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THE CO-MOVEMENT OF CHINA AND US STOCK INDICES: A PORTFOLIO DIVERSIFICATION ANALYSIS

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ABSTRACT

The aim of this article is to find diversification opportunities by examining the time-varying and time-scale-based volatility and correlation of the US and Chinese stock market indices with crude oil, gold and Bitcoin price returns, as well as the exchange rate of the Chinese Yuan Renminbi against the US Dollar (CNY/USD) using a vector error correction model (VECM), namely, maximum overlap discrete wavelet transformation (MODWT). Furthermore, individual and institutional investors may also reduce the risk of their investment portfolio by investing in commodities and stock markets from countries with a negative or substantially low correlation. Our

VECM result shows that Bitcoin, crude oil and CNY/USD lead the other variables under consideration, indicating that changes in the prices of Bitcoin, crude oil and CNY/USD affect the US and Chinese stock market indices, as well as gold. Our research utilising the MODWT technique shows that Bitcoin leads crude oil at almost all levels, indicating that crude oil prices will respond to Bitcoin price movement in the long and medium term. However, investors may be deterred from using Bitcoin as a diversification tool due to its extreme volatility. The research also indicates that diversification with gold may help US investors. However, the continuous wavelet transformation finding shows that the diversification benefit effects will persist for a holding period of little more than 64 days. Our study results tend to emphasise the significance of using reasonably modern methods to identify diversification possibilities for investors with diverse investment horizons or holding stocks for various periods.

Keywords: Bitcoin, Gold, Crude Oil, CWT, MGARCH-DCC.

JEL: C22, C58, E44, G15.

INTRODUCTION

Research on the co-movement of stock markets is crucial for portfolio risk management. Investors would like to avoid a high correlation of assets in any portfolios that will lower gains. The literature has acknowledged that the co-movement of stock returns fluctuates with different periods. Therefore, investors must identify risks associated with a different time horizon to manage their risk. In the meantime, the different objectives and motives between heterogeneous (short-term and long-term) investors should be considered. Short-term investors are more inclined to short-term gains from price movement in their investment portfolio. By contrast, long-term investors are interested in long-term price fluctuation. The risk associated with investors depends on the investors' type, whether they focus on the long or short term. Therefore, due to the degree of the co-movement of stock returns varying across time horizons, the risk associated with investors will also be different. Modern techniques such as wavelet analysis can help us identify the "time-varying" and "scale-dependent" volatilities and correlations of stock indices and commodities price return.

Volatilities of stock indices and commodities price return refer to the drastic changes, either increasing or decreasing, in the value of the variables under review within a specific period. Investors need to evaluate the volatilities of variables before making their decision to buy or sell. The period of volatility tends to be long and can last for months. High or low volatility stages can continue for months, depending on internal and external shocks. For example, the high volatility of stock returns inclines to last for years during economic recessions. Stock market volatility is positively connected to economic data such as debt level, inflation, and industrial production (Schwert, 1989). We can summarise that investors need to understand the volatilities of variables under review to make an informed decision regarding their portfolio management. In addition to volatility, correlation is an essential factor for decision making in portfolio diversification management. Successful investment strategies rely heavily on understanding the correlation among assets under a portfolio, as Markowitz (1959) explained. Since his study, stock return correlation has played a significant role in risk estimation for portfolio diversification.

Several studies have tested international diversification’s benefits since the 1960s and 1970s, such as Grubel (1968) and Levy and Sarnat (1970). Grubel (1968) investigated stock indices of 11 industrialised countries, finding enormous advantages for portfolio diversification. According to his study, an investor can increase their annual returns by 65 per cent and, at the same time, ensure their risk is manageable if they diversify their portfolio in international markets. However, Bekaert and Harvey (1995) asserted that globalisation activities have increased due to the lower foreign investment barrier, reducing profit for international portfolio diversification.

Portfolio investment that solely focuses on international stock markets for diversification benefits will be exposed to fluctuation in the currency exchange rate. This fluctuation will increase the portfolio risk and force investors to hedge their currency exposure, leading to increased investment costs. Therefore, Bitcoin is the appropriate investment alternative for investors because Bitcoin transactions have no fees and commissions and are not subject to exchange rate fluctuations. Several studies employ conventional models (such as correlation analysis and linear regression) to emphasise the low correlation between Bitcoin and stock market indexes and possible diversification advantages (Baur et al., 2017; Guesmi et al., 2018). Bitcoin has been found to be a good stock diversifier (Bouri et al., 2017). However, several other researchers have questioned Bitcoin's diversification advantages (e.g., Chowdhury, 2016) and expressed concerns about Bitcoin's chances as an alternative currency. Disagreement remains about whether Bitcoin has inherent value and if its meteoric price increase results from an irrational bubble (Li & Wang, 2017). The link between Bitcoin and stock markets seems minimal, most likely because the two markets have different participant criteria (Filtz et al., 2017). Financial institutions are reluctant to invest directly in Bitcoin for various legal, tax, and accounting reasons (Tan & Low, 2017). Most Bitcoin market participants are inexperienced, youthful individuals who seem to stray from logical behaviour while processing information and making investment decisions. As a result, they drove Bitcoin prices up by thousands of per cent in only a few years, while gold and other commodity prices exhibited minor fluctuations (Bouri et al., 2018). Another reason explaining the shaky connection between Bitcoin and stock markets is that the price drivers in each market are distinct, as Kristoufek (2015) and Bouoiyour et al. (2016) showed. Owing to the ambiguous findings on the relationship between Bitcoin and stock markets, we would want to delve further into the subject to contribute to the literature.

International investment also exposes investors to many risks. For example, the correlation coefficients rise significantly during the crisis period, leading to diminishing diversification benefits. Investors also have to consider other risks, such as political risk, unfamiliarity with local laws and regulations, and different accounting standards and tax rules (Solnik & McLeavey, 2003). Therefore, other investment alternatives, such as commodities, are becoming popular with investors who seek new diversification instruments. This study also selected a few commodities, such as gold, crude oil, and Bitcoin, as variables to be investigated to find their volatilities and correlation with the US and China stock indices. According to Lee and Zulkefli's (2021) research, the fundamental cause of the trade war between the US and China was not due to trade imbalances or unfair practices, as previous literature has indicated, but rather the US determination to avoid the erosion of the US hegemony. By doing this study, we could determine whether the US is still leading or behind China in the stock market in recent years.

The primary objective of the research is to analyse the lead-lag relationship between China and the US stock indices with other variables such as Bitcoin, gold, crude oil, and the exchange rate of CNY/USD (as a control variable) to assess the possible advantages that investors could realise from diversifying their holdings in the stock markets and commodities markets. We want to find out how the six variables under analysis are related and determine the granger-causality direction.

The paper is unique in its empirical analysis contribution to the "time-varying" and "multi-scale" volatilities and correlations, enhancing the present literature. The paper can identify unique portfolio diversification potentials for heterogeneous investment intervals by incorporating scale dependence. The particular research questions are as follows:

i. Do the increase or decline in the US and China stock indices affect the other variable being reviewed, in which past indices values have improved the prediction of the other variables?
ii. Among the variables, which one at different time horizons is more exogenous?
iii. Which commodities and stock indices would investors benefit from market diversification?
iv. How would the advantages of diversification change, considering the various investment horizons?

