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Effects of Lightning Electromagnetic Field due to Lightning Strikes on a Tall Structure

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***Abstract*— It is a fact that tall towers have a high chance of being struck by lightning, resulting in an induced voltage on a piece of a conductor through the coupling effect of the electromagnetic field. As a result, it disrupts the power supply on the distribution power line. This paper proposes new expressions of the lightning electromagnetic field (LEMF) and further investigates their effects due to lightning strikes on a tall structure. The new expression is calculated using trapezoidal and finite-difference time-domain (FDTD) methods and programmed through MATLAB. The expression was verified by literature measurement work and obtained with a reasonably accurate match. Also, the effects of LEMF due to the lightning current front time and the ground impedance were investigated. The result indicated that at least 50% of the LEMF peaks experience a reduction as the current front time increases and declines as the ground impedance increases. Hence, these results significantly impact the assessment of lightning-induced voltages in the distribution power line. It may help determine an appropriate solution for lightning protection, including selecting a line arrester rating, improving the basic insulation level (BIL) and the ground system.**

***Index Terms*—Electromagnetic field, ground impedance, lightning, tall tower**

I. Introduction

The electromagnetic field is an essential component of lightning that contributes to large disturbances in power lines once they couple to conductor lines, resulting in lightning-induced overvoltages (LIOV). According to [1-3], tall towers are likely to be struck by lightning and are estimated to have higher LEMF values than indirect strikes. Therefore, investigating the LEMF effect in the presence of tall towers at close distances becomes essential as it affects the performance of the associated power lines.

LEMF can be calculated using different methods, such as a monopole, dipole, finite-difference time-domain (FDTD), and method of moments (MoM) [4-7]. Each method represents a particular knowledge-based calculation difference, leading to several advantages and disadvantages. In addition, the evaluating LEMF is based on numerical integration and the propagation of lightning current along the channel and tower. This paper proposes new expressions by considering the straightforward numerical integration, which gives accurate results. The validation of the proposed expression is demonstrated by comparing it with the literature measurement work.

Moreover, the proposed expressions are expanded to investigate LEMF effects due to the lightning current front time and ground impedances. The tall tower's ground impedances are the reflection factor parameters that can influence lightning current propagation and LEMF [8,9]. In reality, the ground impedance value depends on various types of grounding arrangements of the tower itself. The relationship between these parameters can cause changes in the ground impedance value, leading to variations in tall towers' ground-reflection factor (GRF) value due to the correlation between the ground impedance and the tower.

Hence, this paper presents a new LEMF expression and the effects of LEMF due to lightning strikes on tall structures. This study may interact with the interest of electrical engineers when considering lightning protection schemes for their systems. It should be emphasized that the LEMF is related to the LIOV resulting from the coupling event. Therefore, any effect on LEMF will, in turn, affect the LIOV.

Some basic assumptions are considered in this paper: 1) the lightning channel is a vertical channel without any branches, and the subsequent return stroke is considered; 2) the tower is lossless and a perfect conductor; and 3) the ground is flat and has perfect soil conductivity.

II. Literature Review

The LEMF can be calculated by considering the lightning current input, which is the channel-base current and the return stroke current [10, 11]. These two currents can be expressed in mathematical expressions in subsections A and B. Subsection C reviewed the expression of the LEMF method.

A. Evaluation of Channel-Base Current

Each lightning channel-base current expression presents some advantages and disadvantages [12,13]. The Bruce-Golde (BG's) function provides fewer parameters that need to be considered. However, the main problem of the BG's function is the generation of discontinuity values for the first of current derivatives with respect to the time derivative, $\frac{dy}{dx}$ at time equal

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to zero. The drawbacks of BG's function are overcome by the other channel-based current function development. The Heidler, Pulse, Diendorfer and Uman (DU), and Nucci's current function represent the continuity value for the time equal to zero. The current derivative, with respect to the time derivative, will affect the calculation of LEMF and LIOV. Moreover, even though the newly devised current functions have been proposed, several additional parameters have been contributed that may be too complex to be considered in the calculation of LEMF and LIV. Equation (1) shows the DU expression.

