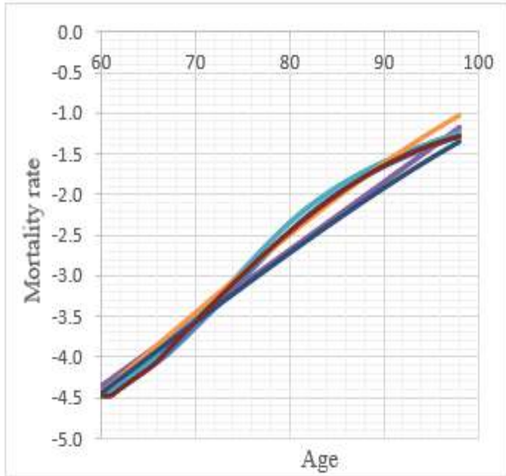
(i) Male 2014



(j) Female 2014



(k) Male 2015



(l) Female 2015

(a) Male 2016 (b) Female 2016

Figure 3 Fitted logistic model using the mortality models in for old age from 2010 to 2016

Apart from the assessment of accuracy, this research also measures the forecastability of the mortality models over the period of 2010 to 2016. Again, the models' goodness of fit tests are evaluated based on RMSE and MAPE and are tabulated in Table 4 and Table 5. A low value in Table 4 and low rank in Table 5 indicate that the model is the best mortality model for Malaysian dataset. Likewise, the results agree with Table 2, Table 3 and Figure 3. The Khaliludin model is the most accurate model to describe the future mortality rates of the older age in Malaysia. This is mainly because of the parameters of this model represent inflection points at which the deceleration of age starts as well as the flexibility of the curve and the limiting rates so that the produced rates will not be far from the crude mortality rates.

As can be seen from Table 4, the other logistic models, Beard and Kannisto model also produce less error. This is mainly because of the frailty component of these models.

Table 4 Forecast error measurement for males and females

Gender	Male		Female	
Model	RMSE	MAPE	RMSE	MAPE
Khaliludin	0.0017	0.1503	0.0022	0.1254
Gompertz	0.0511	0.3578	0.0215	0.1701
Makeham	0.0673	0.3821	0.0341	0.1573
Beard	0.0156	0.2673	0.0215	0.1701
Kannisto	0.0346	0.3200	0.0195	0.1824
Heligman Pollard	0.0666	0.3527	0.0257	0.1377
Wilmoth	0.0714	0.3316	0.0253	0.1391

Table 5 Ranking of model based on future RMSE and MAPE

Gender	Male		Female	
Model	RMSE	MAPE	RMSE	MAPE
Proposed	1	1	1	1
Gompertz	4	6	4	6
Makeham	6	7	7	4
Beard	2	2	3	5
Kannisto	3	3	2	7
Heligman Pollard	5	5	6	2
Wilmoth	7	4	5	3

CONCLUSION

The progressive decrement in the late-life mortality and rapid population ageing is of vital interest to policymakers, retirement providers and insurers due to the higher expenditure on retirement and insurance payments as well as greater social care costs. However, modelling of old age mortality is challenging not only because of the low quality of data but more because the observed numbers of deaths are low because of the low number of people alive at old ages. Hence, a simple parametric model would be a better choice as it allows borrowing strength across ages, which is important when the mortality data is sparse.

This paper compares seven well-known mortality models such as the Gompertz model, Makeham model, Beard Model, Kannisto model, Heligman Pollard model, Wilmoth model and Khaliludin model using the measurement error namely Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). The validation for all the mortality models are based on approximate k-fold Cross-Validation (CV) performance as this method has been proven to test the bias-variance trade-off. The results show that Khaliludin model significantly improves the estimation of the older age mortality in Malaysia in terms of accuracy and forecast performance.

All of the approaches are applied to the males and females in Malaysia from age 60 to 98 years old for the period of 2010 until 2016. The mortality data are separated into male and female as they both behave differently at certain ages such as during late teens where male mortality inherits a phenomenon known as the 'accident hump' and the male mortality has higher rate compared to the female up to an age where the female finally has higher death rate than male.

The data on the death and population number used in this research is for the years 2010 until 2016 for each male and female in the age of one to 98 years. Thus, more data from recent years can be utilised to compare these models. It is also recommended to extend these mortality model by incorporating more explanatory variables to investigate the relationship of the mortality rates with several death causes mainly smoking status and medical history.

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