The findings from this study should have a considerable effect on investors in portfolio allocation and investment strategies. In brief, this paper humbly aims to address investors' technical analytical needs with recent data and modern methodologies that intend to diversify their investments in the US and China stock indices, Bitcoin, gold, crude oil, and CNY/USD.

The article structure is organised as follows. Section 2 discusses the literature relevant to Bitcoin, the diversification of the commodity sector and stock market index in time-varying and scale-dependence, and the discussion of the theoretical basis utilised for this article. Section 3 offers information on the methodologies for achieving this paper's research goals. Section 4 provides an extensive analysis of the evidence and analytical observations. Lastly, section 5 addresses the conclusion from the previous sections with reasonable explanations and historical research observation.

LITERATURE REVIEW

Studies on portfolio diversification with emerging markets have gained momentum due to the high potential benefits in this area. Emerging markets have less connection with developed markets, implying diversification opportunities for international investors. For instance, Markellos and Siriopoulos (1997) argued that investors who diversified their investment across European emerging markets would generate more significant profits than those with non-diversified portfolios. Dunis and Shannon (2005) studied equity markets of Southeast Asia (Malaysia, Indonesia, and the Philippines) and Central Asia (China, Korea, Taiwan, and India) and reported that US investors experienced international diversification benefits from both markets. Another study by Middleton et al. (2007) found that Central and Eastern European investments remain beneficial even during the financial crisis.

Foreign investment is not immune to foreign exchange risk, which might affect international diversification benefits to some extent (De Roon et al., 2003; Dunis & Shannon, 2005; Eun & Resnick, 1988). Passive and active international investors spend much time finding ways to hedge foreign exchange risk (Eun & Resnick, 1997; Papadamou & Tsopoglou, 2002). Nevertheless, empirical findings related to hedging strategies studies show inconclusive results. Undeniably, currency hedging would significantly stabilise the international cash flow of diversified cross-border portfolios (Bekaert & Harvey, 2002; Eun & Resnick, 1988; Eun & Resnick, 1997; Solnik, 1993). Nevertheless, Walker (2008) exerted that hedging activities might increase volatility in the emerging market's international assets. In any instance of negative global equity returns, market currencies tend to move downward and vice versa.

Presently, businesses accept Bitcoin as a payment method and consider it similar to other currencies. According to Gajardo et al. (2018), among the many advantages of cryptocurrencies are their ability to decrease transaction costs, provide high security in online transactions and potentially reduce exchange rate risk. Despite its high price volatility, the trading ability of Bitcoin units on specialised trading platforms has made it an investment asset (Polasik et al., 2015). The introduction of Bitcoin-linked funds by significant investment banks has broadened access to the Bitcoin market. Notably, the introduction of futures contracts based on Bitcoin prices in late 2017 has increased Bitcoin's credibility as an investment and pushed it closer to the heart of the financial world. This development signifies that the investment community should not dismiss Bitcoin as a potential safe haven like gold in the future (Shahzad et al., 2019). According to Hanley (2013), Bitcoin has a weak correlation with other currencies. Macroeconomic factors that alter daily have little effect on Bitcoin price swings. Consequently, Hanley contended that Bitcoin's volatility is uncontrollable, and investors cannot hedge their Bitcoin holdings. He further emphasised that Bitcoin has no intrinsic value and that its volatility concerning other currencies is speculative because it is decided by market pricing.

In summary, the literature examining the co-movements of stock prices and portfolio diversification strategies (especially with Bitcoin) is limited and inconclusive. Therefore, this topic requires deeper study.

THEORETICAL BACKGROUND

Markowitz introduced the theory identified for this research, namely, portfolio diversification theory. Markowitz formed the modern portfolio theory (MPT) in which portfolio volatility is lower than the weighted average of securities volatility that it contains, given that the assets underlying the portfolio have the least correlation in return (Markowitz, 1959). The earlier models of MPT assumed that portfolio variances have a normal distribution. The normally distributed variation is, however, inadequate to quantify risk, according to Markowitz. Subsequent models should have asymmetrical and fat-tailed properties as it mimics real-world data. In this analysis, we used the multivariate generalised autoregressive conditional heteroscedastic dynamic conditional correlation (MGARCH-DCC) approach and adopted a Student's *t*-distribution variance that is preferable for detecting the fat-tailed nature of index return (Pesaran & Pesaran, 2010). In and Kim (2013) clarified that implementing wavelet transformation methodologies does not make a priori assumptions of distributions leading to more realistic results. We further clarify in the next section the data and methods that were used.

METHODOLOGY

Data

Our data include Bitcoin, crude oil, gold prices, CNY/USD, and China and US conventional stock indices from 1 September 2011 to 28 June 2019.

Table 1

Description of Variables

Variable	Description	Source
BITCOIN	Exchange rate of Bitcoin to USD	Thomson Reuters Datastream
OIL	Crude oil prices	
GOLD	Gold prices	
CHINCON	MSCI Conventional Index - China	
USCON	MSCI Conventional Index - US	
CNY/USD	Exchange Rate of the China Yuan to USD	

Time Series Techniques

The study adopted traditional time series methods of error correction modelling to effectively capture the empirical relationship between Bitcoin price, commodities price, and stock indices. Standard time series methods were introduced to test the hypothesis of Bitcoin leading (or lagging) other commodities and market indexes under study. Recent cointegration-based time series experiments have implemented either vector error correction modelling (VECM) and/or variance decomposition (VDC) approaches to examine the Granger causality relationship. Several standard procedures to run lead-lag interaction included unit root test, identification of VAR lag order, and Johansen cointegration test. Cointegration analysis provides long-term information but is insufficient to indicate a lead-lag relationship. Therefore, to determine Granger causality covering long-term and short-term dynamics relationships, the VECM analysis was conducted (Masih et al., 2009).

VECM analysis alone did not offer information on the nature of the variable's exogeneity or endogeneity. VDC methodology was the proper approach for determining the most exogenous and endogenous variables. However, VDC testing was constrained to 150 observations only. At the time of this study, our data covered 2042 observations. If we used only 150 observations (five months of data), the results might not be sufficient to provide a valid opinion. Therefore, the study opted for maximum overlap discrete wavelet transformation (MODWT) analysis to cater to a causal relationship between defined variables across various time scales.