$$i_{subs}(z',t) = \left[\frac{i_{01}}{\eta_1}\frac{\left(\frac{t}{\Gamma_{11}}\right)^{n_1}}{1+\left(\frac{t}{\Gamma_{11}}\right)^{n_1}} exp\left(\frac{-t}{\Gamma_{12}}\right) + \frac{i_{02}}{\eta_2}\frac{\left(\frac{t}{\Gamma_{21}}\right)^{n_2}}{1+\left(\frac{t}{\Gamma_{21}}\right)^{n_2}} exp\left(\frac{-t}{\Gamma_{22}}\right)\right] \quad (1)$$

Where i_{01} and i_{02} are the amplitude of the channel base current, Γ_{11} and Γ_{12} are the front time constants, Γ_{21} and Γ_{22} are the decay-time constants, n_1 and n_2 are exponents in the range of 2~10 and η_1, η_2 are the amplitude correction factors.

B. Evaluation of Lightning Return Stroke

Once lightning has struck the top of a tower, the lightning current begins to propagate upward through the lightning channel at return stroke velocity speed and downward through the tower path at the speed of light as shown in Fig.1. The channel-base current is presented in the mathematical expression in Subsection A. Both of these currents experience a reflection event. However, the first lightning current striking the top of the tower is assumed to be undisturbed and not influenced by any reflection event.

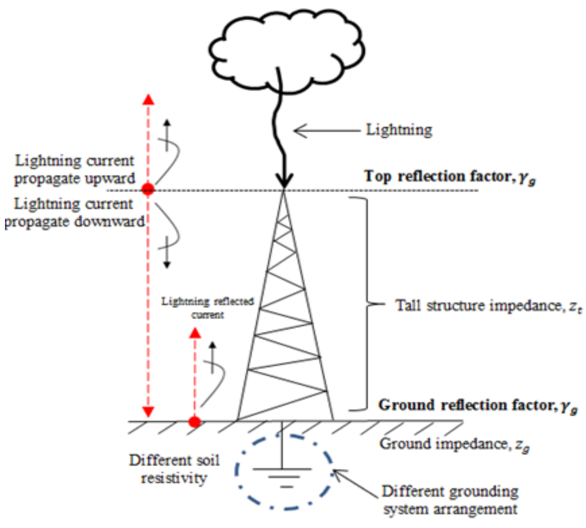


Fig. 1. Lightning return stroke current illustration

Moreover, this current begins to flow into the channel and tower path and experiences a reflection event. The reflection event refers to the movement of current along the tower, which experiences a reflection at the bottom of the tower (due to the difference in impedance between the tower and the ground), travels up along the tower, and again is faced with reflection at the top of the tower (due to the difference in impedance between the tower and the return stroke channel). Hence, some of this current will enter the channel path. The same event occurs in the current along the channel. This current experiences a reflection at the top of the channel travels down along the channel and again experiences a reflection. This process is known as the lightning return stroke along the channel and tower.

As reviewed and discussed by [13-15], the lightning return stroke current can be evaluated along the channel and tower by considering the effect of multiple reflections and the reflection coefficient of the tower. A few mathematical models can express this situation. The models are identical to each other apart from the differences in defining the term for the injected lightning current. Equations (2) to (3) describe the lightning return stroke current along the channel and tower, respectively.

$$i(z',t) = \left[\begin{array}{c} P(z'-h)i_o\left(h, t-\frac{z'-h}{v}\right) - \gamma_t i_o\left(h, t-\frac{z'-h}{c}\right) \\ +(1-\gamma_t)(1+\gamma_t)\sum_{n=0}^{\infty}\gamma_g^{n+1}\gamma_t^{n} i_o\left(h, t-\frac{z'+h}{c}-\frac{2nh}{c}\right) \end{array}\right] \quad (2)$$

The distribution current along the channel $h < z' < H$

$$i(z',t) = (1-\gamma_t)\sum_{n=0}^{\infty}\left[\gamma_t^n\gamma_g^n i_o\left(h, t-\frac{h-z'}{c}-\frac{2nh}{c}\right) + \gamma_t^n\gamma_g^{n+1} i_o\left(h, t-\frac{h+z'}{c}-\frac{2nh}{c}\right)\right] \quad (3)$$

The distribution current along the tower $0 \le z' \le h$

Where γ_t and γ_g are the top and ground current reflection coefficients, respectively, n is the number of reflection currents inside the tower, h is the tower's height, i_o is a current function, c is the speed of light for the waves that propagate in the tower, v is the speed of the upward lightning channel for the waves that propagate in the lightning channel, $P(z'-h)$ is the model-dependent attenuation function and $u(t)$ is the Heaviside unit-step function. It should be noted that $P(z'-h)$ is a model-dependent attenuation function known as the return stroke current model as presented in Table I.