MODWT

Based on past studies, the discrete wavelet transform (DWT) and MODWT will fragment the sample time series variance by squared wavelet transform coefficient in a well-defined scale. Given that the MODWT estimator is superior to the DWT estimator (Percival, 1995; Gallegati, 2008), the study adopted MODWT analysis. Whitcher et al. (1999, 2000) expanded the traditional MODWT modelling by introducing wavelet covariance and correlation along with its point estimators and confidence intervals. The wavelet covariance needs to be utilised to determine the extent of co-movement between X and Y series across different scales. Following Gençay et al. (2001) and Gallegati (2008), the wavelet covariance can be explained as the covariance between wavelet coefficients between variables X and Y on a scale-by-scale basis such that $\gamma_{XY,j} = \mathrm{Cov}[\tilde{\omega}^{X}_{j,t}\tilde{\omega}^{Y}_{j,t}]$. The following equation calculates unbiased wavelet estimator covariance after satisfying boundary conditions (Gallegati, 2008):

$$\tilde{\gamma}_{XY,j} = \frac{1}{\tilde{N}_j}\, N - 1 \sum_{t=L_{J-1}}^{N-1} \tilde{\omega}^{X}_{j,t}\tilde{\omega}^{Y}_{j,t}$$

Subsequently, MODWT wavelet cross-correlation coefficients for scale j at lag τ can be calculated as the ratio between the cross-covariance wavelet $\tilde{\gamma}_{\tau,XY,j}$, and the wavelet standard errors $\tilde{\sigma}_{X,j}$ and $\tilde{\sigma}_{Y,j}$ as:

$$\tilde{\rho}_{\tau,XY,j} = \frac{\tilde{\gamma}_{\tau,XY,j}}{\tilde{\sigma}_{X,j}\tilde{\sigma}_{Y,j}}$$

The magnitude of wavelet coefficients $\tilde{\rho}_{\tau,XY,j}$ at scale j has a value between 0 and 1, signifying the strength of the relationship between X and Y series at a specific level j. Starting from spectrum $S_{\omega\ X,j}$ of wavelet coefficients at scale j, we can estimate the asymptotic variance V_j of the MODWT wavelet variance (covariance). Subsequently, 100(1 − 2p)% random confidence interval could be constructed. The formulation for the 100(1 − 2p)% confidence interval MODWT estimator robust to non-Gaussianity for $\tilde{\upsilon}^2_{X,j}$ was explained in detail by Gençay et al. (2002) and Gallegati (2008). Following past studies related to wavelet variance (Whitcher et al., 2000; Gallegati, 2008), the number of observations N=128 was sufficient to provide an excellent approximation to the wavelet coefficients.

MGARCH-DCC

To address the fourth research question, we adopt the multivariate generalised autoregressive conditional heteroscedastic (MGARCH) as proposed by Pesaran and Pesaran (2010). We examined both the normal and *t* distributions to find optimal specifications. The unconditional correlation coefficients address the fourth research question. Nevertheless, calculating conditional cross-asset correlations to validate our results was necessary. We used the MGARCH-DCC measurement as follows:

$$\tilde{\rho}_{ij,t-1}(\phi) = \frac{q_{ij,t-1}}{\sqrt{q_{ii,t-1}q_{jj,t-1}}}$$

Where $q_{ij,t-1}$ is represented as:

$$q_{ij,t-1} = \bar{\rho}_{ij}(1 - \phi_1 - \phi_2) + \phi_1 q_{ij,t-2} + \phi_2 \tilde{r}_{i,t-1}\tilde{r}_{j,t-1}$$

$\bar{\rho}_{ij}$ denotes the (*i,j*) th unconditional correlation, ϕ_1 and ϕ_2 represent parameters such that $\phi_1 + \phi_2 < 1$ and $\tilde{r}_{i,t-1}$ denotes historical asset returns. The mean-reversion process can be tested by estimating $(1 - \lambda_{i1} - \lambda_{i2})$. Some diagnostic tests were conducted to substantiate our estimates, as suggested by Pesaran and Pesaran (2010).

Continuous Wavelet Transformation (CWT)

We adopted CWT to address the fifth research question. Recently, research interest has been growing in adopting CWT analysis in economics and finance, such as Abdullah et al. (2016). The CWT connects one variable time function into time-frequency variables. CWT's significant advantage relative to DWT/MODWT has no specific a priori requirement to specify the number of time scales where the scales would be endogenously determined by CWT analysis according to its data length. CWT also explains the series correlations into a two-dimensional figure, allowing patterns and hidden information to be identified and interpreted easily. CWT analysis is easier to understand than a discrete approach because its redundancy reinforces the features and makes all information more noticeable. The analysis, therefore, becomes more straightforward to understand.

The Daubechies (1992) least asymmetric wavelet filter of length L=8 (denoted by LA (8)) was used to decompose time series for both CWT and MODWT. The moderate filter of L=8 seemed to be ideal for high-frequency time series data (Gençay et al., 2001; In & Kim, 2013). The previous findings suggested that an LA (8) filter creates smoother wavelet coefficients compared to other wavelet filters like Haar wavelet transformation.

The CWT term $W_X(u,s)$ was extracted by projecting a mother wavelet ψ onto the sample time series $x(t) \in L^2(\mathbb{R})$, which is:

$$W_X(u,s) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)dt$$

u represents the position of the wavelet in the time domain, while the frequency domain is given by *s*. The wavelet transform involves mapping the original series into a function of *u* and *s*, providing simultaneous details on time and frequency. To study the co-movement between two variables, we applied bivariate wavelet coherence to capture a linear transformation that relates X with Y. The wavelet coherence of the time series is defined as:

$$R_n^2(s) = \frac{\left|S(s^{-1}W_n^{xy}(s))\right|^2}{S(s^{-1}|W_n^x(s)|^2.S(s^{-1}\left|W_n^y(s)\right|^2}$$

S represents the smoothing operator, wavelet scale is denoted by *s*, $W_n^x(s)$ and $W_n^y(s)$ are the continuous wavelet transform of variables X and Y, respectively, and $W_n^{xy}(s)$ is the cross wavelet transform between variables X and Y (Madaleno & Pinho, 2012). Further references may be found in Gençay et al. (2001), Gençay et al. (2002), and In and Kim (2013).

Empirical Findings and Interpretations

Descriptive of Data

Figure 1 illustrates raw time-series data for all variables selected. The Bitcoin price is highly volatile, with the price increasing and decreasing substantially in a short period in 2017. A decreasing oil and gold prices trend was also shown, suggesting a lack of demand for these commodities. When the China stock index was compared with the US stock index, the China stock index was more volatile than the US stock index. From 2011 to 2015, the China equity index return was on an upward trend. However, the index plummeted dramatically between 2015 and 2017. The index gradually rose until it reached its peak in 2018. The high volatility of the China stock index implies an uncertain economy, which is mirrored in the CNY/USD exchange rate. The US stock index was more stable than the China stock index because it rose steadily from year to year with no interruption. The stability of the US stock index indicates a robust economy throughout the period.

Figure 1

Dynamics of Raw Time-Series Data

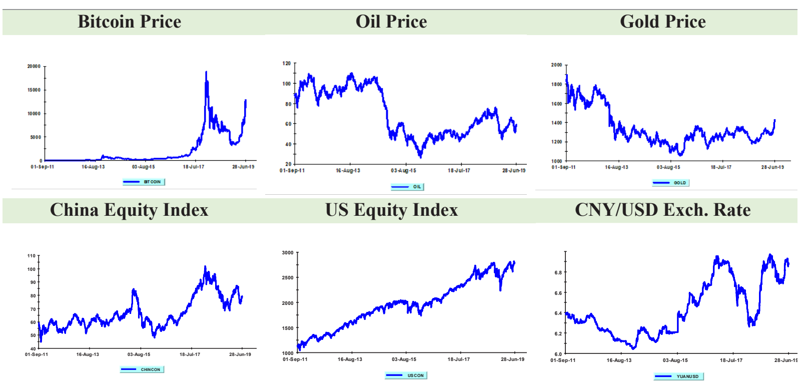


Table 2 shows the descriptive statistics for the returns series, presented as $rt = \ln(Pt/Pt\ 1)$, where rt is the series return calculated using the natural log and Pt is the price index at time t. Bitcoin's mean return exceeds the returns of US and China stock indices and other commodities. Bitcoin is also the most volatile asset compared to gold, crude oil, and the US and China stock indices. As a result, Bitcoin seems to be riskier, with a higher potential return than stock indices and other commodities in risk-return analysis.

Table 2

Descriptive Statistics

Variable	Mean	Standard Deviation	Min	Max	Skewness	Kurtosis	Number of observations
BITCOIN	0.00156	0.02558	-0.28835	0.2105	-0.95300	20.22617	2,042
OIL	-0.00009	0.00907	-0.04658	0.0505	0.02700	3.28375	2,042
GOLD	-0.00005	0.00415	-0.04413	0.0236	-0.79887	9.50625	2,042
CHINA	0.00006	0.00551	-0.02887	0.0282	-0.11339	2.83758	2,042
US	0.00002	0.00077	-0.00499	0.0079	0.39627	11.49093	2,042
CNY/ USD	0.00019	0.00371	-0.01796	0.0211	-0.33810	3.36847	2,042

Empirical Findings of Standard Time-Series Techniques

Stationary testing for all variables was examined using the augmented Dickey-Fuller specification where all variables were integrated of order 1, I(1). Furthermore, the optimal order of lag of the vector autoregressive (VAR) was found to be 1 based on Akaike's information criterion and Schwarz information criterion statistics. The leading or lagging properties of variables can be determined through the vector error correction model (VECM). Table 3 shows that Bitcoin, crude oil and the exchange rate of CNY/USD are exogenous, but the China and US conventional stock indices and gold are endogenous. Therefore, the gold and conventional stock indices of China and the US will respond to the Bitcoin prices, crude oil and exchange rate of CNY/USD. The VECM provides short-term and long-term Granger causality. The lagged error correction term captures the long-term relationship, while the significant joint F-test of the lagged differenced variables provides short-term causality. The diagnostics of the error correction model (which involved testing autocorrelation, heteroscedasticity, and functional forms) imply that the model is somewhat well specified. The relative exogeneity and endogeneity would be specified under the proportion of variance decomposition. Owing to constraints in the limit of observations (t=150) in VDC analysis, although our data consists of 2042 observations, we adopted MODWT to capture the causal relationship between variables.

Table 3

Error Correction Model of Bitcoin and Other Variables

Dependent Variable	DBitcoin		DOil		DGold		DChincon		DUScon		DYuanusd	
ECM (-1)	0.0057	(0.018)*	-0.0085	(0.006)*	-0.0061	(0.003)*	0.021	(0.004)	-0.011	(0.003)	-2E-05	(0.0006)*
Chi - square SC (1)	4.0991	(0.043)	14.3222	(0.000)	0.75159	(0.386)	10.9	(0.001)	0.383	(0.536)	4.1817	(0.041)
Chi - square FF (1)	2.7174	(0.099)	0.00634	(0.937)	0.41277	(0.521)	2.423	(0.120)	7.206	(0.007)	2.4964	(0.114)
Chi - square N (2)	34851	(0.000)	905.857	(0.000)	7978.5	(0.000)	668.7	(0.000)	1001	(0.000)	11211	(0.000)
Chi - square Het (1)	5.6395	(0.018)	14.6517	(0.000)	2.473	(0.116)	17.64	(0.000)	161.5	(0.000)	0.0529	(0.818)

Notes: Parenthesis values refer to SEs. The diagnostic checks are tested based on chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroskedasticity (Het).

* Indicate significance at the 5% level.

Empirical Findings of MODWT

Figure 2 displays MODWT wavelet cross-correlation estimates and confidence intervals for bitcoin and crude oil relationships against all scales' time lead and lag. The individual cross-correlation functions correspond to its wavelet scales from λ_1 …, λ_8 representing 1-2 days and 2-4 days until 128-156 days, respectively. The red lines for wavelet cross-correlation are bound around a 95 per cent confidence interval. The first variable is considered to lead if the curve is significant on the graph's left side and vice versa. The wavelet cross-correlation is significantly positive if both 95 percent confidence intervals are above horizontal axes, and negative wavelet cross-correlation is captured if both 95 percent confidence intervals are below the horizontal lines.

Figure 2 illustrates the wavelet cross-correlation between Bitcoin price return and crude oil price changes. We observed the following:

i) No solid causal relationship exists between these two variables at wavelet scales 1 and 2.
ii) The graph skewed to the left at wavelet levels of 3, 4, 5, 6, 7, and 8, indicating that the Bitcoin price return leads crude oil price return.

Figure 2 shows that Bitcoin leads crude oil at all levels except 1 and 2, suggesting that the two variables may be used to diversify, particularly in the long term. Even though Bitcoin leads crude oil, the two variables have a low correlation, as shown in Table 6, suggesting that both variables are suitable as portfolio investment instruments. According to Baur et al. (2018), Bitcoin is the most speculative commodity and does not serve as a money replacement or means of exchange. As a result, additional measures must be taken while contemplating Bitcoin as part of an investment portfolio.

Figure 2

MODWT: Bitcoin Price Return vs Crude Oil Price Return

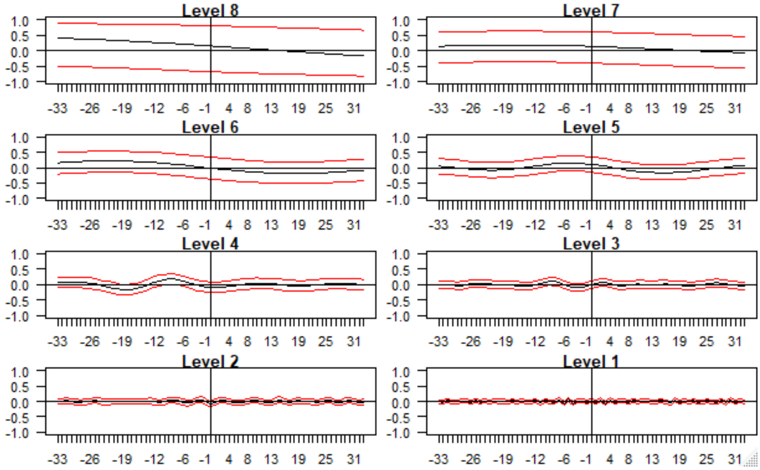


Figure 3 demonstrates the wavelet cross-correlation between Bitcoin price return and CNY/USD return. The figure indicates the following:

i) At the wavelet scales 1, 2, 3, 4, and 5, no clear lead-lag relationship exists between the two variables.
ii) At the wavelet scales of 6 and 7, the graph skewed to the left, indicating that Bitcoin leads the CNY price return.
iii) At level 8, the graph skewed to the right-hand side, suggesting that CNY/USD leads Bitcoin.

We may deduce a few observations from Figure 3, such as the lack of clear direction of which variable leads or lags at lower levels. However, at higher levels (levels 6 and 7), Bitcoin leads CNY/USD, and at the highest level (level 8), CNY/USD leads Bitcoin. This finding suggests diversification possibilities occur between these two variables, especially in the long term. Bitcoin is regarded as money and is now widely used in online transactions. Therefore, investors exposed to CNY/USD may opt to include Bitcoin in their portfolios for diversification purposes.