TABLE I
MODEL-DEPENDENT ATTENUATION FACTOR BASED ON THE RETURN STROKE CURRENT MODEL [11, 16]

Return stroke current model	v	P(z′)
Bruce-Golde, BG	∞	1
Traveling Current Source, TCS	$-c$	1
Transmission Line, TL	v	1
Modified Transmission Line with Exponential Decay, MTLE	v	EXP $\left(-\frac{z}{\lambda}\right)$
Modified Transmission Line with Linear Exponential Decay, MTLL	v	$1 - z'/H_{TOT}$

This model affects the lightning return stroke current along a channel as described in (2). This paper employs the Transmission Line (TL) model to describe the return stroke current along the lightning channel. The TL model is used because it can estimate the initial field peak for the initial period and provides the simplest mathematical model [16].

Furthermore, as presented in (2) and (3), both of the lightning return stroke currents are dependent on the input of the injected lightning current (channel-base current) and the reflection coefficient factor (RCF) parameters. Hence, both currents can be observed based on the variation in the value of the current front time for the lightning current injection (channel-base current), which can be described as a "speedy," "fast," or "slow," current front time. Also, the effect of the variation in the value of the ground impedance, particularly for the GRF for the RCF on the LEMF, is obtainable. It should be noted that the ground impedances are accounted by the circular grounding arrangement as presented in (4) with different levels of soil resistivity as presented in Table II [17]. In this paper, the soil resistivity values were selected as 30 Ω.m, 80 Ω.m, 130 Ω.m, 2 kΩ.m, 10 kΩ.m, and 20 kΩ.m.

$$z_g = \frac{1}{n}\frac{\rho}{2\pi L}\left[\left(\ln\frac{4L}{a}\right) - 1 + \frac{L}{S}\left(\ln\frac{2n}{\pi}\right)\right] \quad (4)$$

Where ρ is the soil resistivity, n is the number of rods, L is the conductor length, a is the conductor radius and S is the space between two rods.

TABLE II
THE SOIL RESISTIVITY VALUE RANGES [17]

Type of soil	Soil resistivity range, Ω.m
Lake and river water	100 to 400
Commercially distilled water	1000 to 4000
Clay	25 to 100
Sandy clay	40 to 300
Peat, marshy soil	50 to 300
Limestone	100 to 5000
Sand, granites	100 to 10,000
Moraine	1000 to 10,000
Calcareous remains	3000 to 30,000

C. *Evaluation of Lightning Electromagnetic Field*

Once a tower is struck by lightning, the LEMF responses are created. In mathematical modeling, the LEMF response can be determined by calculating the lightning current distribution along the channel and tower, $i(z', t)$ at an observation point, z_o, a few meters separated from the tall tower, r and h height of tall tower, as presented in Fig. 2. Several methods can be employed to calculate the LEMF, such as monopole, dipole, FDTD, and Hybrid [18,19]. The monopole method is a method that considers the current and charge densities as a function of time and space. This method is primarily used and can be applied to simple lightning models. In addition, dipole is a method that considers the lightning current as a function of time and position. Hence, this model is suitable for straightforward applications using the return stroke current. Also, this model represents different terms of the LEMF concerning various range dependencies, which allows the researcher to simplify the work based on the specific range. Table III summarizes the calculation knowledge method, advantages, and disadvantages.

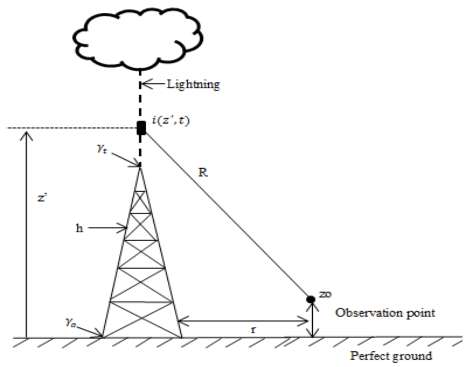


Fig. 2. Geometry for the calculation of LEMF concerning a lightning strike on a tall tower

TABLE III
SUMMARY OF THE LEMF METHOD [18, 19]

LEMF method	Based on knowledge of	Advantages and Disadvantages
Monopole	Charge and current densities	- Simpler solution - It only supports perfect ground conductivity cases - Does not support complex lightning models
Dipole	Current distribution	- It can support complex applications, such as the upward return stroke current - Provides straightforward formulation that is based on the current distribution - Able to differentiate range dependencies of the LEMF components - Allows specification of specific ranges that require the LEMF to calculate - The integral limit is assumed to be the same during the time period
FDTD method (finite-difference time domain)	Maxwell's equations	- Field calculation in the time domain - It can support non-perfect ground conductivity - Applicable to short distances only - Estimation of the fields in the division of the plane - Requires a high memory capacity and longer processing time
Method of moment (MoM)	Maxwell's equations	- Can support non-perfect ground conductivity - Field calculation in the frequency domain

III. METHODOLOGY

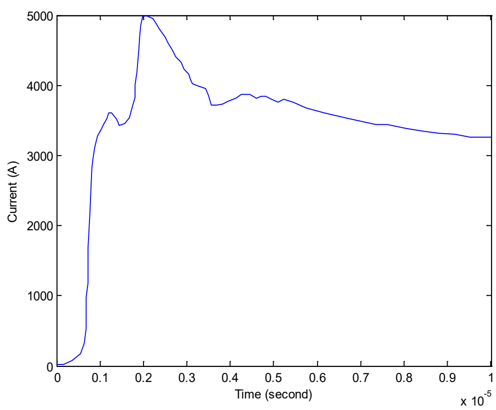
A detailed explanation regarding the method of this work is presented in this section. It comprises an evaluation of the lightning return stroke current due to the channel and the tower as well as proposing a new expression of the LEMF.

A. *Channel-Base Current*

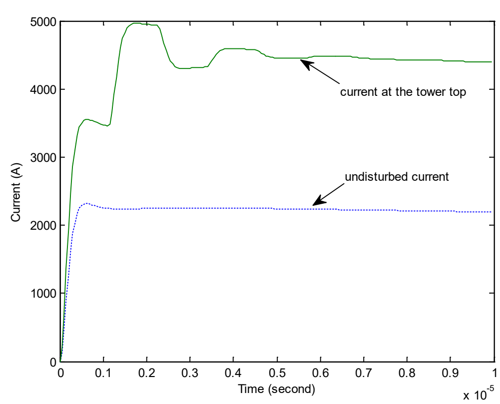
The work begins with the evaluation of the channel-base current. In the presence of a tall tower, the channel-base current is seen as an undistributed current, $i_o(t)$, which is assumed to be an ideal current, with both of the reflection coefficients for the tower equal to zero and the return stroke wavefront reflections being disregarded. Hence, the channel-base current can be seen as an injected lightning current, $i_o(t)$, at the top of the tower. In this work, the DU channel-base current is selected to model the injected current at the top of the tall tower due to the ability of this current function to present a similar waveshape of the measured current. The expression of DU function is shown in Subsection A, (1).

B. Return Stroke Current and Verification

As reviewed in Subsection B (in Literature Review), an engineering model based on a distributed source representation expressed by (2) and (3) was adopted in this work. This model provides a straightforward formulation in which the specific channel-base current function, current distribution along the channel and the tower, and return stroke current model are considered. In addition, the return stroke current model of the MTLE model was selected to describe the return stroke current along the lightning channel, which represents a decreasing current in the exponential factor at an increasing channel height. The mathematical expressions of the channel-base current and return stroke current were programmed and validated with the measured data found in the literature [20]. Fig. 3 presents the validation work whereby a good agreement was observed between the measured and simulated waveshape.



a) Measured current at the top of Peissenberg tower



b) Simulated current

Fig. 3. Return stroke current

C. A New LEMF Expression and Verification

In addition, the work continues by proposing a new LEMF expression using the dipole and FDTD methods known as a Hybrid solution. An equation of LEMF of the dipole method as presented in Eqs. (5) to (7) was used as basic of LEMF equations.