Figure 3

MODWT: Bitcoin Price Return vs. CNY/USD Exchange Rate Return

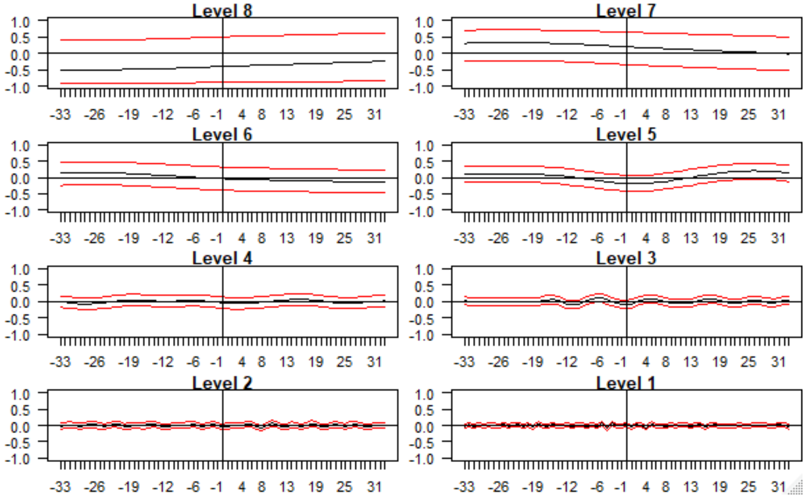


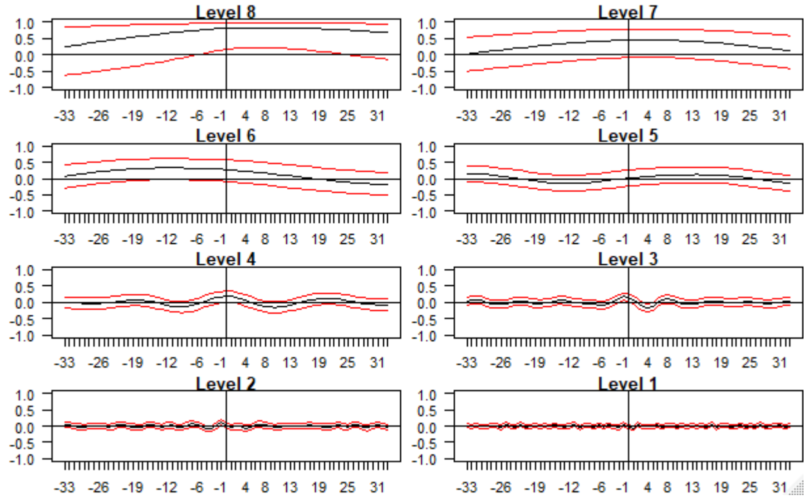
Figure 4 displays the wavelet cross-correlation between crude oil price return and CNY/USD return. Several noteworthy findings are as follows:

i) At the wavelet scales 1, 2, 3, and 4, no clear causal relationship exists between the two variables.
ii) At the wavelet scales of 5, 7, and 8, the graph skewed to the right, indicating that the CNY/USD leads the crude oil price return.
iii) At the wavelet scale 6, the graph skewed to the left, indicating that the crude oil price return leads CNY/USD price return.

This result implies that the CNY/USD leads the crude oil price return in the long run with a positive correlation. Furthermore, CNY does not rely heavily on the fluctuation of oil prices but vice versa.

Figure 4

MODWT: Crude Oil Price Return vs. CNY/USD Exchange Rate Return



Empirical Findings of MGARCH-DCC

MGARCH-DCC was used to examine the diversification benefits between the stock indices and commodities in this section. Table 4 displays the maximum likelihood estimates of commodities price returns (λ_{i1}) and stock indices (λ_{i2}). Furthermore, δ_1 and δ_2 were used to compare the multivariate normal distribution and multivariate Student's *t*-distribution.

For *t*-distribution, the maximised log-likelihood value is [49736], which is relatively more significant than the normal distribution [48893.3]. Furthermore, the degree of freedom for the *t*-distribution [6.3342] was below 30, implying that the *t*-distribution was much more appropriate in capturing the fat-tail of price returns. Therefore, the *t*-distribution was adopted for the rest of the analysis.

The diagonal elements of Table 5 illustrate the estimated unconditional volatilities, while the off-diagonal estimates infer unconditional correlations of the six variables. The numbers in the parentheses represent the ranking of unconditional volatility (from highest to lowest). Accordingly, Bitcoin and crude oil prices tend to face many speculative trades based on the unconditional volatility in Table 5.

The CNY/USD exchange rate has the lowest volatility, signifying that China's economy is stable and robust in the Asia-Pacific region, equipped with a stable currency. Referring to Table 5, the US stock index is the second least volatile. The index's stability shows that the US is still regarded as a safe haven for investors worldwide as the world's largest economy,

Table 4

Estimates of λ_{i1} and λ_{i2}, and δ_1 and δ_2, for the Six Variables Under Review

		Multivariate Normal Distribution		Multivariate *t*-Distribution	
		Estimate	T-Ratio	Estimate	T-Ratio
Lamda 1 (λ1)	Bitcoin	0.87098	58.4927	0.80892	43.0955
	Crude oil	0.9286	87.8063	0.94093	92.7363
	Gold	0.93849	92.6062	0.96716	147.1485
	Chincon	0.93827	78.8262	0.93652	62.2531
	CNYUSD	0.3687	5.772	0.87979	41.1235
	UScon	0.76814	31.2355	0.81161	31.1512
Lamda 2 (λ2)	Bitcoin	0.11467	9.2838	0.17863	10.6234
	Crude oil	0.058385	7.5514	0.04802	6.4743
	Gold	0.053751	6.6746	0.02948	5.4338
	Chincon	0.052307	5.9105	0.04896	4.8164
	CNYUSD	0.13418	9.699	0.09464	6.7881
	UScon	0.17813	9.9882	0.14652	7.7563
Delta 1(δ1)		0.98774	456.048	0.98859	443.5373
Delta 2(δ2)		0.00774	8.0035	0.00798	7.0104
Maximised log-likelihood		48893.3		49736.0	
Degree of freedom (df)		-		6.3372	

Note: λ_1 and λ_2 are decay factors for variance and covariance, respectively.

Table 5

Estimated Unconditional Volatility Matrix for China and US Stock Indices Return and Other Variables

	Bitcoin	Oil	Gold	Chincon	CNYUSD	UScon
Bitcoin	0.0255 (1)	0.0132	0.0487	0.0275	0.0128	0.0250
Oil	0.0132	0.0090 (2)	0.1110	0.1096	-0.0526	0.3151
Gold	0.0487	0.1110	0.0041 (4)	0.0420	-0.1153	0.0184
Chincon	0.0275	0.1096	0.0420	0.0054 (3)	-0.2103	0.2834
Cnyusd	0.0128	-0.0530	-0.1150	-0.2100	0.0008 (6)	-0.0763
UScon	0.0250	0.3151	0.0184	0.2834	-0.0763	0.0037 (5)

The third objective of the study focused on the correlations between commodity prices and stock indices. A quick analysis of the unconditional correlations listed in Table 6 highlights essential information that Bitcoin price exhibits the lowest correlation relative to other variables. The unconditional correlations are ordered from highest to lowest in Table 6 to provide a clearer picture of the relative correlation between variables.

Table 6

Ranking of Unconditional Correlations among China and US Stock Indices Return and Other Variables

Bitcoin (BITCOIN)	Crudeoil (OIL)	Gold (GOLD)	China CSIR (CHINCON)	CNY/USD CNY/NUSD	US CSIR (USCON)
GOLD	USCON	CNYUSD	USCON	CHINCON	OIL
CHINCON	GOLD	OIL	CNYUSD	GOLD	CHINCON
USCON	CHINCON	BITCOIN a	OIL	USCON	BITCOIN a
OIL	CNYUSD	CHINCON	GOLD	OIL	CNYUSD
CNYUSD	BITCOIN a	USCON	BITCOIN a	BITCOIN a	GOLD

Two critical facts are explained by the rankings. First, the Bitcoin price return has the lowest correlation for most variables (refer to notation ‘a’ in Table 6). This finding implies that Bitcoin should be included in the portfolio to realise the advantages of diversity. However, as shown in Table 5, the Bitcoin price return is the most volatile, and investors may choose gold as an alternative to avoid the high volatility of Bitcoin. Historically, gold has been known as the best safe haven instrument, and it is also regarded as an excellent inflation hedge (Worthington & Pahlavani, 2007). The US stock index has the least correlation with gold. Therefore, choosing gold as the primary tool of diversification is appropriate for investors in the US stock index. Currently, the US is the third largest importer of gold globally (Workman, 2021), suggesting that gold is widely used in the US for its safe haven property. For an investor in the China stock index, crude oil is ranked as the second least correlated after Bitcoin, and as a result, investors in the China stock index may select both instruments to diversify their portfolio. The findings are consistent with previous research indicating that Bitcoin is a good asset diversifier (Corbet et al., 2018; Guesmi et al., 2018). However, several other researchers have questioned Bitcoin's diversification advantages (e.g., Chowdhury, 2016) and expressed concerns about Bitcoin's potential as an alternative investment instrument. As a result, for investors in the China stock index, diversifying their holdings with gold and Bitcoin is prudent.

Second and more relevant, crude oil price return, China stock index return and CNY/USD exchange rate change have the lowest correlation with Bitcoin, as shown in Table 6. Therefore, any investor exposed to crude oil, the China stock index, and CNY/USD and pursuing full diversification benefits should select Bitcoin as a diversification instrument.

Thus far, the assumptions on the volatilities and correlations analyses are made on an unconditional basis. Unconditional basis refers to historical volatility and correlations with no weightage in estimation. However, the assumption of constant correlation and variance across the sample does not meet economic intuition. Volatility and correlation are more likely to be dynamic. For this scenario, the dynamic differentiation coefficient model was used.

Figures 5 and 6 display the temporal dimension of volatility and conditional volatilities for the six variables. During the eight-year observation, we found that Bitcoin's price return has the highest volatility relative to others. The CNY/USD exchange rate return shows the lowest volatility during the period. The chart indicates that Bitcoin investment is very risky due to its unpredictability in movements. Furthermore, the CNY/USD exchange rate exhibits better stability compared to the rest, followed by the US stock index returns, as shown in Figure 6. The results are consistent with our prior findings, as shown in Table 5.

Figure 5

Conditional Volatilities of China and US Stock Indices Return and Other Variables

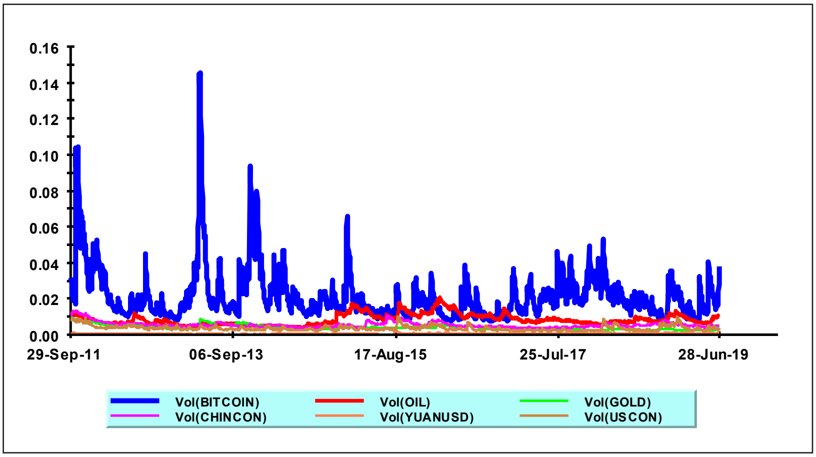


Figure 6

Conditional Volatilities of China and US Stock Indices Return and Other Variables (excluding Bitcoin)

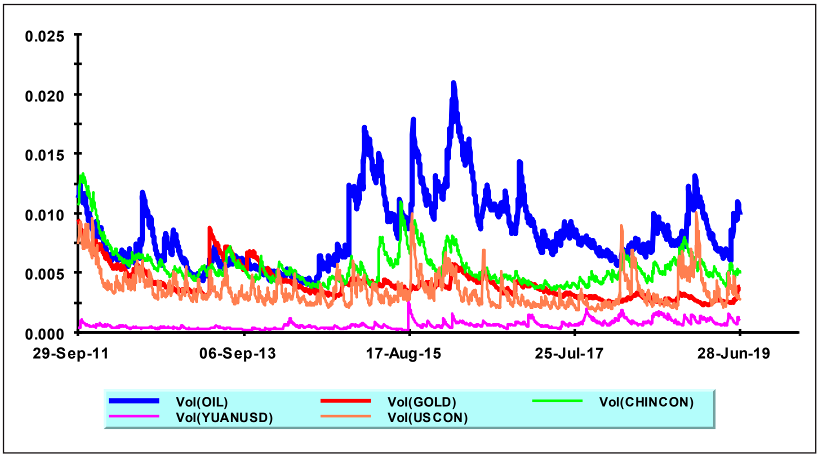


Figure 7

Conditional Correlation of Bitcoin Price Return with China and US Indices Return

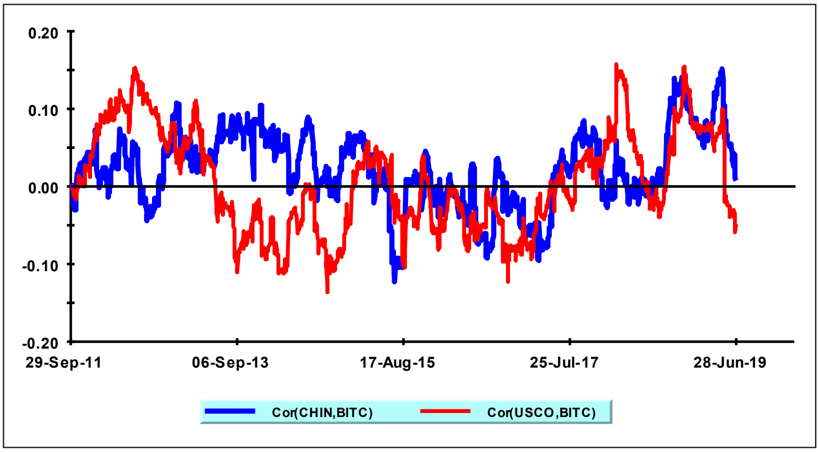
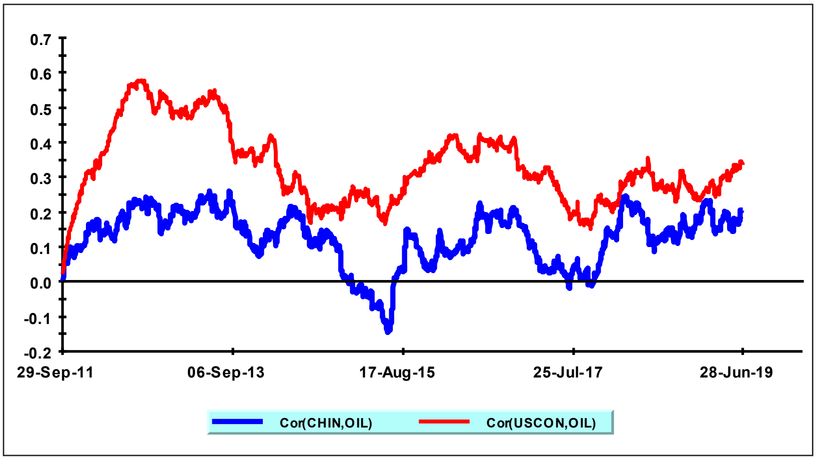