$$B_{\emptyset}(r,z,t) = \frac{\mu_o}{4\pi}\left[\int_{-H}^{H}\frac{r}{R^3}\, i\left(z',\tau-\frac{R}{c}\right)dz' + \int_{-H}^{H}\frac{r}{cR^2}\frac{\partial i\left(z',t-\frac{R}{c}\right)}{\partial t}dz'\right] \tag{5}$$

$$dE_z(r,z,t) = \frac{1}{4\pi\varepsilon_o}\left[\int_{-H}^{H}\frac{2(z-z')^2-r^2}{R^5}\int_0^t i\left(z',\tau-\frac{R}{c}\right)d\tau dz' + \int_{-H}^{H}\frac{2(z-z')^2-r^2}{cR^4}\, i\left(z',t-\frac{R}{c}\right)dz' - \int_{-H}^{H}\frac{r^2}{c^2R^3}\frac{\partial i\left(z',t-\frac{R}{c}\right)}{\partial t}dz'\right] \tag{6}$$

$$dE_r(r,z,t) = \frac{1}{4\pi\varepsilon_o}\left[\int_{-H}^{H}\frac{3(z-z')^2}{R^5}\int_0^t i\left(z',\tau-\frac{R}{c}\right)d\tau dz' + \int_{-H}^{H}\frac{3r(z-z')}{cR^4}\, i\left(z',t-\frac{R}{c}\right)dz' - \int_{-H}^{H}\frac{r(z-z')}{c^2R^3}\frac{\partial i\left(z',t-\frac{R}{c}\right)}{\partial t}dz'\right] \tag{7}$$

Where r is denotes the radial distance, the observation height, is z', the height of the return stroke path is R, the distance between the height at the return stroke path and the observation point is $R = \sqrt{r^2 + (z'-z)^2}$, c is the speed of light, ε_o is the permittivity of free space, H and –H are the integral limit on the real and image, respectively.

Moreover, for solving the non-integral function, $i(z',t)$ as presented in (2) and (3), numerical integration is applied. In this work, the trapezoidal numerical integration solution was used. This solution provides a straightforward formulation and gives a good result compared with the measured value. The derivative electric fields were calculated as expressed in (8) to (9).

$$dE_z(r,z,t_n) = \sum_{m=0}^{k}\left\{a_{cr,m}F\left(x,y,z,t=t_n,z'=H_{cr,m}\right) - a'_{ci,m}F\left(x,y,z,t=t_n,z''=H_{ci,m}\right)\right\} \tag{8}$$

$$dE_r(r,z,t_n) = \sum_{m=0}^{k}\left\{a_{cr,m}F\left(x,y,z,t=t_n,z'=H_{cr,m}\right) - a'_{ci,m}F\left(x,y,z,t=t_n,z''=H_{ci,m}\right)\right\} \tag{9}$$

where $a'_{ci,m}$ and $a'_{cr,m}$ are the coefficients of trapezoid respecting the channel for real and image at the number of trapezoid segments, $H_{cr,m}$ and $H'_{ci,m}$ is an integral limits respecting to channel for real and image at different height of the channel path as well at the number of trapezoid segment, F, the elemental current, $i(z',t)$ along the channel and t_n is the time, $t_n = (n-1)\Delta t\ n = 1,2, \ldots n_{max}$

The FDTD method was employed in (8) and (9) to solve the derivative of the electric fields, $\frac{\partial E}{\partial t}$ whereby layer by layer, the electric fields were calculated. In addition, the vertical electric field is a significant field that should be determined when considering the coupling evaluation. Fig. 4 shows the validation result of the vertical electric field, 185 m from the tower. Moreover, the wave shape between the simulated and measured values is in good agreement, and the percentage differences are less than 10 %.

Overall, the vertical electric fields exhibited good agreement with the measured values. The percentage difference between these values for first of 2 μs is in the acceptable range at less than 10%, and the wave shape of the result is seen to be practically the same as the measured values.

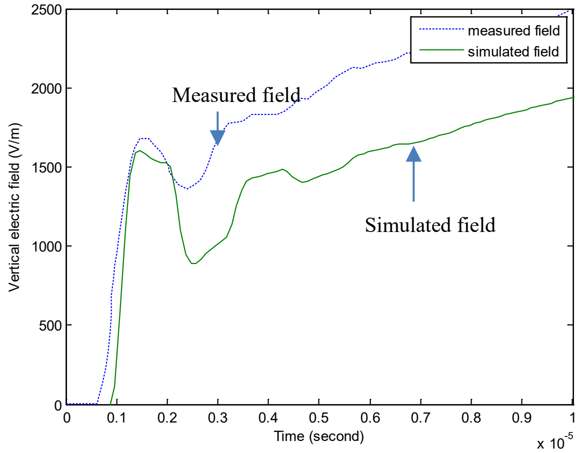


Fig. 4. Comparison between the measured and simulated vertical electric field at 185-m distance from Peissenberg tower

D. Lightning Current Front Time and Ground Impedances

The parameter values of (1) were varied to generate a different lightning current front time, as expressed in Fig. 5. The front time was classified as "very fast," "fast," or "slow" at 0.15 µs, 0.70 µs, and 1.20 µs, respectively. Moreover, as shown in (4), the ground impedance was varied based on a circle grounding arrangement and soil resistivity as tabulated in Table II, and the tower impedance was assumed to be 180 Ω. The ground impedance was defined as having a low or high value, as shown in Table IV.

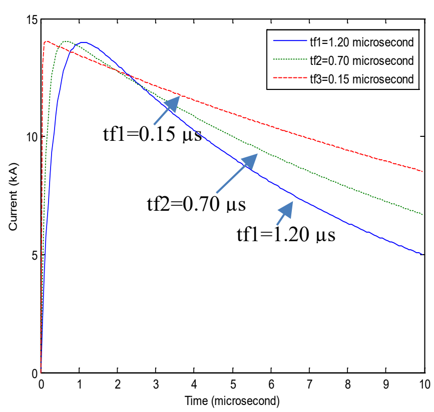


Fig. 5. Lightning current for different values of the front time

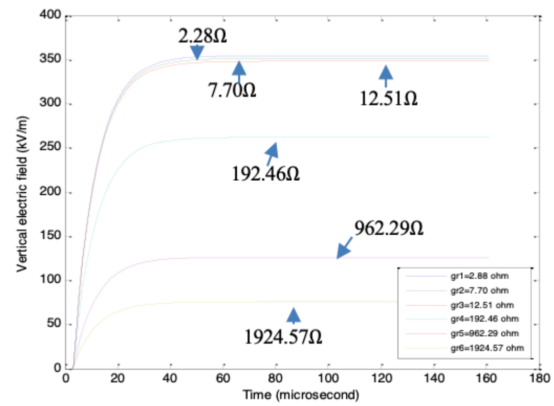
TABLE IV
VARIATION OF GROUND IMPEDANCES

Tower impedance (Ω)	Ground impedance (Ω)	Ground impedance description
180	2.88	Low value
	7.70	
	12.51	
	192.46	High value
	962.29	
	1924.57	

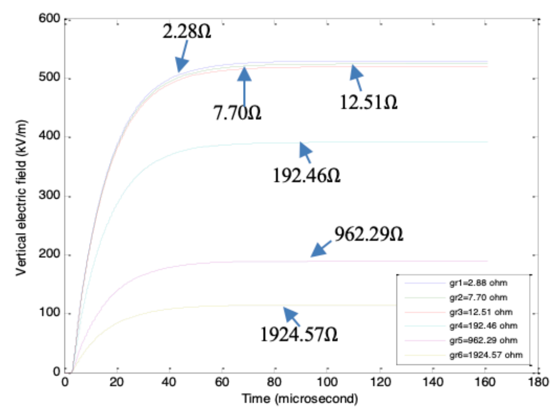
IV. RESULT AND DISCUSSION

Effects of vertical electric field due to lightning current front time and ground impedances

The wave shape of vertical electric field as shown in Fig. 6 was observed. Result indicates that a non-fluctuation in the wave shape for the initial time was observed for an increasing current front time and ground impedance. This was due to the simulated field at a very close distance in which the electrostatic effect dominated instead of induction or radiation. However, flattening of the fields occurred during the tail time.