Figure 7 depicts the conditional correlations used to compare the correlation between Bitcoin price returns and China and US conventional stock indices returns. According to the graph, from 2011 to 2016, the correlation of Bitcoin with US and China stock indices returns was in a declining trend. However, the trend from 2017 to 2019 was upward, suggesting that opportunities for diversification benefits will be fewer in the future. Throughout the eight years under consideration, the correlation of Bitcoin with the US stock index return seems to be somewhat greater than the correlation of Bitcoin with the China stock index return. This finding implies that the US stock index and Bitcoin are riskier as portfolio diversifiers than Bitcoin and the China stock index. As a result, an investor exposed to Bitcoin may diversify their investment by holding a position in the US stock index rather than the China stock index. The outcome is consistent with our prior results in Table 6.

Figure 8

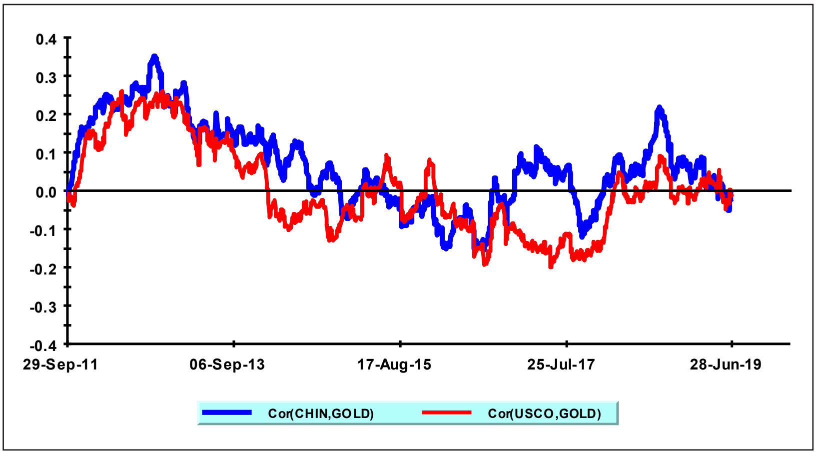
Conditional Correlation of Crude Oil Price Return with China and US Indices Return



Based on Figure 8, the correlation of crude oil price return with the US stock index return is consistently higher throughout the eight years under study when compared with the China stock index. As a result, the China stock index is better for a diversified investment portfolio with crude oil. The strong connection between the US stock index and crude oil prices is mostly attributed to the petrodollar effect, in which crude oil transactions use the US dollar as the main currency. Therefore, fluctuations in crude oil prices will directly affect the US currency and the US economy. Referring to Table 6, crude oil has the highest correlation with the US stock index. We also note that the correlation of crude oil with both indexes is on a downward trend from 2012 to 2015. However, beginning in 2015, the correlations indicate a modest upward trend until 2019. In the future, the rising trend suggests a reduced potential for the diversification advantage between the stock indices and crude oil.

Figure 9

Conditional Correlation of Gold Price Return with China and US Indices Return



As shown in Figure 9, the gold price return is strongly correlated with the China stock index compared with the US stock index during the eight years under consideration. An investor with a gold portfolio would be better diversifying the US stock index than the China stock index. The result also accords with our earlier results in Table 6 that used MGARCH-DCC. Our findings are consistent with earlier research by Hood and Malik (2013), who found that gold serves as a hedge (negatively correlated with equities) and a weak safe haven (negatively correlated with stocks in extreme stock market declines). Figure 9 also shows that the correlation of gold with both indices returns is decreasing from 2011 to 2017. However, the trend is increasing from 2017 to 2019. This finding implies a reduction in the potential for diversification advantage between the gold and the stock indices.

Empirical Findings of CWT

Figures 10 to 15 provide in-depth information on the estimated CWT and phase difference from scale 1 (one day) up to the scale of 8 (approximately two years, 512 trading days). The horizontal axis represents trading days, while the vertical axis denotes the investment holding interval. Monte Carlo simulation was adopted to produce a 5 percent significance curved line below. The figure displays different colour codes with intensity levels from low correlation (blue) to high correlation (red).

Any investor who wants to diversify their portfolio by investing in the China stock index and Bitcoin should know the connection between the two assets. According to Figure 10, any investor interested in investing in the China stock index while also having exposure to Bitcoin may retain their investment because no short-term or long-term correlation exists between Bitcoin and the China stock index. Figure 11 shows that the connection between Bitcoin and the US stock index is also low, with some moderate correlations in the investment horizon of 128 days to 256 days. Therefore, investors who have exposure to the US stock index may diversify their holdings by investing in Bitcoin (especially in the short-term below 128 days).

Investors who want to diversify their portfolios using crude oil and the China stock index should hold the investment for no more than one year (between 1 day to 256 days). If the investment lasts longer than 12 months or more than 256 days, the portfolio will experience a significant correlation (Figure 12). Figure 12 further shows that the correlation of crude oil with the China stock index is lower than the correlation with the US stock index (further details are shown in Figure 13). The US economy is more vulnerable to crude oil price changes than China’s because the US currency is used for crude oil transactions and the US economy depends significantly on oil for energy. This outcome is consistent with our results in Table 6, which shows that the strongest correlation is between the US stock index and crude oil. Figure 13 further shows that the diversification benefits between the US index and the crude oil price can be obtained only within six months (approximately 64 days). The figure shows a significant correlation after 64 days, indicating that diversification benefits decrease for investments more than 64 days.

China’s stock index is vulnerable to gold prices in the same way that it is vulnerable to crude oil prices. Figure 14 shows that investors would gain diversification benefits on a smaller scale below 256 days due to the weak correlation between the stock index and gold prices. The correlation between the variables is high at certain times between the investment horizons of day 1 and day 256, although it is low for most parts throughout that period. When the correlation between the two variables exceeds 256 days, the correlation between the two subjects is strong, limiting the portfolio’s diversification potential. However, the correlation between the China stock index and gold is slightly higher than the correlation between the China stock index and crude oil, consistent with our findings in Table 6.

Figure 15 shows that the correlation between the price of gold and the US stock index is lower than the correlation between the price of gold and the Chinese stock index. The correlation is very weak for the entire holding period, except for the 64-day holding period, where a strong correlation is centred between 2015 and 2017. The result is consistent with our findings that used MGARCH-DCC, where we found that the correlation of the US stock index with gold is the lowest. Therefore, investors may utilise gold as a diversification instrument if they have exposure in the US stock index.