(a) t_f=0.15 µs



(b) t_f=0.70 µs

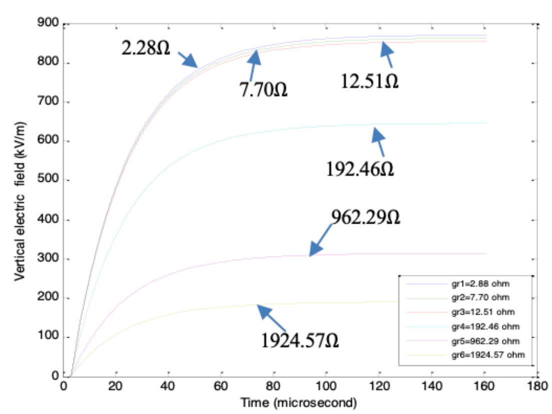


Fig. 6. Vertical electric field for different lightning current front times and GRF at a 20-m distance from the struck tower; a) front time t_f=0.15 µs, (b)t_f=0.70 µs, and (c)t_f=1.2 µs

In addition, the field peak demonstrated a reducing trend as the current front time and ground impedance increased, as presented in Table V. By referring to the value at a current front time of 0.15 µs and a ground impedance of 2.88 Ω, the vertical electric field was reduced from 809.70 kV/m to 338.90 kV/m, which is equivalent to at least 0 % to 58.16 % as the current front times increase. However, the percentage reduction of the peaks increased as the ground impedances varied. The field peaks were reduced from 0.89% to 58.91% for the case of low ground impedance, equivalent to 802.70 kV/m to 332.80 kV/m. For the high impedances, the percentage reduction in the field peaks became more apparent at 25.9% to 91.50%, equivalent to 600.10 kV/m to 68.88 kV/m.

TABLE V
VERTICAL ELECTRIC FIELD PEAK VALUES

Front time t_f (μs)	Vertical electric field peak (kV/m) Ground impedance (Ω)					
	2.88	**7.70**	**12.51**	**192.46**	**962.29**	**1924.57**
0.15	809.70	802.70	795.70	600.10	291.40	176.80
0.70	498.00	493.50	489.20	368.70	177.90	107.30
1.20	338.90	335.90	332.80	250.70	115.00	68.88

Overall, the results indicated that the lightning current front time and ground impedance affect the wave shape of the LEMF in the case of striking a tall tower. The wave shapes of the vertical electric field was hardly affected. The wave shape fluctuated in "speedy" and "fast" front time cases, and the wave shape was slightly smooth for the "slow" front time cases. Moreover, different ground impedances tended to generate various wave shapes. Multiple field peaks were observed for low values of ground impedance and slightly reduced for high values of ground impedance. However, in all cases, a very smooth wave shape could be observed for the 192 Ω ground impedance. Furthermore, the lightning current front time and ground impedance strongly affected the field peak for all the LEMFs. The field peak reduced as the front time and ground impedances increased, suggesting the following:

1) At least in the 0% to 58% range, all the peaks showed a reduction for an increasing lightning current front time and the lowest ground impedance.
2) At least in the 30% to 91 % range, all the peaks showed a reduction for the highest ground impedance.

Hence, this study can be helpful for electrical engineers to decide on the appropriate protection for evaluating the LIV when the LEMF has interacted with the conductor line in which the factor of the lighting current front time and the variation in the value of the ground impedances need to be considered.

V. CONCLUSION

In this paper, the effects of LEMF due to the lightning strike to a tall structure have been investigated, and the results are discussed accordingly. The results indicate that the lightning current front time and ground impedance significantly affect the wave shape and value of the LEMF. The wave shape and value fluctuation slightly reduce as the lightning current front time increases. Also, the wave shape of the LEMF delivers a smooth wave shape and shows a reduced peak for an increase in ground impedance. Therefore, the influence of the lightning current front time and ground impedance on the LEMF should be considered when predicting the LEMF, which may be affected when evaluating the LOIV and is known as a major issue for power distribution lines.

REFERENCES

[1] T. Shindo, "Lightning striking characteristics to tall structures," *IEEJ Trans. On Electrical and Electronic Eng*., vol. 13, no.2, pp. 938–947, Nov, 2017.

[2] L. Cai *et al*., "Differences of electric field parameters for lightning strikes on tall towers and nonelevated objects," *IEEE Trans. on Electromagnetic Compatibility*, 2022.

[3] V. Cooray, Lightning electromagnetics, IET Pub; 2015.

[4] M. Rubinstein and M. A. Uman, "Methods for calculating the electromagnetic fields from a known source distribution: application to lightning," *IEEE Trans. on Electromagnetic Compatibility*, vol. 31, no. 2, pp. 183-189, May 1989.

[5] C. Yang and Bihua Zhou, "Calculation methods of electromagnetic fields very close to lightning," *IEEE Trans. on Electromagnetic Compatibility*, vol. 46, no. 1, pp. 133-141, Feb. 2004.

[6] T. Takeshima *et al*., "FDTD calculation of lightning-induced voltages on an overhead two-wire distribution line," *2010 Asia-Pacific International Symposium on Electromagnetic Compatibility*, 2010.

[7] A. Shoory, R. Moini, S. H. H. Sadeghi and V. A. Rakov, "Analysis of lightning-radiated electromagnetic fields in the vicinity of lossy ground," *IEEE Trans. on Electromagnetic Compatibility*, vol. 47, no. 1, pp. 131-145, Feb. 2005.

[8] F. H. Heidler and C.Paul, "Field enhancement by lightning strikes to tall tower versus lightning strikes to flat ground," *IEEE Trans. On Electromagnetic Compatibility*, vol. 63, no. 2, 2021.

[9] Y. Baba and V. A. Rakov, "On the interpretation of ground reflections observed in small-scale experiments simulating lightning strikes to towers," *IEEE Trans. on Electromagnetic Compatibility*, vol. 47, no. 3, pp. 533-542, Aug. 2005.

[10] V. A. Rakov and M. A. Uman, "Review and evaluation of lightning return stroke models including some aspects of their application," in *IEEE Trans. on Electromagnetic Compatibility*, vol. 40, no. 4, pp. 403-426, Nov. 1998.

[11] F. H. Heidler, "Review and extension of the TCS model to consider the current reflections at ground and at the upper end of the lightning channel," *IEEE Trans. on Electromagnetic Compatibility*, vol. 61, no. 3, pp. 644-652, June 2019.

[12] Y. Chen and L. Wang, "Research on channel-base current of lightning return Stroke," in *Second International Conference on Mechanic Automation and Control Engineering (MACE),* pp. 7579-7582, 2011.

[13] G. Ramarao and K.Chandrasekaran, "Evaluation of an approximate channel-base-current and its analytical function parameters based on the measured lightning magnetic field," *IEEE Trans. on Electromagnetic Compatibility,* vol. 62, no. 1, Febrauray 2020.

[14] F. Rachidi, V.A Rakov, C.A Nucci, J.L. Bermudez, "Effect of vertically extended strike object on the distribution of current along the lightning channel," *J Geophys Res: Atmosph*. December, 2002.

[15] Y. Baba and V.A Rakov, "On the use of lumped sources in lightning return stroke models," *J Geophys Res: Atmosph*. February, 2005.

[16] V.A. Rakov and M.A Uman, "Review and evaluation of lightning return stroke models including some aspects of their application," *IEEE Trans Electromagn Compat*., pp. 403-426, 1989.

[17] V. Cooray. Lightning protection, IET Pub;2010.

[18] Y. Zhang, J. Gu, E. Zhang, C. Zhao and W. Luo, "Electromagnetic fields of accelerating charges: applications in lightning strikes to a tall tower," *IEEE Transactions on Electromagnetic Compatibility*, vol. 63, no. 6, pp. 1998-2006, Dec. 2021.

[19] M.Izadi, M.Z. Ab Kadir and M.Hajikhani, "An algorithm for evaluation of lightning electromagnetic fields at different distances with respect to lightning channel," Mathematical Problems in Engineering, vol. 2014.

[20] S. Guerrieri, F. Heidler, C. A. Nucci, F. Rachidi and M. Rubinstein, "Extension of two return stroke models to consider the influence of elevated strike objects on the lightning return stroke current and the radiated electromagnetic field: comparison with experimental results," in *Proceedings of International Symposium on Electromagnetic Compatibility*, 1996.