Table 7

Date for Horizontal Axis

Horizontal Axis	Date
200	June 2012
400	March 2013
600	December 2013
800	September 2014
1000	July 2015
1200	April 2016
1400	January 2017
1600	October 2017
1800	July 2018
2000	May 2019

Figure 10

CWT – Bitcoin PR vs. China SIR

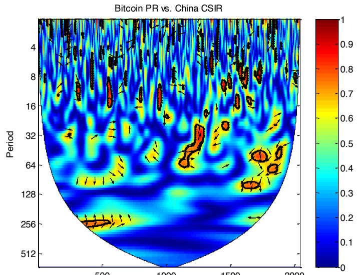


Figure 11

CWT – Bitcoin PR vs. US SIR

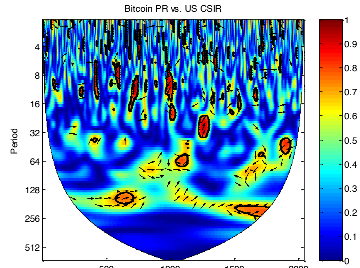


Figure 12

CWT – Crude Oil PR vs. China SIR

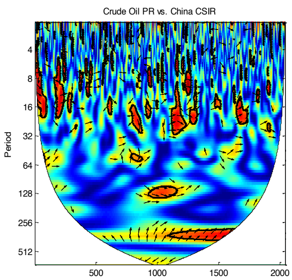


Figure 13

***CWT** – Crude Oil PR vs. US SIR*

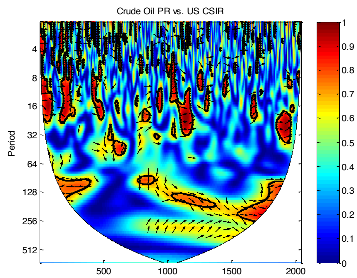


Figure 14

CWT – Gold PR vs. China SIR

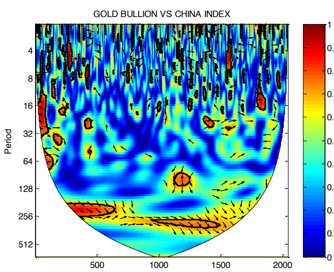
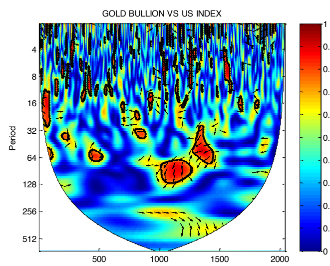


Figure 15

CWT – Gold PR vs. US SIR



CONCLUDING REMARKS

Theoretical Contributions

The essence of MPT by Markowitz is the establishment of an efficient portfolio which should be assessed based on the overall portfolio's risk and return properties. MPT suggests the diversification of constituent assets to maximise overall portfolio return with the possible lowest level of combined risks. Another important contribution of MPT is the reduction of the overall portfolio's volatility with the selection of assets having a negative correlation. To construct an efficient portfolio, the estimation heavily relies on the statistical measures of variance and correlation. The most significant criticism against MPT is the evaluation of variance rather than downside risk (fat-tailed properties). In other words, MPT indicates that two portfolios having the same level of variance and returns are equally important, a scenario that is not always true in reality. The prior assessments of variance and correlation under MPT have been largely concentrated on the use of models with strict a priori assumptions on normality and symmetrical tail distribution properties. This study contributes to the existing evaluation of MPT by adopting wavelet transformation analysis which does not require a priori distributional properties and suits well with our fat-tailed index return data. Leveraging the enhanced practicality of this method, the wavelet technique provides more meaningful inputs for investors having heterogeneity on investment holding periods and potential diversification benefits from the co-movement of China and US stock indices in response to the movements of Bitcoin, crude oil, and bilateral foreign exchange rate CNY/USD.

Practical Contributions

In this article, we assessed the possibility of diversification advantages among the variables we selected. Many contemporary statistical methods were used to address the study goals, including VECM, MGARCH-DCC, MODWT, and CWT. The VECM results indicate that Bitcoin, crude oil, and the CNY/USD exchange rate are exogenous, whereas China and US stock indexes and gold are endogenous. Thus, the China and US stock indexes and gold will respond to Bitcoin, crude oil prices, and the CNY/USD exchange rate.

Based on MODWT's findings, we can deduce that Bitcoin leads crude oil at nearly all levels, implying that crude oil prices will react to Bitcoin price movement. Given that the two variables have a low correlation compared with the other variables, a diversification opportunity arises. MODWT results also show that at higher levels (levels 6 and 7), Bitcoin leads CNY/USD, but at the highest level (level 8), CNY/USD leads Bitcoin, indicating room for diversification between these two variables, particularly in the long run. The result from MODWT also indicates that at higher levels (levels 6 and 7), Bitcoin leads CNY/USD, but at the highest level (level 8), CNY/USD leads Bitcoin, suggesting that diversification possibilities arise between these two variables, especially in the long term. We also found that CNY/USD leads crude oil at higher levels, indicating that these two variables provide long-term diversification advantages. In comparison with the other variables in our sample, the two variables show a weak correlation. As a result, low correlation increases the potential for portfolio diversification advantages.

According to the MGARCH-DCC results, the CNY/USD exchange rate return has the lowest volatility, suggesting China is a stable economy with a stable currency in the Asia-Pacific region. The CNY/USD pair also has the least correlation with the Bitcoin price return. Therefore, investors interested in Bitcoin should invest in CNY/USD to profit from a diversified portfolio. Given that the US stock index return has the most negligible correlation with gold, we can infer that gold is still the best option for portfolio diversification for the US stock index return. The China stock index has the lowest correlation with Bitcoin and the second-lowest correlation with crude oil. Consequently, if they have exposure to China's stock index, investors may utilise Bitcoin and crude oil as an investment diversification strategy.

Finally, the results from CWT indicate that diversification gains can be realised between gold and US stock indexes over a short and long period, as the correlation between these variables is very low. This finding is in line with the outcome that utilised MGARCH-DCC, where the correlation between the US stock index and gold is the lowest. Gold is a stable investment instrument, making it a perfect diversification portfolio for the US stock index. An investor in the China stock index may choose Bitcoin and crude oil to diversify their investment, but the holding period should not exceed 256 days if they want to gain diversification benefits.

The policy consequence of this study is that investors should consider the results of this study when deciding whether to invest in commodities or stock indexes in the United States and China. Risk-averse investors who wish to minimise systematic risk in their portfolio may diversify by investing in gold and the US stock index, both of which have extremely low volatility and correlation. A risk-seeking investor should invest in the China stock index and Bitcoin, which are highly volatile and have the lowest correlation. Investors who wish to protect themselves against stock market volatility must invest a percentage of their entire investment in gold and Bitcoin. Such investment is made to safeguard the investor from the stock market's uncertainties because the stock market, the China stock market, is highly volatile. This study's results should help develop portfolio strategies for investors and others engaged in commodities and stock markets. From this analysis, we can appreciate the modern techniques' contributions to consider the potential for diversification of investments for investors with varied investment goals over different time lengths.

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